Frontiers in Analysis and Probability
5th Strasbourg / Zurich - Meeting
Université de Strasbourg and University of Zurich

Abstracts

Place: Université de Strasbourg,
Institut de Recherche Mathématique Avancée (IRMA), Conference Room

March 23 - 24, 2017

The Time-Dependent Hartree-Fock-Bogoliubov Equations for Bosons

by Sébastien Breteaux (BCAM Bilbao)

Abstract: It was first predicted in 1925 by Einstein (generalizing a previous work of Bose) that, at very low temperatures, identical Bosons could occupy the same state. This large assembly of Bosons would then form a quantum state of the matter which could be observed at the macroscopic scale. The first experimental realization of a gas condensate was then done in 1995 and awarded by the 2001 Nobel price, and this motivated numerous works on Bose-Einstein condensation.

In particular, we are interested in the dynamics of such a condensate. To describe the dynamics of such a condensate, the first approximation is the time dependent Gross-Pitaevskii equation, or, in an other scaling, the Hartree equation. To precise this description, we derive the time-dependent Hartree-Fock-Bogoliubov equations describing the dynamics of quantum fluctuations around a Bose-Einstein condensate via quasifree reduction. We prove global well posedness for the HFB equations for sufficiently regular interaction potentials (including Coulomb). We show that the HFB equations have a symplectic structure and a structure similar to an Hamiltonian structure, which is sufficient to prove the conservation of the energy.
Joint work with V. Bach, T. Chen, J. Fröhlich, and I. M. Sigal.

Discrete spectrum in bands and zeta function of contact Anosov flows

by Frédéric Faure (Grenoble)

Abstract: The geodesic flow on a negatively curved manifold (non necessary constant) is a model of chaotic dynamics. Using semiclassical analysis, we show that the generating vector field has an intrinsic discrete spectrum in specific Sobolev spaces. This spectrum, called Ruelle resonances, governs asymptotic expansion of dynamical correlation functions. It is structured in vertical bands separated by gaps. A semiclassical zeta function (or Gutzwiller Voros zeta function) expressed this spectrum in terms of closed periodic orbits and generalizes the Selberg zeta function to the case of non constant curvature. We can interpret theses results as an emergence of quantum dynamics in classical correlation functions.
Collaboration with Masato Tsujii.

Effective Dynamics in Quantum Theory: Systems under repeated measurement

by Jürg Fröhlich (ETH Zürich)

Abstract: We start by presenting a short summary of examples of effective dynamics in quantum theory. We then study more closely the effective quantum dynamics of systems interacting with a long chain of independent probes, one after another, which, afterwards, are subject to a projective measurement and are then lost. This leads us to develop a theory of indirect measurements of time-independent quantities (non-demolition measurements). Next, the theory of indirect measurements of time-dependent quantities is outlined, and a new family of diffusion processes - "quantum jump processes" - is described. Some open problems are proposed.

Mean field evolution of fermionic systems

by Marcello Porta (University of Zurich)

Abstract: In this talk I will consider the dynamics of interacting fermionic systems in the mean field regime. As the number of particles goes to infinity, the dynamics is expected to be well approximated by the time-dependent Hartree-Fock equation. In this talk, I will present results on the rigorous derivation of this effective evolution equation, at zero temperature (pure states) and at positive temperature (mixed states). Under the assumption that a suitable semiclassical structure of the initial datum is propagated along the Hartree-Fock flow, I will discuss the extension of these results to the case of particles interacting via a Coulomb potential.

Spectral cluster bounds for orthonormal functions

by Julien Sabin (Orsay)

Abstract: Sogge's L^p bounds are a way to measure the concentration of eigenfunctions of the Laplace-Beltrami operator on compact Riemannian manifolds associated to large eigenvalues. We generalize these bounds to systems of orthonormal functions, building a bridge between Sogge's result about concentration and the Weyl law, which in some sense is a manifestation of non-concentration. The optimality of these new bounds is also discussed. These spectral cluster bounds follow from Schatten-type bounds on oscillatory integral operators.
Joint work with Rupert Frank (Caltech).

Gibbs measures of nonlinear Schrödinger equations as limits of quantum many-body states in dimension $d \leq 3$

by Vedran Sohinger (University of Zurich)

Abstract: Gibbs measures of nonlinear Schrödinger equations are a fundamental object used to study low-regularity solutions with random initial data. In the dispersive PDE community, this point of view was pioneered by Bourgain in the 1990s. We prove that Gibbs measures of nonlinear Schrödinger equations arise as high-temperature limits of appropriately modified thermal states in many-body quantum mechanics. We consider bounded defocusing interaction potentials in dimensions $d = 1, 2, 3$ and we work either on the d-dimensional torus or on $\mathbb{R}^d$ with a confining potential. The analogous problem for $d = 1$ and in higher dimensions with smooth non translation-invariant interactions was recently studied by Lewin, Nam, and Rougerie by means of entropy methods. In our work, we apply a perturbative expansion of the interaction, motivated by ideas from field theory. The terms of the expansion are analyzed using a diagrammatic representation and their sum is controlled using Borel resummation techniques. When $d = 2, 3$, we apply a Wick ordering renormalization procedure. The latter allows us to deal with the rapid growth of the number of particles. Moreover, in the one-dimensional setting our methods allow us to obtain a microscopic derivation of time-dependent correlation functions for the cubic nonlinear Schrödinger equation.
This is a joint work with Jürg Fröhlich, Antti Knowles, and Benjamin Schlein.

Spectral asymptotics along interfaces between allowed and forbidden regions

by Steve Zelditch (Northwestern University)

Abstract: Interfaces arise in spectral asymptotics when the asymptotics abruptly change on a boundary between two regions. For the Schrodinger equation, the boundary is the caustic separating the allowed and forbidden region. In the Bargmann-Fock (Kaehler) setting,

i.e. in phase space, it is an energy level. In the real domain, there is a thin transition region around the caustic where Airy functions dominate. In the complex domain there is a Gaussian transition. My talk will go over interface asymptotics in one or both setting(s).