

Review 1

1. Find limits:

- (a) $\lim_{n \rightarrow \infty} \frac{n^3 - 2n + 1}{2n^3 + n}$.
 (b) $\lim_{n \rightarrow \infty} \frac{n + \sin n}{e^n}$.
 (c) $\lim_{n \rightarrow \infty} (1 + n)^{1/n}$.
 (d) $\lim_{n \rightarrow \infty} (\sqrt{n^2 + 2n} - \sqrt{n^2 + n})$.
 (e) $\lim_{n \rightarrow \infty} \frac{\tan^{-1} n}{n}$.

2. Determine if the series converges:

- (a) $\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 2n - 1}$.
 (b) $\sum_{n=1}^{\infty} \frac{n}{2^n}$.
 (c) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.
 (d) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$.
 (e) $\sum_{n=1}^{\infty} \frac{1}{2^n + n}$.

3. Test if the series converges absolutely or conditionally:

- (a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \sqrt{\ln n}}$.
 (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$, and find its sum.

4. Find interval of convergence of the power series:

- (a) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$.
 (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n 2^n} x^n$.

5. Find Taylor series for:

- (a) e^{-x^2} .
 (b) $\ln \frac{1}{1-x}$.
 (c) $\sin^2 x$ (Hint: $(\sin^2 x)' = \sin 2x$).

6. Compute with error 0.01:

- (a) $\sin 1$.
 (b) $\sqrt{26}$.

7. Using the first 2 terms of the Taylor series for $\cos(x^2)$, compute $\int_0^1 \cos(x^2) dx$.

8. Using Taylor series for $\cos x$, find $\lim_{x \rightarrow 0} \frac{\cos x - 1 + x^2/2}{x^4}$.