Braids

This lecture will be about braids. A braid with n strands is a set of strings going from n points on a disk to n parallel points on a parallel disk. In general, we use diagrams to describe braids, i.e. a planar picture where we just have to specify uppercrossings and undercrossings (see Fig. 2). The strands are allowed to be "isotoped", i.e. to move but not to cross each other.

These objects are richer and harder to understand than their simple definition suggests. For example, we see on Fig. 3 how a trivial braid (vertical strands) can look complicated. The kind of question we will study in the lecture is "when are two given braids equivalent ?

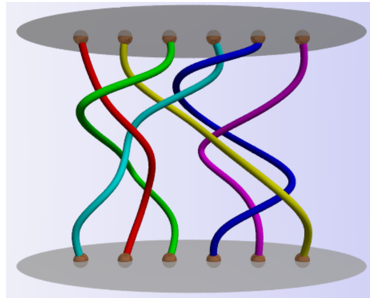

Figure 1: A braid with 6 strands

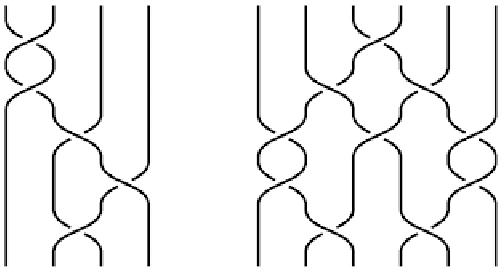

Figure 2: Diagrams of braids

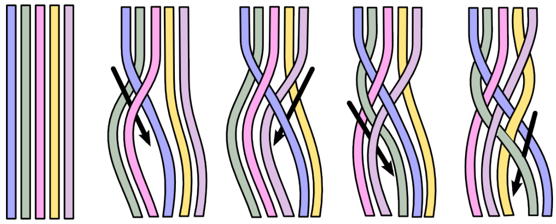

Figure 3: Trivial braids

Braids have many important properties and appear in many places in modern mathematics. In particular it can be applied to the study of knots, via the closure construction (see Figure 4). We will study when two different braids give the same knot/link.

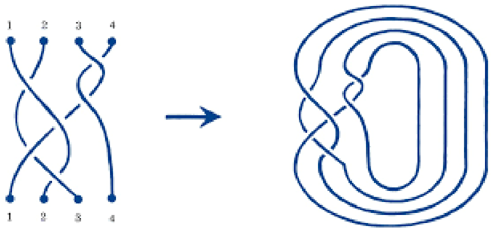

Figure 4: Getting a knot via a braid

Furthermore, braid have the advantage that you can plug them together (Fig. 5), getting what is called a group structure on braids. In this sense, braids generalize the group of permutations, as one can see in Fig. 6 (forgetting the crossings in braid diagrams gives a permutation). This is one of the reason why they appear in different field of mathematics. In some sense, in any mathematical process where "swapping" two objects twice modify the objects, you get an application of braid theory.

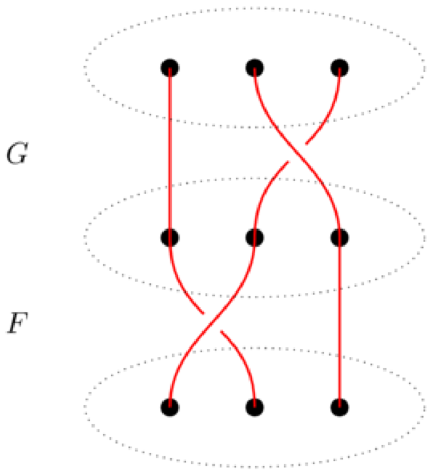

Figure 5: Plugging the braids G and F

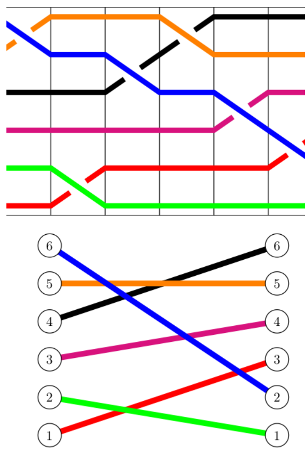

Figure 6: Braids induce permutations