Wednesday, September 14

14:00–15:00: Nicolas Monod (EPFL)

A Type I Conjecture and Boundary Representations of Hyperbolic Groups

In harmonic analysis, "Type I" groups refers to those groups for which a description of unitary representations can be attempted at all. For instance, Fourier analysis tells us that abelian groups are Type I. It is known since the sixties which discrete groups are Type I. However, for locally compact groups, despite a very rich and deep representation theory in special cases, there is not even a conjectural characterization of Type I.

We propose a conjectural restriction which would have far-reaching classification consequences. In joint work with Caprace and Kalantar, we establish this conjecture in the special case of hyperbolic locally compact groups. Our method relies essentially on dynamical and geometric considerations for the boundary actions of these groups.

15:30–16:30: Thi Dang (Orsay)

Equidistribution of periodic flat tori

Bowen and Margulis in the 70s proved that closed geodesics on compact hyperbolic surfaces equidistribute towards the measure of maximal entropy. From a homogeneous dynamics point of view, this measure is the quotient of the Haar measure.

In a joint work with Jialun Li, we study a higher rank generalization of this homogeneous dynamics problem. In the compact case, what we consider instead of closed geodesics are periodic flat tori of dimension ≥ 2. I will present our equidistribution formula and sketch the proof. We follow Roblin's counting strategy with a quantitative shortcut due to Gorodnik–Nevo which allow us to deduce the main term and an exponential error term.

16:45–17:45: Cagri Sert (UZH)

Stationary measures on projective spaces without irreducibility

A probability measure μ on the general linear group $\mathrm{GL}_d(\mathbb{R})$ induces a non-commutative random walk R_n on $\mathrm{GL}_d(\mathbb{R})$ and a Markov chain on the projective space $P(\mathbb{R}^d)$. The associated stationary measures on $P(\mathbb{R}^d)$ encodes various information on R_n and their understanding is crucial for the study of asymptotic properties of the random walk. While the pioneering works of Furstenberg, Kifer, Guivarc'h, Hennion etc. have given a satisfactory description of stationary measures (particularly when the measure μ is irreducible), many natural questions remain to be studied in the reducible case. In this ongoing series of works, we give a description of stationary measures which refines that of Furstenberg–Kifer and Hennion from '80s and also generalizes recent work by Aoun–Guivarc'h and Benoist–Bruère. After reviewing the well-known aspects of the theory, we will discuss our results, techniques and further consequences. Joint work with R. Aoun.

Thursday, September 15

9:30–10:30: Uri Shapira (Technion)

Stationary measures on hybrid spaces

Given a homogeneous space $X = G/S$ and a probability measure μ on G, we wish to classify the ergodic μ-stationary measures on X.

This question has been studied extensively, but mostly in two completely different settings.

> (Type 1) When the homogeneous space is projective (i.e. embedded in the projective space $P(V)$ of a vector space).
>
> (Type 2) When the homogeneous space is a quotient of a Lie group by a lattice in it.

We suggest a setting which is a hybridization of the two, namely, when the homogeneous space is a bundle over a Type 1 space where the fibers are of Type 2. A prime example of such a space is the space $X(k, n)$ of rank-k discrete subgroups of $\mathbb{R}^n$ identified up to homothety. Here the base of the bundle is the grassmannian of k-planes and the fibers are copies of $\mathrm{SL}(k, \mathbb{R})/\mathrm{SL}(k, \mathbb{Z})$. A natural conjecture is that under some reasonable assumptions on the measure μ, all the ergodic stationary measures are "homogeneous lifts" (definitions will be given in the talk). In this talk I will discuss work in progress (joint with Uri Bader and Oliver Sargent) in which we prove special cases of this conjecture.

11:00–12:00: Alex Gorodnik (UZH)

Analysis and Arithmetic on Homogeneous Spaces

We explore interplays between harmonic analysis on homogeneous spaces and questions regarding the distribution of rational points.

13:30–14:30: Alexander Kolpakov (Neuchâtel)

Subspace stabilizers in hyperbolic lattices

We show that immersed totally geodesic m-dimensional suborbifolds of n-dimensional arithmetic hyperbolic orbifolds correspond to finite subgroups of the commensurator given a very simple condition on their dimension. We call such totally geodesic suborbifolds "finite centralizer subspaces" (or fc-subspaces) and use them to formulate an arithmeticity criterion: a hyperbolic orbifold is arithmetic if and only if it has infinitely many fc-subspaces.

We also analyze the relation between Vinberg's commensurability invariants of an arithmetic hyperbolic orbifold and those of its totally geodesic suborbifolds, and provide examples of 3-dimensional arithmetic hyperbolic orbifolds which cannot be geodesically immersed in higher dimensional arithmetic ones.

This is a joint work with Mikhail Belolipetsky (IMPA, Brazil), Nikolay Bogachev (University of Toronto) and Leone Slavich (University of Pavia).

14:45–15:45: Ursula Hamenstädt (Bonn)

Entropy at infinity

For a flow on a non-compact space X, define the entropy at infinity as the lim sup of the entropies of invariant probability measures whose supports exit all compact sets. In the case of the moduli space of abelian differentials, we relate this to dynamics of an extension of the flow to a compactification of X. Time permitting, we give a conjectural picture for locally symmetric spaces of higher rank.

16:15–17:15: Michael Magee (Durham)

The maximal spectral gap of a hyperbolic surface

A hyperbolic surface is a surface with metric of constant curvature -1. The spectral gap between the first two eigenvalues of the Laplacian on a closed hyperbolic surface contains a good deal of information about the surface, including its connectivity, dynamical properties of its geodesic flow, and error terms in geodesic counting problems. For arithmetic hyperbolic surfaces the spectral gap is also the subject of one of the biggest open problems in automorphic forms: Selberg's eigenvalue conjecture.

A conjecture of Buser from the 1980s stated that there exists a sequence of closed hyperbolic surfaces with genera tending to infinity and spectral gap tending to $1/4$. (The value $1/4$ here is the asymptotically optimal one.) We proved that such a sequence does exist. I'll discuss the very interesting background of this problem in detail as well as some ideas of the proof.

This is joint work with Will Hide.

Friday, September 16

9:00–10:00: Barbara Schapira (Rennes)

Twisted Patterson-Sullivan measures and amenability of covers

Let $\Gamma' \triangleleft \Gamma$ be two discrete groups acting properly by isometries on a hyperbolic space X. We prove that their critical exponents coincide if and only if Γ/Γ' is amenable in Γ, under the assumption that the action of Γ on X is strongly positively recurrent, i.e. has a growth gap at infinity. The proof relies on the construction of twisted Patterson-Sullivan measures on the boundary at infinity of the hyperbolic space.

In this talk, I will give the flavour of this common work with R. Coulon, R. Dougall, S. Tapie.

10:30–11:30: Emmanuel Breuillard (Oxford)

Limit theorems and harmonic functions for random walks on nilpotent Lie groups

We establish the local limit theorem for biased aperiodic random walks on nilpotent Lie groups. The presence of a bias 'flattens' the walk so that at large scale the renormalized process behaves like a hypoelliptic left invariant diffusion on a certain new graded Lie group that we determine.

As consequences we obtain a probabilistic Ratner type theorem (the Cesaro equidistribution of unipotent random walks on homogeneous spaces) and we derive the Choquet–Deny theorem on nilpotent Lie groups under a moment condition. The methods are based on Fourier analysis, via a Gaussian replacement scheme and a swapping argument inspired by work of Diaconis and Hough. Joint work with Timothée Bénard.