Belief propagation is a method, or better a bunch of methods, to analyze complex probabilistic models, and to derive algorithms for extracting information, for instance on marginal distributions, averaged properties of the model, and on many others. A key property of these models is that there is a graphical representation of the dependency structure. The belief propagation equations (sometimes also called message passing equations) describe the local properties of the model. In “easy” and essentially uninteresting cases, the equations determine completely the properties of the model, for instance the marginal distributions, and they can be used to compute them quickly. That is the case only if the underlying graph is a tree. In more interesting cases, this is approximately true, and in even more interesting cases, it is not true at all, but the set of solutions, which may be large, gives interesting partial information on the models. This last case is connected with a breaking of an abstract symmetry property, often called “replica symmetry breaking”.

We will give an introduction to the method, and will present a number of applications. Particular focus will be given to so-called "approximate message passage" which has found powerful applications recently, for instance in statistics and machine learning (compressed sensing).

There will be time for one other application which will be chosen based on the preference of the audience of the course: Here the possible choices:

- **Error correcting codes** (Gallager or LDPC codes), which was actually the first occurrence of belief propagation methods in decoding algorithms (Gallager 1968)
- **Disordered media** in statistical physics, in particular mean-field spin glasses. (The method there is usually called **“cavity method”**). There would not be enough time to completely cover that, but key mathematical aspects of the cavity method could be presented.
- Random **satisfiability problems** from theoretical computer science. This is a case where the symmetry breaking mentioned above is important. The standard K-SAT problem is mathematically too difficult, but there is a kind of toy version which is much easier and still exhibits some of the most interesting aspects. (See Ch 18 of the monograph by Mézard-Montanari.)