The argument given for Proposition 4.1 is incorrect. In particular, after equation (25) we claim that, since $h''_{t,x}$ changes sign at most once in $[u^-, u^+]$, $h_{t,x}(u^-) = h_{t,x}(u^+) = 0$ and $h_{t,x} \geq 0$ on $[u^-, u^+]$, then $h''_{t,x} \leq 0$. This is, however, an obviously false claim. I do not know whether Proposition 4.1 holds in the generality claimed by the paper. Therefore I do not know whether conclusion (c) holds in the generality claimed by Theorem 1.1. However, I can supply a proof of Proposition 4.1 (and hence of Theorem 1.1(c)) under the following stronger assumption:

(A) f'' does not vanish at infinite order, that is, if $f''(x_0) = 0$, then there are $c_0 \neq 0$ and $j \in \mathbb{N}$ such that $f''(x) = c_0(x - x_0)^j + o(x - x_0)^j$ in a neighborhood of x_0.

Condition (A) is obviously fulfilled by an analytic function. Variations of the argument given below lead to weaker conditions then (A).

Proof. Following the arguments of Proposition 4.1 till (25), what we need to show is the following. Fix a point x_0 with $f''(x_0) = 0$. Then there are constants C and ε with the following property: if there is an admissible shock wave with a, b as traces such that

- $b - a < \varepsilon$;
- $a < x_0 < b$;

then

$$\int_a^b |f''| \leq C|f'(b) - f'(a)|\,. \tag{1}$$

The condition of admissibility of the shock is that f lies above the line connecting $(a, f(a))$ and $(b, f(b))$, in case b is the right and a the left trace, and that f lies below otherwise.

We can therefore assume, without loss of generality, that

- $x_0 = 0$;
- $f(0) = f'(0) = f''(0) = 0$.

From (A) we know that $f(x) = c_0 x^n + o(x^n)$ with $c_0 \neq 0$. If n is even, the function f is convex (or concave) in a neighborhood of 0 and the proof of (1) is obvious. We therefore assume $n = 2k+1$ with $k \geq 1$ and $c_0 = \pm 1$. The case $c_0 = -1$ can be handled similarly and we therefore assume $c_0 = 1$. Analogously we assume $a < 0 < b$. The inequality (1) reduces then to

$$f'(b) + f'(a) \leq C(f'(b) - f'(a)) \tag{2}$$

which in turn is equivalent to the existence of a $\delta > 0$ such that

$$f'(b) \leq (1 - \delta) f'(a)\,. \tag{3}$$

Note that $f'' > 0$ on some interval $]0, \alpha[$, and hence it suffices to prove the inequality (3) for the largest possible b in a neighborhood of 0 which can be connected to a by an admisible shock. This is achieved when the line r passing through $(a, f(a))$ and $(b, f(b))$ is tangent to the graph of f in $(b, f(b))$. In this case we have

$$f'(b) = \frac{f(b) - f(a)}{b - a} . \tag{4}$$

The inequality (3) becomes then

$$f(b) - f(a) \leq (1 - \delta) f'(a)(b - a) . \tag{5}$$

Recalling the assumption (A), for any fixed $\eta > 0$, there is an $\varepsilon > 0$ such that (4) implies

$$\begin{aligned} (2k+1)(1-\eta)b^{2k} &\leq \frac{(1+\eta)b^{2k+1} + (1+\eta)|a|^{2k+1}}{b + |a|} \\ &\leq \frac{(1+\eta)\left(b^{2k+1} + |a|^{2k+1}\right)}{b} . \end{aligned}$$

This in turn gives

$$\frac{2k - (2k+2)\eta}{1 + \eta} b^{2k+1} \leq |a|^{2k+1} .$$

Choosing $\eta \leq \frac{2}{3}$, we conclude $|a| \geq b$.

Using this last information and (A), we can estimate

$$f(b) - f(a) \leq (1 + \eta)\left(b^{2k+1} + |a|^{2k+1}\right) \leq 2(1 + \eta)|a|^{2k+1}$$

and

$$f'(a)(b - a) \geq (2k + 1)(1 - \eta)|a|^{2k}(b + |a|) \geq 3(1 - \eta)|a|^{2k+1} .$$

Choosing $\eta = \frac{1}{7}$ and $\delta = \frac{1}{9}$ we get inequality (5). □

Acknowledgments. I thank Alessio Figalli for pointing out the mistake to me.