Review Problems

1. Let the position of a particle at the time t is $x(t) = t^3 - 3t + 1$. Find the total distance traveled from $t = 0$ to $t = 2$.

2. Find local maximums, local minimums, global maximum, global minimum, and infection points:

   (a) $f(x) = x^3 - 3x^2 + 2x + 5$, $0 \leq x \leq 3$.

   (b) $f(x) = \frac{x}{x^2+1}$.

   (c) $f(x) = xe^{-\pi x^2}$.

   (d) $f(x) = e^{-x} \sin x$, $x \geq 0$.

3. The equation $x^4 + y + xy^4 = 1$ defines y implicitly as a function of x. Use the tangent line approximation at the point $(-1, 1)$ to estimate the value of $y(-0.9)$.

4. A 15 feet ladder rests vertically against the side of a barn. A pig that has been hitched to the ladder starts to pull the base of the ladder away from the wall. When the base of the ladder 3 ft. away from the wall, the base moves with a rate of 0.6 ft./sec. Find the rate of change of the height of the top of the ladder at this moment.

5. Find the equation of the tangent line to the curve $x + \tan(xy) = 2$ at the point $(1, \frac{\pi}{4})$.

6. A little boy buys a spherical balloon of total volume 1 $ft.^3$ He starts blowing to fill the balloon at a rate of 0.1 $ft.^3$/min. How fast is the radius of the balloon increasing when he has the balloon halfway blown up?

7. Find all points on the curve $y = 3x^2$ where the tangent line passes through the point $(2, 9)$.

8. At 08 : 00, an aircraft carrier is 400 miles directly south of Honolulu and is sailing south at 15 miles/hour. A submarine is 300 miles east of Honolulu and is sailing west at 20 miles/hour. At what rate is the distance between the ships changing?

9. A ball is dropped from a height of 49 meters above the ground. The height of the ball at time t is $h(t) = 49 - 4.9t^2$ meters. A light which is also 49 meters above the ground is 10 meters to the left of the ball's original position. As the ball drops, the shadow of the ball caused by the light moves across the ground. How fast is the shadow moving one second after the ball is dropped?

10. Without using calculator, find approximately:

    (a) $\sin \frac{401\pi}{100}$.

    (b) $65^{1/3}$.