1 Scientific and Social Program

Thursday, September 30

Time	Speaker	Title
14:45	Arne Ogrowsky Universität Paderborn	Attractors of random differential equations with random delay
15:15	Andreas Gegg KU Eichstätt-Ingolstadt	Change-Point in Profile-Data and Residual Partial Sums in Multivariate Regression
15:45	Hella Timmermann Universität zu Köln	Sequential testing of gradual changes in the drift of a stochastic process
16:15	Coffee break	
16:45	Lorenz Pfeifroth <i>TU München</i>	About the Uniqueness of the Mixing Measure for a Random Walk in a Random Environment on the Integers
17:15	Hadrian Heil Universität Tübingen	Branching Random Walks in Random Envi- ronment: where are the particles?
17:45	Laura Vinckenbosch EPF Lausanne	Pushing a Brownian particle out of an interval subject to a switching cost
18:45	Dinner at Irchel in 13M Foyer	

Friday, October 1

Time	Speaker	Title
09:00	Chiranjib Mukherjee MPI MIS Leipzig	Large deviations for Brownian intersection measures
09:30	Patrick Schmid Universität Leipzig	Brownian motion in a truncated Weyl chamber
10:00	Philipp Thomann Universität Zürich	Numerical Simulations of Random Walks in Random Environments
10:30	Coffee break	

11:00	Wael Mohammed Universität Augsburg	Amplitude Equations for SPDEs with Cubic Nonlinearities
11:30	Matteo Casserini ETH Zürich	A characterization of Widder's theorem via Hermite polynomials
12:00	Discussion session	
12:30	Lunch and leisure time	
16:15	Guided tour by Martin Herdegen	
17:45	Boat excursion and dinner on MS Glärnisch	

Saturday, October 2

Time	Speaker	Title
09:30	Roman Muraviev ETH Zürich	A Limit Theorem for a Double Stochastic Integral
10:00	Sandra Haas Université de Lausanne	Ruin Probabilities with Excess of Loss Reinsurance and Reinstatements
10:30	Coffee break	
11:00	Le Chen EPF Lausanne	Intermittence Properties for Stochastic Heat and Wave Equations in One Space Dimension
11:30	Adrian Schnitzler <i>TU Berlin</i>	Time Correlations for the Parabolic Anderson Model
12:00	End of conference	

2 Abstracts

Matteo Casserini: A characterization of Widder's theorem via Hermite polynomials

It is a well known consequence of a classical result by Widder that continuous positive Brownian martingales of the form $(g(t, W_t))_{t\geq 0}$ can be represented as $g(t, W_t) = \int_{\mathbf{R}} \mathcal{E}(a \cdot W)_t \mu(da)$, for some measure μ . However, there is no explicit characterization of the measure μ .

In this work, we consider stochastic integrals with respect to a complex Brownian motion, and we obtain an explicit representation of their predictable projections on the real line. By applying these results to series of Hermite polynomials, we then provide a characterization of the exponential moments of Widder's measure μ .

Le Chen: Intermittence Properties for Stochastic Heat and Wave Equations in One Space Dimension

In this talk, I will present some our recent results on the intermittence properties for stochastic heat and wave equations driven by space-time white noise in one-space dimension. In particular, we shall consider the following model

$$\begin{cases} \left(\frac{\partial}{\partial t} - \frac{\nu}{2}\frac{\partial^2}{\partial x^2}\right)u(t,x) = u(t,x)\dot{W}(t,x), & x \in \mathbb{R}, \ t > 0\\ u(0,x) = g(x), & x \in \mathbb{R} \end{cases}$$
(1)

where \dot{W} is space-time white noise, g(x) is some initial data, and $\nu > 0$ is some constant. When dealing with stochastic integral, we shall use standard Walsh's integral. We are interested in the second moment of the solution u(t, x) of the above equation, which is denoted by

$$f(t,x) \stackrel{\Delta}{=} \mathbb{E}(u(t,x)^2)$$
.

One aim of my research is to impose assumptions on the initial condition g as weak as possible such that the intermittence properties continue to hold, i.e., the second Lyapunov exponent

$$\lambda_2(x) \stackrel{\Delta}{=} \lim_{t \to \infty} \frac{\log f(t, x)}{t}$$

is strictly greater than zero. If time is permitted, I shall also present some work on the stochastic wave equation case.

A standard reference for intermittency problem is [CM94]. Our analytical method recovers some results in [BC94], where the same property is proved by a probabilistic

method. A closely related recent work is [CK10], where we hope to refine their results at least in this simple case. See also some recent papers [DMT06, DM09, FK08] and references therein for more references.

References

- [BC94] Lorenzo Bertini and Nicoletta Cancrini. The stochastic heat equation: Feynman-Kac formula and intermittence. *Journal of Statistical Physics*, 78(5-6):1377–1401, 1994.
- [CK10] Daniel Conus and Davar Khoshnevisan. On the existence and position of the farthest peaks of a family of stochastic heat and wave equations. *Probability Theory and Related Fields*, to appear, 2010.
- [CM94] René A. Carmona and S. A. Molchanov. Parabolic Anderson Problem and Intermittency. Mem. Amer. Math. Soc., 1994.
- [DM09] Robert C. Dalang and Carl Mueller. Intermittency properties in a hyperbolic Anderson problem. Annales de l'Institut Henri Poincaré, 2009.
- [DMT06] Robert C. Dalang, Carl Mueller, and Roger Tribe. A Feynman-Kactype formula for the deterministic and stochastic wave equations and other P.D.E.'s. *Transactions of the American Mathematical Society*, 360(9):4681–4703, 2006.
- [FK08] Mohammud Foondun and Davar Khoshnevisan. Intermittence and nonlinear parabolic stochastic partial differential equations. *Electr. J. Probab.*, 14(14):548–568, 2008.

Andreas Gegg: Change-Point in Profile-Data and Residual Partial Sums in Multivariate Regression

We are interested in testing the constancy of regression parameters in a linear profile data set (panel data in econometrics). For that, we use residual partial sums in several dimensions.

We introduce the one-dimensional partial sums operator and cite some well-known results concerning residual partial sums processes for univariate linear regression. Particularly, we show that the residual partial sums limit process is a function of Brownian motion – no matter which distribution is used in the linear model .

In a second step, we extend this technique to the multivariate case and show that the corresponding residual partial sums limit process is a function of multivariate Brownian motion. Using this result, we propose a change-point approach for parameter vectors of linear profile data. In our setting, each sample collected over time consists of several multivariate observations for which a linear regression model is appropriate. The question now is whether all of the profiles follow a linear regression model with the same parameter vector or whether a change occurred in one or more model parameters after a special sample. We use the partial sums operator in several dimensions to test the null hypothesis " H_0 : no change–point occurred" and propose a size α -test.

Sandra Haas: Ruin Probabilities with Excess of Loss Reinsurance and Reinstatements

Joint work with Hansjörg Albrecher¹.

The present paper studies the probability of ruin of an insurer, if excess of loss reinsurance with reinstatements is applied. In the setting of the classical Cramer-Lundberg risk model, piecewise deterministic Markov processes (PDMP) are used to describe the surplus process in this more general situation. It is shown that the finitetime ruin probability is both the solution of a partial integro-differential equation and the fixed point of a contractive integral operator. We exploit the latter representation to develop and implement a recursive algorithm for numerical approximation of the ruin probability that involves high-dimensional integration. Furthermore we study the behavior of the finite-time ruin probability under various levels of initial capital and security loadings and compare the efficiency of the numerical algorithm with the computational alternative of stochastic simulation of the risk process.

The presented probabilistic approach for the numerical solution of a deterministic equation can also be employed in other applications.

References

ALBRECHER, H., KAINHOFER, R., TICHY, R. (2003): Simulation methods in ruin models with non-linear dividend barriers, *Math. Comput. Simulation* 62(3-6), 277-287.

DASSIOS, A., EMBRECHTS, P. (1989): Martingals and insurance risk, *Stochastic Models*, 5(2), 181-217.

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WALHIN, J.F., PARIS, J. (2000): The effect of excess of loss reinsurance with reinstatements on the cedent's portfolio, *Blätter der Deutschen Gesellschaft für Versicherungsmathematik*, 24, 616-627.

Hadrian Heil: Branching Random Walks in Random Environment: where are the particles?

We treat branching random walks in a space-time-i.i.d. random environment. There is a phase-transition between regulare growth (where the number of particles grows as fast as its expectation) and slow growth (where it does not). In spatial dimensions three or larger, diffusive behaviour holds in the entire regular growth phase. On the other hand, in the slow growth phase, localization occurs under weak assumptions on the environment. The talk is about joint work with Makoto Nakashima and Nobuo Yoshida.



References

- [HN] H.H. and Makoto Nakashima. A Remark on Localization for Branching Random Walks in Random Environment. Preprint.
- [HNY] H.H., Makoto Nakashima, and Nobuo Yoshida. Branching Random Walks in Random Environment in d > 3 are Diffusive in the Regular Growth Phase.

Wael Mohammed: Amplitude Equations for SPDEs with Cubic Nonlinearities

We consider a quite general class of SPDEs with cubic nonlinearities and derive rigorously amplitude equations and their higher order corrections, using the natural separation of time-scales near a change of stability. We show that degenerate additive noise has the potential to stabilize the dynamics of the dominant modes. We focus on equations with cubic nonlinearity and give applications to the Swift-Hohenberg equation, the Ginzburg-Landau / Allen-Cahn equation and a model from surface growth.

Chiranjib Mukherjee: Large deviations for Brownian intersection measures

We consider a number of independent Brownian motions running in the d-dimensional Euclidean space until their first exit from a domain. Classical results from the 1950's show that Brownian paths have mutual intersections. Le Gall and others constructed an object which measures the intensity of the path intersections. This measure was called the "Brownian intersection local time", keeping track of the notion of Brownian local time for the case of one single path. Donsker and Varadhan developed the celebrated theory of large deviations for a single path Brownian local time (in fact, the occupation measures). In a joint project with Wolfgang Koenig, we study large deviations for the (many-path) intersection local time (as measures).

Roman Muraviev: Heterogeneous Equilibrium with Habit Formation

We study Arrow-Debreu equilibrium in a pure exchange economy populated by heterogeneous investors represented by habit forming utility functions. The heterogeneity might be reflected in the following parameters: beliefs concerning the economy, risk aversion, impatience (time preference rate) and strength of the habits.

Arne Ogrowsky: Attractors of random differential equations with random delay

We investigate a random differential equation with random delay. First the nonautonomous case is considered. We show the existence and uniqueness of a solution that generates a cocycle. In particular, the existence of an attractor is proved. Finally we look at the random case. We pay special attention to the measurability. This allows us to prove that the solution to the random differential equation generates a random dynamical system. The existence result of the attractor can be carried over to the random case.

Lorenz Pfeifroth: About the uniqueness of the mixing measure for a random walk in a random environment on the integers

In this talk we will consider a Random walk in a random environment on \mathbb{Z} . First, I introduce the correct model. Then we have a look on the mixing measure of a random walk in a random environment and prove that under a certain condition this measure is unique. In the last part of this talk I give an example that if the condition fails I can construct two mixing measures for the same random walk in a random environment. About the uniqueness of the mixing measure for a random walk in a random environment on the integers.

Patrick Schmid: Brownian motion in a truncated Weyl chamber

Joint work with Wolfgang König.

We examine the non-exit probability of a multidimensional Brownian motion from a growing truncated Weyl chamber. Different regimes are identified according to the growth speed, ranging from polynomial decay over stretched-exponential to exponential decay. Furthermore we derive associated large deviation principles for the empirical measure of the properly rescaled and transformed Brownian motion as the dimension grows to infinity. Our main tool is an explicit eigenvalue expansion for the transition probabilities before exiting the truncated Weyl chamber.

Adrian Schnitzler: Time Correlations for the Parabolic Anderson Model

We consider asymptotics of time correlations for the parabolic Anderson model, i.e. the Cauchy problem for the heat equation on the lattice with a random potential. We show how to derive exact formulae for the case of a potential that consists of an i.i.d. field of nonnegative random variables with tails that decay more slowly than those of a double-exponential distribution. Furthermore, we apply these results to investigate intermittency and ageing properties of the model.

Philipp Thomann: Numerical Simulations of Random Walks in Random Environments

Random Walks in Random Environments are a well established area of Probability Theory. Even though they are not difficult to treat numerically there seems to be quite few published knowledge. We use the new Schroedinger Cluster in Zurich to calculate exit probabilities of lattices up to side length 2^{15} therefore solving linear equations with over a billion of unknowns. We use this numerical evidence to get some insight into theoretical conjectures.

Hella Timmermann: Sequential testing of gradual changes in the drift of a stochastic process

I will describe and analyze some sequential monitoring procedures for detecting a gradual change in the drift parameter of a general stochastic process satisfying a certain (weak) invariance principle. It is shown that the tests can be constructed such that the false alarm rate attains a prescribed level and that the tests have asymptotic power one. A more precise analysis of the procedures under the alternative proves that the stopping times, suitably normalized, have a standard normal limit distribution. A few results from a small simulation study are also presented in order to give an idea of the finite sample behavior of the suggested procedures.

Laura Vinckenbosch: Pushing a Brownian particle out of an interval subject to a switching cost

The game we are considering is to force a Brownian particle out of the interval [0, 1] as quickly as possible using two opposite constant forces, either upwards or downwards. The player is allowed to change between the two forces at any time, subject to a time penalty c > 0.

More precisely, we consider a stochastic control model where the state of the system is driven by the stochastic differential equation

$$dX_t^A = A_t \mu \, dt + dB_t,$$

where $\mu > 0$, (B_t) is a Brownian motion and (A_t) is the control process. We require the control to be piecewise constant and to take values in $\{\pm 1\}$. The value function of the game is then defined by

$$V^{c}(x,a) = \inf_{A} \mathbb{E}_{x,a} \left(\tau^{A} + cN_{\tau^{A}}(A) \right)$$

where $\tau^A = \inf\{t \ge 0 : X_t^A \notin [0,1[]\}$ is the exit time of the controlled process and

$$N_t(A) = \sharp \{ s \in (0, t] : A_{s_-} \neq A_s \}$$

is the number of times the drift is changed up to time t. We compute this function and exhibit the optimal strategy (A_t^*) as a function of the switching cost c. The method of proof makes use of a free boundary problem and the principle of smooth-fit, as well as a local time-space formula.