

Tree-Based Construction of LDPC Codes

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Abstract— We present a construction of LDPC codes that have minimum pseudocodeword weight equal to the minimum distance, and perform well with iterative decoding. The construction involves enumerating a d -regular tree for a fixed number of layers and employing a connection algorithm based on mutually orthogonal Latin squares to close the tree. Methods are presented for degrees $d = p^s$ and $d = p^s + 1$, for p a prime, – one of which includes the well-known finite-geometry-based LDPC codes.

I. INTRODUCTION

Low Density Parity Check (LDPC) codes are widely acknowledged to be good codes due to their near Shannon-limit performance when decoded iteratively. However, many structure-based constructions of LDPC codes fail to achieve this level of performance, and are often outperformed by random constructions. (Exceptions include the finite-geometry-based LDPC codes (FG-LDPC) of [1], which were later generalized in [2].) Moreover, there are discrepancies between iterative and maximum likelihood (ML) decoding performance of short to moderate blocklength LDPC codes. This behavior has recently been attributed to the presence of so-called *pseudocodewords* of the LDPC constraint graphs, which are valid solutions of the iterative decoder which may or may not be optimal [3]. Analogous to the role of minimum Hamming distance, d_{\min} , in ML-decoding, the minimal pseudocodeword weight, w_{\min} , has been shown to be a leading predictor of performance in iterative decoding. Furthermore, the error floor performance of iterative decoding is dominated by minimal weight pseudocodewords. Although there exist pseudocodewords with weight larger than d_{\min} that have adverse effects on decoding, pseudocodewords with weight $w_{\min} < d_{\min}$ are especially problematic [4].

The Type I-A construction and certain cases of the Type II construction presented in this paper are designed so that the resulting codes have minimal pseudocodeword weight equal to the minimum distance of the code, and consequently, these problematic low-weight pseudocodewords are avoided. The resulting codes have minimum distance which meets the lower tree bound originally presented in [5], and since w_{\min} shares the same lower bound [4], [6], and is upper bounded by d_{\min} , the proposed constructions have $w_{\min} = d_{\min}$. It is worth noting that this property is also a characteristic of some of the FG-LDPC codes [2], and indeed, the projective-geometry-based codes of [1] arise as special cases of our Type II

construction. Furthermore, the Type I-B construction presented herein is a modification of the Type I-A construction, and it yields a family of codes with a wide range of rates and blocklengths that are comparable to those obtained from finite geometries.

We now present the tree bound on w_{\min} derived in [6].

Theorem 1.1: Let G be a bipartite LDPC constraint graph with smallest left (variable node) degree d and girth g . Then the minimal pseudocodeword weight w_{\min} (for the AWGN/BSC channels) is lower bounded by

$$w_{\min} \geq \begin{cases} 1 + d + d(d-1) + d(d-1)^2 + \dots + d(d-1)^{\frac{g-6}{4}}, & \frac{g}{2} \text{ odd} \\ 1 + d + d(d-1) + \dots + d(d-1)^{\frac{g-8}{4}} + (d-1)^{\frac{g-4}{4}}, & \frac{g}{2} \text{ even} \end{cases}$$

This bound is also the tree bound on the minimum distance established by Tanner in [5]. And since the set of pseudocodewords includes all codewords, we have $w_{\min} \leq d_{\min}$. In the following sections we present two construction techniques of LDPC codes wherein for certain cases, $w_{\min} = d_{\min}$.

II. PRELIMINARIES

The connection algorithms for the tree constructions Type I-B and Type II are based on mutually orthogonal Latin squares. A well-known construction of a family of mutually orthogonal Latin squares of order p^s , a power of a prime, may be found in [7]. Let $M^{(1)}, M^{(2)}, \dots, M^{(p^s-1)}$ denote $p^s - 1$ mutually orthogonal Latin squares (MOLS) of order p^s . Let the rows (and columns) of each square be indexed by the integers $0, 1, 2, \dots, p^s - 1$. Without loss of generality, assume that the first column of each of the Latin squares is $[0, 1, 2, \dots, p^s - 1]^T$. In addition, define a new square of size $p^s \times p^s$, denoted $M^{(0)}$, where each column of $M^{(0)}$ is $[0, 1, 2, \dots, p^s - 1]^T$.

III. TREE-BASED CONSTRUCTION: TYPE I

In the Type I construction, first a d -regular tree of alternating variable and constraint node layers is enumerated from a root variable node (layer L_0) for ℓ layers. If ℓ is odd (respectively, even), the final layer $L_{\ell-1}$ is composed of variable nodes (respectively, constraint nodes). Call this tree T . The tree T is then reflected across an imaginary horizontal axis to yield another tree, T' , and the variable and constraint nodes are reversed. That is, if layer L_i in T is composed of variable nodes, then the reflection of L_i , call it L'_i , is composed of constraint nodes in the reflected tree, T' . The union of these two trees, along with edges connecting the nodes in layers $L_{\ell-1}$ and $L'_{\ell-1}$ according to a connection algorithm that is

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described next, comprise the graph representing a Type I LDPC code. We now present two connection schemes that can be used in this Type I model, and discuss the resulting codes.

A. Type I-A

For $d = 3$, the Type I-A construction yields a d -regular LDPC constraint graph having $1 + d + d(d - 1) + \dots + d(d - 1)^{\frac{g-4}{2}}$ variable and constraint nodes, and girth g . The tree T has $\frac{g}{2}$ layers. To connect the nodes in $L_{\frac{g}{2}-1}$ to $L'_{\frac{g}{2}-1}$, first label the variable (resp., constraint) nodes in $L_{\frac{g}{2}-1}$ (resp., $L'_{\frac{g}{2}-1}$) when $\frac{g}{2}$ is odd, as $v_0, v_1, \dots, v_{2^{\frac{g}{2}-2}-1}$, $v_{2^{\frac{g}{2}-2}}, \dots, v_{2 \cdot 2^{\frac{g}{2}-2}-1}, v_{2 \cdot 2^{\frac{g}{2}-2}}, \dots, v_{3 \cdot 2^{\frac{g}{2}-2}-1}$ (resp., $c_0, c_1, \dots, c_{2^{\frac{g}{2}-2}-1}$). The nodes $v_0, v_1, \dots, v_{2^{\frac{g}{2}-2}-1}$ form the 0th class, the nodes $v_{2^{\frac{g}{2}-2}}, \dots, v_{2 \cdot 2^{\frac{g}{2}-2}-1}$ form the 1st class, and the nodes $v_{2 \cdot 2^{\frac{g}{2}-2}}, \dots, v_{3 \cdot 2^{\frac{g}{2}-2}-1}$ form the 2nd class; classify the constraint nodes in a similar manner. In addition, define three permutations $\pi(\cdot), \tau(\cdot), \tau'(\cdot)$ of the set $\{0, 1, \dots, 2^{\frac{g}{2}-2} - 1\}$ as follows. The nodes in $L_{\frac{g}{2}-1}$ and $L'_{\frac{g}{2}-1}$ are connected as follows:

- 1) For $i = 0, 1$, and $j = 0, 1, \dots, 2^{\frac{g}{2}-2} - 1$, the variable node $v_{j+i \cdot 2^{\frac{g}{2}-2}}$ is connected to nodes $c_{\pi(j)+i \cdot 2^{\frac{g}{2}-2}}$ and $c_{\tau(j)+(i+1) \cdot 2^{\frac{g}{2}-2}}$.
- 2) For $i = 2$, and $j = 0, 1, \dots, 2^{\frac{g}{2}-2} - 1$, the variable node $v_{j+i \cdot 2^{\frac{g}{2}-2}}$ is connected to nodes $c_{\pi(j)+2 \cdot 2^{\frac{g}{2}-2}}$ and $c_{\tau'(j)}$.

The permutations for the cases $g = 6, 8, 10, 12$ are given below. The above construction can be extended for higher g in a natural way and we are working on an explicit closed form expression for the permutations π, τ, τ' for higher g .

$$g = 6, \pi = \tau = \tau' = (0)(1), \text{ the identity permutation.}$$

$$g = 8, \pi = (0)(2)(1, 3), \tau = (0)(2)(1, 3), \tau' = (0, 2)(1)(3).$$

$$g = 10, \pi = (0)(2)(4)(6)(1, 5)(3, 7), \tau = (0)(2)(4)(6)(1, 7)(3, 5),$$

$$\tau' = (0, 4)(2, 6)(1, 3)(5, 7).$$

$$g = 12, \pi = (0)(4)(8)(12)(2, 6)(10, 14)(1, 9)(3, 15)(5, 13)(7, 11),$$

$$\tau = (0)(4, 12)(8)(2, 6, 10, 14)(1, 15, 13, 11)(3, 9, 7, 5),$$

$$\tau' = (0, 8)(4, 12)(2, 14)(6, 10)(1, 3, 5, 7)(9, 11, 13, 15).$$

When $\frac{g}{2}$ is odd, the minimum distance of the resulting code meets the tree bound, and hence, $d_{\min} = w_{\min}$. When $\frac{g}{2}$ is even, d_{\min} is strictly larger than the tree bound; we believe however, that w_{\min} is equal to d_{\min} in this case as well. Figure 1 illustrates the general construction procedure and Figure 2 shows a girth 10 Type I-A LDPC constraint graph.

B. Type I-B

For $d = p^s, p$ a prime, the Type I-B construction yields a d -regular LDPC constraint graph having $1 + d + d(d - 1)$ variable and constraint nodes, and girth 6. The tree T has 3 layers L_0, L_1 , and L_2 . L_2 (resp., L'_2) is composed of p^s sets $\{S_i\}_{i=0}^{p^s-1}$ of $p^s - 1$ variable (resp., constraint) nodes in each set; the set S_i corresponds to the children of branch i of the root node. Let S_i (resp., S'_i) contain the variable (resp., constraint) nodes $v_{i,1}, v_{i,2}, \dots, v_{i,p^s-1}$

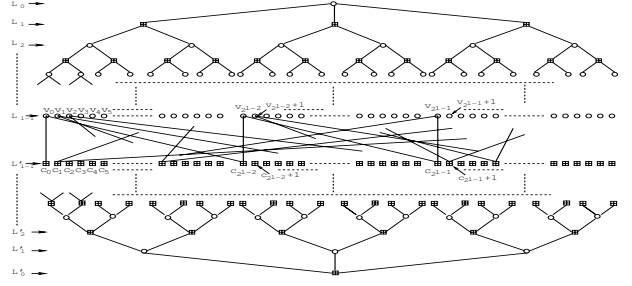


Fig. 1. Tree construction of Type I-A LDPC code.

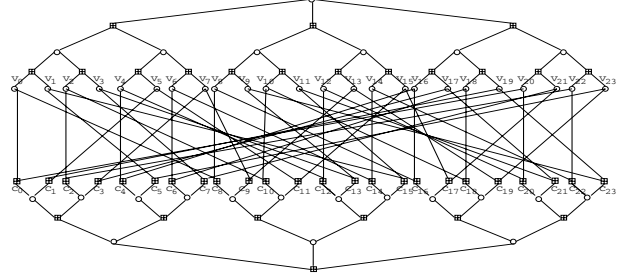


Fig. 2. Type I-A LDPC constraint graph having degree $d = 3$ and girth $g = 10$.

(resp., $c_{i,1}, c_{i,2}, \dots, c_{i,p^s-1}$). To use MOLS of order p^s in the connection algorithm, an imaginary node, $v_{i,0}$ (resp., $c_{i,0}$) is temporarily introduced into each set S_i (resp., S'_i). The connection algorithm proceeds as follows:

- 1) Let $x_t(i, j)$ denote the $(j, t)^{th}$ entry of the square $M^{(i)}$ defined in Section II. For $i = 0, \dots, p^s - 1$ and $j = 0, \dots, p^s - 1$, connect variable node $v_{i,j}$ to constraint nodes $c_{0,x_0(i,j)}, c_{1,x_1(i,j)}, \dots, c_{p^s-1,x_{p^s-1}(i,j)}$.
- 2) Delete all imaginary nodes $\{v_{i,0}, c_{i,0}\}_{i=0}^{p^s-1}$ and the edges incident on them.
- 3) For $i = 1, \dots, p^s - 1$, delete the edge connecting $v_{0,i}$ to $c_{0,i}$.

The resulting d -regular constraint graph represents the Type I-B LDPC code. Figure 3 illustrates the construction procedure and Figure 4 provides a specific example of a Type I-B LDPC constraint graph with $d = 4$; the squares used for constructing this graph are

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 & 3 & 1 \\ 1 & 3 & 2 & 0 \\ 2 & 0 & 1 & 3 \\ 3 & 1 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 3 & 1 & 2 \\ 1 & 2 & 0 & 3 \\ 2 & 1 & 3 & 0 \\ 3 & 0 & 2 & 1 \end{bmatrix}.$$

The Type I-B algorithm yields LDPC codes having a wide range of rates and blocklengths that are comparable to, but different from, the two-dimensional LDPC codes from finite Euclidean geometries [1], [2]. The Type I-B LDPC codes are p^s -regular with girth six, blocklength $N = p^{2s} + 1$, and distance $d_{\min} \geq p^s + 1$. For degrees of the form $d = 2^s$, the resulting Type I-B codes have very good rates, above 0.5, and perform well with iterative decoding.

IV. TREE-BASED CONSTRUCTION: TYPE II

In the Type II construction, first a d -regular tree T of alternating variable and constraint node layers is enumerated

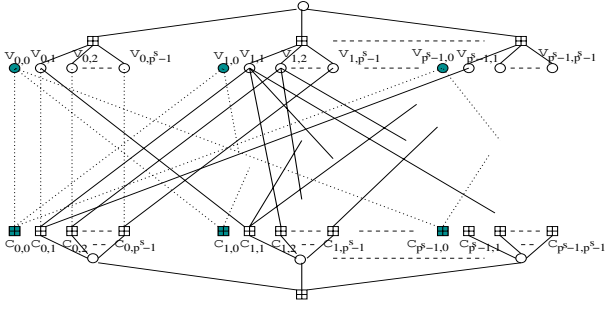


Fig. 3. Tree construction of Type I-B LDPC code. (Shaded nodes are imaginary nodes and dotted lines are imaginary lines.)

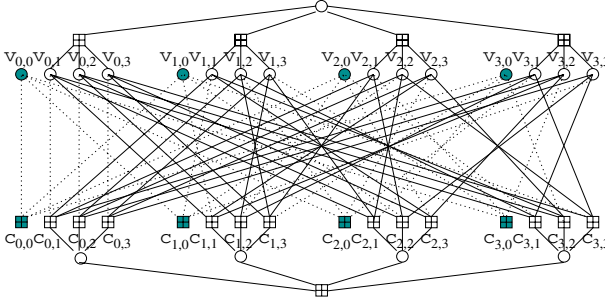


Fig. 4. Type I-B LDPC constraint graph having degree $d = 4$ and girth $g = 6$.

from a root variable node (layer L_0) for ℓ layers, as in Type I. The tree T is not reflected; rather, a single layer of $(d-1)^{\ell-1}$ nodes is added to form layer L_ℓ . If ℓ is odd (resp., even), this layer is composed of constraint (resp., variable) nodes. The union of T and L_ℓ , along with edges connecting the nodes in layers $L_{\ell-1}$ and L_ℓ according to a connection algorithm that is described next, comprise the graph representing a Type II LDPC code. We now present the connection scheme that is used for this Type II model, and discuss the resulting codes. The connection algorithm for $\ell = 3$ and $\ell = 4$ proceeds as follows.

A. $\ell = 3$

The d constraint nodes in L_1 are labeled B_0, B_1, \dots, B_{p^s} to represent the d branches stemming from the root node, and the $d(d-1)$ variable nodes in the third layer L_2 are labeled as $B_{0,0}, B_{0,1}, \dots, B_{0,p^s-1}, B_{1,0}, \dots, B_{1,p^s-1}, \dots, B_{p^s,0}, \dots, B_{p^s,p^s-1}$. The p^{2s} constraint nodes in the final layer $L_\ell = L_3$ are labeled $A_{0,0}, A_{0,1}, \dots, A_{0,p^s-1}, A_{1,0}, A_{1,1}, \dots, A_{1,p^s-1}, \dots, A_{p^s-1,0}, A_{p^s-1,1}, \dots, A_{p^s-1,p^s-1}$.

- 1) The constraint nodes in L_3 are grouped into $d-1 = p^s$ classes of $d-1 = p^s$ nodes in each class. Similarly, the variable nodes in L_2 are grouped into $d = p^s + 1$ classes of $d-1 = p^s$ nodes in each class. Those nodes descending from B_0 form the 0^{th} class, those descending from B_1 form the first class, and so on.
- 2) Each of the variable nodes descending from B_0 is connected to all the constraint nodes of one class.

That is, $B_{0,0}$ is connected to $A_{0,0}, A_{0,1}, \dots, A_{0,p^s-1}$, $B_{0,1}$ is connected to $A_{1,0}, A_{1,1}, \dots, A_{1,p^s-1}$, and in general, $B_{0,k}$ is connected to $A_{k,0}, A_{k,1}, \dots, A_{k,p^s-1}$ which correspond to the constraint nodes in the k^{th} class.

- 3) Let $x_t(i, j)$ denote the $(j, t)^{th}$ entry of $M^{(i-1)}$.
- 4) Then connect the variable node $B_{i,j}$ to the constraint nodes

$$A_{0,x_0(i,j)}, A_{1,x_1(i,j)}, A_{2,x_2(i,j)}, \dots, A_{p^s-1,x_{p^s-1}(i,j)}.$$

Figure 5 illustrates the construction procedure and Figure 6 provides an example of a Type II LDPC constraint graph with degree $d = 4$ and girth $g = 6$; the squares used for constructing this example are

$$M^{(0)} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}, M^{(1)} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}, M^{(2)} = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}.$$

The ratio of minimum distance to blocklength of the codes is at least $\frac{2+p^s}{1+p^s+p^{2s}}$, and the girth is six. For degrees d of the form $d = 2^s + 1$, the tree bound on minimum distance and minimum pseudocodeword weight [5], [6] is met, i.e., $d_{\min} = w_{\min} = 2 + 2^s$, for the Type II, $\ell = 3$, LDPC codes.

B. Relation to finite geometry codes

The codes that result from this $\ell = 3$ construction correspond to the two-dimensional projective-geometry-based LDPC (PG LDPC) codes of [2]. With a little modification of the Type II construction, we can also obtain the two-dimensional Euclidean-geometry-based LDPC codes of [2].

Since a two-dimensional Euclidean geometry may be obtained by deleting certain points and line(s) of a two-dimensional projective geometry, the graph of a two-dimensional EG-LDPC code [2] may be obtained by performing the following operations on the Type II, $\ell = 3$, graph:

- 1) In the tree T , the root node along with its neighbors, i.e., the constraint nodes in layer L_1 , are deleted.
- 2) Consequently, the edges from the constraint nodes B_0, \dots, B_{p^s} to layer L_2 are also deleted.
- 3) At this stage, the remaining variable nodes have degree p^s , and the remaining constraint nodes have degree $p^s + 1$. Now, a constraint node from layer L_3 is chosen, say, constraint node $A_{0,0}$. This node and its neighboring variable nodes and the edges incident on them are deleted. Doing so removes exactly one variable node from each class of L_2 , and the degrees of the remaining constraint nodes in L_3 are lessened by one. Thus, the resulting graph is now p^s -regular with a girth of six, has $p^{2s}-1$ constraint and variable nodes, and corresponds to the two-dimensional Euclidean-geometry-based LDPC code $EG(2, p^s)$ of [2].

C. $\ell = 4$

- 1) The tree T is now enumerated for four layers, with the nodes in L_0, L_1 , and L_2 labeled as in the $\ell = 3$ case. For $i = 0, \dots, p^s$, the constraint nodes in the i^{th} class of L_3 are labeled $B_{i,0,0}, B_{i,0,1}, \dots, B_{i,0,p^s-1}, B_{i,1,0}, B_{i,1,1}, \dots, B_{i,1,p^s-1}, \dots, B_{i,p^s-1,0}, \dots, B_{i,p^s-1,p^s-1}$.

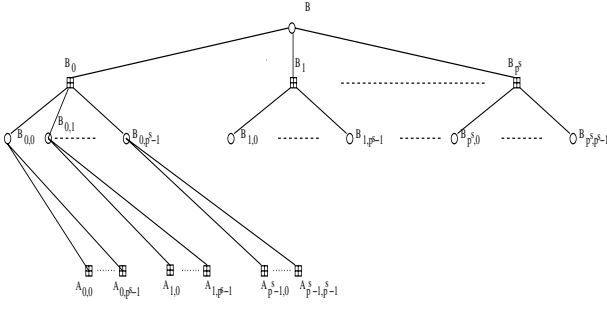


Fig. 5. Tree construction of girth 6 Type II ($\ell = 3$) LDPC code.

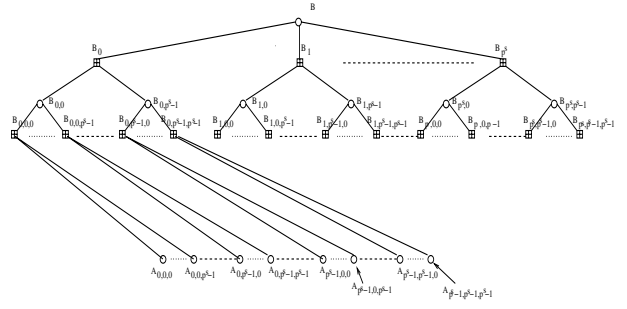


Fig. 7. Tree construction of girth 8 Type II ($\ell = 4$) LDPC code.

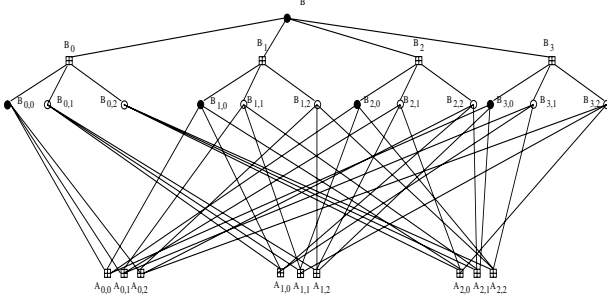


Fig. 6. Type II LDPC constraint graph having degree $d = 4$ and girth $g = 6$. (Shaded nodes highlight a minimum weight codeword.)

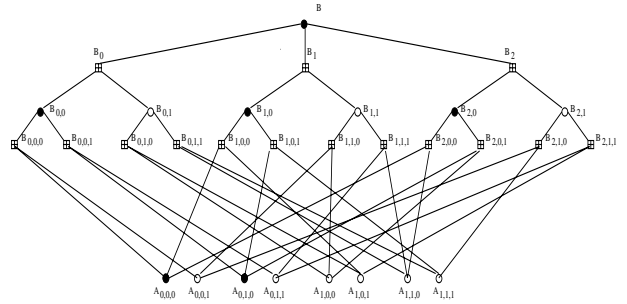


Fig. 8. Type II LDPC constraint graph having degree $d = 3$ and girth $g = 8$. (Shaded nodes highlight a minimum weight codeword.)

- 2) The p^{3s} variable nodes in the final layer L_4 are labeled $A_{0,0,0,0}, A_{0,0,0,1}, \dots, A_{0,0,p^s-1,0}, A_{0,1,0,0}, A_{0,1,0,1}, \dots, A_{0,1,p^s-1,0}, \dots, A_{p^s-1,0,0,0}, A_{p^s-1,0,0,1}, \dots, A_{p^s-1,0,p^s-1,0}, \dots, A_{p^s-1,p^s-1,0,0}, A_{p^s-1,p^s-1,0,1}, \dots, A_{p^s-1,p^s-1,p^s-1,0}, \dots, A_{p^s-1,p^s-1,p^s-1,p^s-1}$.
- 3) For $0 \leq i \leq p^s - 1$, $0 \leq j \leq p^s - 1$, connect the variable node $B_{0,i,j}$, that is in the 0^{th} class of L_3 , to the constraint nodes $A_{i,j,0}, A_{i,j,1}, \dots, A_{i,j,p^s-1}$.
- 4) Let $x_t(i, k) = M^{(i-1)}(k, t)$, the $(k, t)^{th}$ entry of $M^{(i-1)}$, and let $y_t(i, j) = M^{(i)}(j, t)$, the $(j, t)^{th}$ entry of $M^{(i)}$, where $i^* = i \bmod p^s$.
- 5) Then, for $1 \leq i \leq p^s$, $0 \leq j, k \leq p^s - 1$, connect the variable node $B_{i,j,k}$ to the constraint nodes

$$A_{0,x_0(i,k),y_0(j,k)}, A_{1,x_1(i,k),y_1(j,k)}, \dots, A_{p^s-1,x_{p^s-1}(i,k),y_{p^s-1}(j,k)}.$$

The Type II, $\ell = 4$, LDPC codes have girth eight, minimum distance $d_{\min} \geq 2(p^s + 1)$, and blocklength $N = 1 + p^s + p^{2s} + p^{3s}$. (We believe that the tree bound on the minimum distance is actually met for all the Type II, $\ell = 4$, codes, i.e. $d_{\min} = w_{\min} = 2(p^s + 1)$.) Figure 7 illustrates the general construction procedure. For $d = 3$, the Type II, $\ell = 4$, LDPC constraint graph as shown in Figure 8 corresponds to the $(2, 2)$ -Finite-Generalized-Quadrangles-based LDPC (FGQ LDPC) code of [8]; the squares used for constructing this code are

$$M^{(0)} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, M^{(1)} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

We believe that the Type II, $\ell = 4$, construction results in other FGQ LDPC codes for other choices of d . The Type II construction algorithm can be modified for larger ℓ by involving more iterations of the MOLS in the connection scheme, as will be discussed in a forthcoming paper.

V. SIMULATION RESULTS

Figures 9, 10, 11, 12 show the bit-error-rate performance of Type I-A, Type I-B, Type II girth six, and Type II girth eight LDPC codes, respectively over a binary input additive white Gaussian noise channel with min-sum iterative decoding. The performance of regular or semi-regular randomly constructed LDPC codes of comparable rates and blocklengths are also shown. (All of the random LDPC codes compared in this paper have a variable node degree of three and are constructed from the online LDPC software available at

<http://www.cs.toronto.edu/~radford/ldpc.software.html>.)

Figure 9 shows that the Type I-A LDPC codes perform substantially better than their random counterparts. Figure 10 reveals that the Type I-B LDPC codes perform better than comparable random LDPC codes at short blocklengths; but as the blocklengths increase, the random LDPC codes tend to perform better in the waterfall region. Eventually however, as the SNR increases, the Type I-B LDPC codes outperform the random ones since, unlike the random codes, they do not have a prominent error floor. Figure 11 reveals that the performance of Type II girth-six LDPC codes is also significantly better than comparable random codes; these codes correspond to the two dimensional PG LDPC codes of [2]. Figure 12 indicates the performance of Type II girth-eight LDPC codes; these codes perform comparably to random codes at short blocklengths, but at large blocklengths, the random codes perform better as they have larger relative minimum distances compared to the Type II girth-eight LDPC codes.

As a general observation, min-sum iterative decoding of

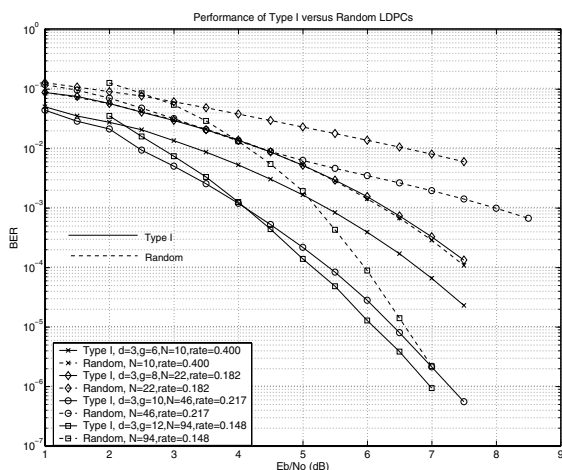


Fig. 9. Performance of Type I-A versus Random LDPC codes with min-sum iterative decoding.

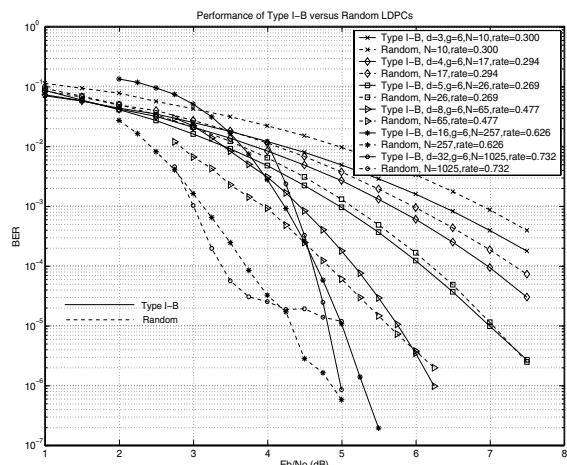


Fig. 10. Performance of Type I-B versus Random LDPC codes with min-sum iterative decoding.

most of the tree-based LDPC codes (particularly, Type I-A, Type II, and some Type I-B) presented here did not typically reveal detected errors, i.e., errors caused due to the decoder failing to converge to any valid codeword within the maximum specified number of iterations. Detected errors are caused primarily due to the presence of pseudocodewords, especially those of minimal weight. We think that the lack of detected errors with iterative decoding of these LDPC codes is primarily due to their good minimum pseudocodeword weight w_{\min} .

VI. CONCLUSIONS

The Type I construction yields a family of LDPC codes that, to the best of our knowledge, do not correspond to any of the LDPC codes obtained from finite geometries or other geometrical objects. The two tree-based constructions presented in this paper yield a wide range of codes that perform well when decoded iteratively, largely due to the maximized minimal pseudocodeword weight. However, the overall minimum distance of the code is relatively small. Constructing codes with larger minimum distance, while still maintaining $d_{\min} = w_{\min}$, remains an open problem.

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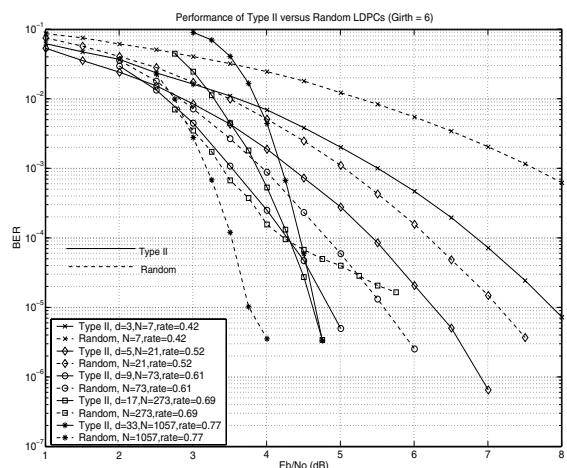


Fig. 11. Performance of girth 6 Type II versus Random LDPC codes with min-sum iterative decoding.

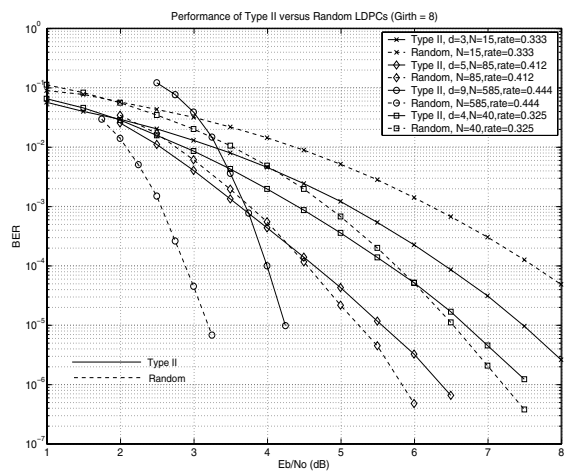


Fig. 12. Performance of girth 8 Type II versus Random LDPC codes with min-sum iterative decoding.