# Frontiers in Analysis and Probability
# 2nd Strasbourg / Zurich - Meeting
# Université de Strasbourg and University of Zurich
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# Abstracts

Place: Institute of Mathematics, University of Zurich

### Beta ensembles at high temperature

by Laure Dumaz

Abstract: In this talk, I will introduce random operators describing the continuum limit of beta ensembles (by Ramirez, Rider, Valko and Virag). I will show how to derive the behavior of the particules at high temperature thanks to this approach.

This is a joint work with Romain Allez.

### Exact small time equivalent for the density of the circular Langevin diffusion

by Jacques Franchi

Abstract: A small time equivalent of the density is obtained for the circular analogue of the Langevin diffusion, which is strictly hypoelliptic (and non-Gaussian), hence of a different nature as the known sub-Riemannian case. The singular case, analogous to the cut-locus of the sub-Riemannian case, is handled too, though more difficultly.

### Examples in relation with a metric Ricci flow

by Nicolas Juillet

Abstract: In a recent paper, Gigli and Mantegazza introduced a procedure that given a metric space $(X, d)$ with a reference measure $L$ and a positive real number $\tau > 0$ produces a "convoluted" metric space $(X, d^\tau)$. The procedure is based on the heat flow associated to $(X, d, L)$ using, equivalently, the theory of Dirichlet forms or optimal transportation.

Losely speaking, $d^\tau$ is a distance between the probability measures $p_x^\tau$ and $p_y^\tau$ obtained as heat kernels at time $\tau$ starting from $x$ and $y$ respectively. Assuming that $(X, d, L)$ is a Riemannian manifold, Gigli and Mantegazza proved that for small numbers $\tau$, the evolution is, in some weak sense, tangent to the Ricci flow.

We investigate the behaviour of this procedure in the case of three nonRiemannian metric spaces. These are three simple modifications of the Euclidean plane that represent typical classes of metric spaces.

This is a joint work with Matthias Erbar.

## Asymptotic growth rates of Riemannian manifolds

by François Ledrappier

Abstract: We present and discuss the properties of different geometric growth rates associated with the universal cover of a compact manifold. We discuss general inequalities, the equality cases and regularity of these numbers when the metric varies. There are many open questions.

## Strong supermartingales and limits of non-negative martingales

by Walter Schachermayer

Abstract: Given a sequence $(M^n)_{n=1}^\infty$ of non-negative martingales starting at $M_0^n = 1$ we find a sequence of convex combinations $(\widetilde{M}^n)_{n=1}^\infty$ and a limiting process $X$ such that $(\widetilde{M}^n_\tau)_{n=1}^\infty$ converges in probability to $X_\tau$, for all finite stopping times $\tau$. The limiting process $X$ then is an optional strong supermartingale. A counter-example reveals that the convergence in probability cannot be replaced by almost sure convergence in this statement. We also give similar convergence results for sequences of optional strong supermartingales $(X^n)_{n=1}^\infty$, their left limits $(X^n_-)_{n=1}^\infty$ and their stochastic integrals $(\int \varphi dX^n)_{n=1}^\infty$ and explain the relation to the notion of the Fatou limit.

## A family of stochastic processes from random matrix theory

by Sasha Sodin

Abstract: We shall discuss some stochastic processes which appear in the scaling limit of random matrices depending on a parameter. The Airy line ensemble from non-equilibrium particle systems appears as a special case. On the other hand, other cases do not seem to be determinantal, and are defined indirectly via a topological expansion.

## Stationary random metrics on self-similar length spaces

by Michele Triestino

Abstract: The Sierpinski gasket is a fractal with the property that it is glued out of three Sierpinski gaskets in a trinagular shape. Inspired by the problem of defining in a rigorous mathematical sense the random metric on a surface given by the exponential of the Gaussian Free Field, we consider a simpler model: given a non-atomic probability measure $m$ over $(0, \infty)$, for any parameter $\lambda$ we define a "renormalization" operator $\Phi_\lambda$ over the space $P$ of probability measures on the space of metrics over the Sierpinski gasket. Given a probability measure $\mu$ in $P$, we choose three independent realizations of metrics of law $\mu$ on three gaskets $(T_1, D_1)$, $(T_2, D_2)$, $(T_3, D_3)$, that we glue together to form a new Sierpinski gasket with a new random metric defined by selecting the shortest path and with all the distances multiplied by the random factor $\lambda\xi$, where the variable $\xi$ has law $m$, and is independent of the metrics $D_1$, $D_2$ and $D_3$.

For the Sierpinski gasket and other self-similar spaces, our main result is that for any measure $m$ there exists a parameter $\lambda_{cr}$ such that the operator $\Phi_{\lambda_{cr}}$ has a fixed point: there a exists a random metric which is invariant (or stationary) under renormalization. In the case of not too simple hierarchical graphs we can also prove that any two invariant random metrics are proportional and actually the half-line where they belong to is a global attractor for the renormalization dynamics.

Moreover, studying the underlying branching process we are able to describe the geometric properties of the invariant random metrics. The main tool is a very intuitive cut-off method that could be used for studying renormalization problems of different nature.

This is a joint work with M. Khristoforov and V. Kleptsyn.

## Topologies of nodal sets of random band limited functions

by Igor Wigman

Abstract: It is shown that the topologies and nestings of the zero and nodal sets of random (Gaussian) b and limited functions have universal laws of distribution. Qualitative features of the supports of these distributions are determined. In particular the results apply to random monochromatic waves and to random real algebraic hyper-surfaces in projective space.

This is a joint work with Peter Sarnak.

## Isotropic Markov processes on ultrametric spaces

by Wolfgang Woess

Abstract: In an ultra-metric space, the usual triangle inequality is replaced by the ultra-metric inequality d(x,y) = max d(x,z),d(z,y). We consider proper ultra-metric spaces (where closed balls are compact-open). Typical examples are:

- direct limits of finite groups,
- the field of p-adic numbers,
- the Cantor set,
- the set of all rooted (finite or infinite) trees with degrees bounded by some finite D,
- the boundary of a locally finite, infinite tree.

We construct a very natural class of isotropic Markov semigroups on ultra-metric spaces. The transparence of the approach leads to a good understanding and very detailed results on the semigroup and the associated Markov process. This leads to a variety of new results and to an improved and simplified approach to various previous constructions of processes on ultra-metric spaces. Among other, the results concern recurrence/transience, transition kernel estimates, a full description of the Markov generator and its spectrum, and the Liouville property for harmonic functions. In the talk, particular emphasis will be given to the duality between random walks on trees and isotropic processes on their boundaries.

Collaboration with Alexander Bendikov, Alexander Grigor'yan, Christophe Pittet.