Frontiers in Analysis and Probability
4th Strasbourg / Zurich - Meeting
Université de Strasbourg and University of Zurich
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Abstracts

Place: University of Zurich, Aula RAA G-01

Hitting times and zeta function

by Philippe Biane (Université Paris-Est Marne-la-Vallée)

Abstract: The Riemann zeta function can be expressed using Mellin tranforms of hitting times of Bessel processes. A related function, considered long ago by Polya and which satisfies a Riemann hypothesis can also be expressed by hitting times of Brownian motion. We show how these functions are related to some basic spectral geometry on spaces of constant curvature.

Normal numbers: recent results and open problems

by Yann Bugeaud (Université de Strasbourg)

Abstract: Let b be an integer greater than or equal to 2. A real number is called simply normal to base b if each digit $0, \ldots, b-1$ occurs in its b-ary expansion with the same frequency $1/b$. It is called normal to base b if it is simply normal to every base b^k, where k is a positive integer (or, equivalently, if, for every positive integer k, each block of k digits from $0, \ldots, b-1$ occurs in its b-ary expansion with the same frequency $1/b^k$). This notion was introduced in by 1909 Émile Borel, who established that almost every real number (in the sense of the Lebesgue measure) is normal to every integer base. We present classical and more recent results on normal numbers and highlight several open problems. Topic discussed include explicit construction of normal numbers, the existence of uncountably many numbers normal to base 2 but not simply normal to base 3 (a result proved independently by Cassels and Schmidt more than fifty years ago), and links between digital properties and Diophantine properties of a real number.

A stochastic process from quadratic Weyl sums

by Francesco Cellarosi (Queen's University)

Abstract: In this talk we will explain the construction of a stochastic process of number-theoretical origins. We will consider suitably rescaled quadratic Weyl sums as a deterministic walk (with a random seed) in the complex plane, and discuss their convergence to a limit process. The existence of the limit (weak invariance principle) uses equidistribution results in homogeneous dynamics, as well as a new dynamical smoothing technique. We will see how some of the properties of the limit process are the same as those of the Brownian motion (e.g. scaling, time inversion, law of large numbers, stationarity, rotational invariance, Hölder continuity of sample paths), while other properties are dramatically different (e.g. tails, dependent increments, modulus of continuity). This is joint work with Jens Marklof (University of Bristol).

Arithmetic statistics, function fields, and matrix integrals

by Jon Keating (University of Bristol)

Abstract: I shall review some recent results concerning the statistical properties of certain arithmetic functions, their function-field analogues, and connections with matrix integrals.

Functional limit theorems in number theory

by Emmanuel Kowalski (ETH Zürich)

Abstract: The unpredictable distribution properties of arithmetic objects leads to many probabilistic statements of convergence in law to classical probability distributions. In a number of cases, the convergence in law applies to functions of arithmetic interests, and holds in suitable function spaces. We will survey some results of this type, with a focus on the shape of paths of exponential sums and on the value distribution of families of L-functions.

Asymptotic Orthogonality of Powers, the MOMO property and small-interval estimations for the Möbius function

by Thierry de la Rue (Laboratoire de Mathématiques Raphaël Salem, CNRS, Université de Rouen Normandie)

Abstract: Talk based on joint work with El Houcein el Abdalaoui (LMRS, Rouen) and Mariusz Lemanczyk (Nicolaus Copernicus University, Torun).

The famous conjecture stated by Peter Sarnak in 2010 states that the behaviour of the Möbius function is so chaotic that this function has no correlation with any sequence produced by a topological dynamicalsystem of zero entropy.

From the point of view of ergodic theory, this conjecture led to interesting properties of a probability-preserving dynamical system, ensuring the validity of Sarnak's conjecture for any uniquely ergodic model of this system. We introduced in particular the AOP property (Asymptotic Orthogonality of Powers), and proved that it is satisfied by quasi-discrete-spectrum systems.

Then we observed that such a property implies in fact a stronger result: the MOMO property (Multiplicative Orthogonality on Moving Orbits). As a consequence, we derived the following small-interval estimation for the Möbius function: for each non constant polynomial $P \in R[x]$ with irrational leading coefficient, we have

$$\frac{1}{M} \sum_{M \leq m < 2M} \frac{1}{H} \left| \sum_{m \leq n < m+H} e^{2\pi i P(n)} \mu(n) \right| \longrightarrow 0$$

as $M \to \infty$, $H \to \infty$, $H/M \to 0$.

Arithmetical processes

by Gérald Tenenbaum (Institut Élie Cartan Nancy)

Abstract: The talk will be devoted to survey the basic models of probabilistic number, recall the fundamental historical results, and to describe classical and less classical convergence results for distribution functions and processes of arithmetic nature.