

Review 1

1. Find the limit or show that it does not exist:

(a) $\lim_{(x,y) \rightarrow (1,0)} \frac{e^{xy} - y}{x}$.

(b) $\lim_{(x,y) \rightarrow (1,1)} \frac{\sin(x-1)}{(x-1)y}$.

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x+y}{\sqrt{x^2+y^2}}$.

(d) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{\sqrt{x^2+y^2}}$.

2. Find f_x and f_y :

(a) $f(x, y) = e^{\sin x} \cos(xy)$.

(b) $f(x, y) = \frac{\ln(x+y)}{xy}$.

(c) $f(x, y) = \frac{y \tan x}{x+y}$.

3. Let $z = e^{x+y^2} - \sin\left(\frac{x}{y}\right)$, $x = \ln t$, $y = t^2$. Compute $\frac{dz}{dt}$.

4. Find the directional derivative of $f(x, y) = 2x2^y$ at $(1, 1)$ in direction $\bar{i} + 2\bar{j}$.

5. Find the direction in which $f(x, y) = x^2 + y^2$ increases most rapidly at $(2, 3)$.

6. Find the normal vector to the graph $\sin x + e^{xy} = 1$ at $(0, 0)$.

7. Find the equation of the tangent plane:

(a) $x^2 + y^2 + z^2 = 1$, $(1, 1, 1)$.

(b) $z = \cos(xy)$, $(0, 0, 1)$.

8. Approximate $\sqrt{(5.1)^2 - (3.9)^2}$.

9. Let $f(x, y) = 3x^2 + 2xy + y^2 - x - y$. Find all critical points and determine which of the critical points is local minimum, local maximum, saddle point.

10. Find the maximum and minimum values of $f(x, y) = e^{xy}$ in the square $[-1, 1] \times [-1, 1]$.

11. Find the volume of the parallelepiped of maximal volume inscribed in the sphere $x^2 + y^2 + z^2 = 1$.

12. Find the point on the sphere $(x - 3)^2 + y^2 + (z - 4)^2 = 1$ which is closest to the origin.

13. We have just enough material to construct a closed cylindrical tank of surface area $20 ft^2$. Find the dimensions of the tank of maximal volume. (Hint: the surface area of a cylinder of radius r and height h is $2\pi r^2 + 2\pi r h$, and the volume is $\pi r^2 h$.)