Schedule of the Workshop

Monday, April 24th - morning

Introductory course on Branched Transport

10	*Registration and welcome*	
10.30 - 12.30	Filippo Santambrogio	*Introductory course on Branched Transport.*

Monday, April 24th - afternoon

Branched structures

14 - 15	Michael Goldman	*An exact self-similar solution to a branched transport problem.*
15 - 16	Alessio Brancolini	*Equivalent formulations for the branched transport and urban planning problems and applications.*
	Coffee break	
16.30 - 17.30	Bozhidar Velichkov	*On the honeycomb conjecture for a class of minimal convex partitions.*

Tuesday, April 25th - morning

Optimal Transport

9 - 10	Alessio Figalli	*Partial regularity of optimal transport maps.*
	Coffee break	
10.30 - 11.30	Connor Mooney	*Counterexamples to Sobolev regularity for degenerate Monge-Ampère equations.*
11.30 - 12.30	Simone Di Marino	*Density Functional Theory and Multi-marginal Optimal Transport: results and perspectives.*

Tuesday, April 25th - afternoon

A glimpse on concrete transport problems

14 - 14.30	Mireia Roca-Riu	*A glimpse on concrete transport problems.*

Young researchers' contributions

14.30 - 15	Alessandra Pluda	*Calibrations for the Steiner problem in a covering space setting.*
15 - 15.30	Luca Ferrari	*Phase-field approximation for the Steiner Problem.*
	Coffee break	
15.50 - 16.20	Paul Pegon	*A fractal optimal shape in branched transport.*
16.20 - 16.50	Yash Jhaveri	*Optimal transportation in polygons.*

Wednesday, April 26th - morning

Numerical applications

9 - 10	Giandomenico Orlandi	*On the convex relaxation and variational approximation of functionals defined on 1-dimensional connected sets.*
	Coffee break	
10.30 - 11.30	Édouard Oudet	*Numerical study of 1D optimal structures.*
11.30 - 12.30	Mario Putti	*Biologically inspired formulation of optimal transportation problems.*

Abstracts of the talks

Alessio Brancolini

Equivalent formulations for the branched transport and urban planning problems and applications

The urban planning and the branched transport are two variational problems which describe a way to transport an initial to a final mass with a preference for networks that collect and transport lots of mass together rather than transporting all mass particles independently.

The strength of this preference determines the ramification patterns and the degree of complexity of optimal networks.

Traditionally, the models are formulated in very different ways, via cost functionals of the network in case of urban planning or via cost functionals of irrigation patterns or of mass fluxes in case of branched transport.

We show here that actually both models can be described by all three types of formulations; in particular, the urban planning can be cast into a Eulerian (flux-based) or a Lagrangian (pattern-based) framework. We also show some application of this new theory.

Simone Di Marino

Density Functional Theory and Multimarginal Optimal Transport: results and perspectives

We will review the known results on the multimarginal optimal transport problem coming from Density Functional Theory, in particular focusing on the issue of the existence of an optimal map and its conjectured form in the radial case. Then we will also show a new representation result for the Kantorovich potentials.

Luca Ferrari

Phase-field approximation for the Steiner Problem

In this talk we consider the branched transportation problem in 2D, associated with a cost per unit length of the form 1 + m, where m denotes the amount of transported mass. We will introduce a family of functionals and show that it Gamma-converges to the above mentioned problem. Finally we present some numerical algorithms based on the Gamma-convergence result to approach solutions of the Steiner Minimal Tree problem.

Alessio Figalli

Partial regularity of optimal transport maps

Michael Goldman

An exact self-similar solution to a branched transport problem

In this talk I will show how one can exactly compute the minimizer of a variant of branched transportation arising as a simplified model for pattern formation in Type-I superconductors. The proof relies on the natural scaling properties of the problem. A 3D version of this model has recently been derived from the full Ginzburg-Landau model together with S. Conti, S. Serfaty and F. Otto.

Yash Jhaveri

Optimal transportation in polygons

We present some examples of regular and singular solutions to the optimal transport problem for quadratic cost on polygons. All of the examples can be viewed as a means to understand the stability of optimal transports, in particular, the stability of the identity map as an optimal transport on a domain to itself.

Connor Mooney

Counterexamples to Sobolev regularity for degenerate Monge-Ampère equations

$W^{2,1}$ estimates for the Monge-Ampere equation $\det(D^2u) = f$ in $\mathbb{R}^n$ were obtained by De Philippis and Figalli in the case that f is bounded between positive constants. Motivated by applications to the semigeostrophic equation, we consider the case that f is allowed to be zero on some set. In this case, in dimensions $n \geq 3$ there are simple counterexamples to $W^{2,1}$ regularity that have a Lipschitz singularity. In contrast, if $n = 2$ a classical result of Alexandrov on the propagation of Lipschitz singularities implies that solutions are C^1. We will discuss a counterexample to $W^{2,1}$ regularity in two dimensions whose second derivatives have nonzero Cantor part, and also a related result on the propagation of Lipschitz/log(Lipschitz) singularities that is optimal by example.

Giandomenico Orlandi

On the convex relaxation and variational approximation of functionals defined on 1-dimensional connected sets

We consider the interpretation of certain variational problems involving 1-dimensional connected sets (e.g. the Steiner-Gilbert and the Steiner tree problem) given in recent works by A. Marchese and A. Massaccesi as a Plateau problem (mass minimization) for currents with coefficients in a normed group G, a subgroup of a given Banach space E. Within this framework we discuss a natural convex relaxation procedure and variational approximations via Gamma convergence by means of functionals of phase transition / Ginzburg-Landau type. This is joint work with Mauro Bonafini (Verona) and Edouard Oudet (Grenoble).

Édouard Oudet

Numerical study of 1D optimal structures

We focus our attention on shape optimization problems in which one dimensional connected objects are involved. Very old and classical problems in calculus of variation are of this kind: euclidean Steiner's tree problem, optimal irrigation networks, cracks propagation, etc.

In a first part we quickly recall some previous work in collaboration with F. Santambrogio related to the functional relaxation of the irrigation cost. We establish a Γ-convergence of Modica and Mortola's type and illustrate its efficiency from a numerical point of view by computing optimal networks associated to simple sources/sinks configurations. We also present more evolved situations with non Dirac sinks in which a fractal behavior of the optimal network is expected.

In the second part of the talk we restrict our study to the euclidean Steiner's tree problem. We recall recent numerical approach which have been developed the last five years to approximate optimal trees: partitioning formulation, relaxation with geodesic distance terms and energetic constraints. We describe the first results obtained in collaboration with A. Massaccesi and B. Velichkov to certify the optimality of a given tree. With our

discrete parametrization of generalized calibration, we are able to recover the theoretical optimal matrix fields which certify the optimality of simple trees associated to the vertices of regular polygons.

Finally, we focus on the delicate problem of the identification of the optimal structure. Based on a recent approach obtained in collaboration with G. Orlandi and M. Bonafini, we describe the first convexification framework associated to the Euclidean Steiner tree problem which provide relevant tools from a numerical point of view.

Alessandra Pluda

Calibrations for the Steiner problem in a covering space setting

In this talk I will present an improvement of the approach to the Steiner problem proposed by Amato, Bellettini and Paolini, which is based on the minimization of the total variation of constrained BV functions on suitably chosen covering spaces. Then, I will define a proper notion of calibration in this setting and I will conclude giving some examples. This is an ongoing research work with Marcello Carioni.

Paul Pegon

A fractal optimal shape in branched transport

Optimal structures in branched transportation seem to exhibit some sort of fractal behavior. In this talk, I will present a shape optimization problem related to branched transport: find a set with unit volume which is closest to a point in the sense of branched transport. One may show existence of an optimal set and relate it to a so-called landscape function, a central tool in branched transport which is essential for the study of this problem. I will present some results suggesting the fractality of its boundary, though the question remains open.

Mario Putti

Biologically inspired formulation of optimal transportation problems

In a joint work with Franco Cardin, Enrico Facca, and Federico Piazzon (University of Padova), we have recently developed an approach, based on an extension of a model proposed by Tero et al (2007), for the simulation of the dynamics of a slime mold (Physarum Polycephalum). We conjecture that the long-time solution of the proposed model approaches the solution of the PDE base Monge-Kantorovich Optimal Transport equations. This new OT formulation assumes that the transport density and potential are time dependent and satisfy the elliptic PDE. The classical constraint on the norm of the gradient is then replaced by an ODE describing transient dynamics of the diffusion coefficient. Analytical results in the case of the Monge-Kantorovich problem, although yet largely incomplete, suggest that indeed the conjecture is true. This is supported by several numerical experiments performed on a number of published test cases. One of the most important advantages of the proposed formulation is that its numerical solution is very efficient and well-defined using simple discretization schemes. Moreover, this dynamical extension allows the reconstruction of the time-history of the process, thus enlarging the range of application of the proposed OT model to a large set of physical and biological processes. Simple modifications of the proposed model yield dynamic versions of the branched and congested transport problems.

We shall discuss our numerical approaches, which are based on either a standard linear (P1) Galerkin method, or a pseudspectral scheme combined with a P0 approximation of the diffusion coefficient, for the discretization of the elliptic PDE and Euler time stepping and Picard iteration to solve the resulting nonlinear differential-algebraic equation. Preliminary

numerical simulations are used to show that the proposed formulation is efficient in finding solutions also of congested and branched transport problems. Although limitations arise when trying to solve highly discontinuous problems, we present experimental convergence results showing robustness of the scheme for a wide range of sample problems. Finally, we will examine models and related numerical results of diverse applications ranging from slime-mold dynamics to geomorphological problems, and discuss current and future progress.

Mireia Roca-Riu

A glimpse on concrete transport problems

The Traffic Engineering group from the Institute for Transport Planning and Systems researches different problems related to traffic management, traffic control and intra-modal interactions in the city. In this talk, an overview of the problems and their applications studied in the group will be presented. A particular focus will be given to two particular topics: the parking slot assignment problem and the use of fusion methods for the estimation of the Macro Fundamental Diagram of traffic flow.

Filippo Santambrogio

Introductory course on Branched Transport

This short course will be a survey on the theory of branched transportation. In particular I will deal with the following topics

1- different models for branched transport: Xia's Eulerian model, Maddalena-Morel-Solimini's Lagrangian model, other formulations with curves of measures (path functionals, Benamou-Brenier...)
2- the distance d_α induced by branched transport (finiteness, topology, and comparison with the Wasserstein distances)
3- the landscape function and its role in branched transport (geophysical meaning and mathematical properties)
4- phase-field approximation of the Eulerian model and Gamma-convergence results.

Bozhidar Velichkov

On the honeycomb conjecture for a class of minimal convex partitions

In this joint work with Dorin Bucur, Ilaria Fragalà, Gianmaria Verzini, we prove that the planar hexagonal honeycomb is asymptotically optimal for a large class of optimal partition problems, in which the cells are assumed to be convex, and the criterion is to minimize either the sum or the maximum among the energies of the cells, the cost being a shape functional $\Omega \mapsto F(\Omega)$ which satisfies a few assumptions. They are:

- monotonicity under inclusions;
- homogeneity under dilations;
- a Faber-Krahn inequality for convex hexagons;
- a convexity-type inequality for the map which associates with every $n \in \mathbb{N}$ the minimizers of F among convex n-gons with given area.

In particular, we obtain the honeycomb conjecture for the Cheeger constant for convex partitions. The method is very general and also applies to the logarithmic capacity (still assuming the cells to be convex) and to the first eigenvalue of the Dirichlet Laplacian, under some appropriate assumptions on the behaviour of λ_1 on the convex pentagons, hexagons, and heptagons with prescribed area.