

# Algebraic Coding Theory e-Summer School - ACT21

## Program

	Monday	Tuesday	Wednesday	Thursday	Friday
14.45-15.00	Opening		ACT Graduate Minisymposium		
15.00-15.50	Martino Borello	Jens Zumbärgel		Alberto Ravagnani	John Sheekey
15.50-16.00	Coffee Break	Coffee Break		Coffee Break	Coffee Break
16.00-16.50	Martino Borello	Jens Zumbärgel		Alberto Ravagnani	John Sheekey
16.50-17.00	Coffee Break	Coffee Break		Coffee Break	Coffee Break
17.00-17.50	Martino Borello	Jens Zumbärgel		Alberto Ravagnani	John Sheekey
17.50-18.30	Gather	Gather		Gather	Gather

The time indicated in the schedule is referred to **Central European Summer Time (CEST)**

## Lectures

### Codes in Group Algebras

Martino Borello

*Abstract:* Group codes over fields are ideals in the group algebra  $KG$ , where  $K$  is a finite field and  $G$  is a finite group. Introduced by Berman and MacWilliams in the late sixties as a generalization of cyclic codes, they are still the subject of intense research. This short course is intended to be an introduction to their theory, presenting their main properties in relation to classical codes. The last part of the lecture will give an overview of current research perspectives and open problems. A good reference (among the rare ones) for these codes is Chapter 16 of the recent Concise Encyclopedia of Coding Theory by Huffman, Kim, and Solé.

### Ring Linear Coding Theory

Jens Zumbärgel

*Abstract:* Codes over ring alphabets gained interest since the 1990s, when it was observed that several extremely good nonlinear binary codes may be viewed as linear codes over the ring  $\mathbb{Z}_{4\mathbb{Z}}$ . In the following years, fundamental questions on code equivalence and the MacWilliams identity have led to a fruitful interplay between ring theory and coding theory. On a more practical side, novel algebraic decoding algorithms have been developed that are able to cope with the existence of zero divisors. More recently, codes over rings also received attention due to new applications in network coding scenarios. In the lectures, I plan to provide an introduction to ring theory with a view towards coding applications, and I will also touch on some recent developments in ring-linear network coding and rank-metric codes.

### Rank-Metric Codes and Their Fundamental Properties

Alberto Ravagnani

*Abstract:* This lecture is an introduction to the theory of rank-metric codes (linear spaces of matrices endowed with the rank metric). I will define these objects and describe their mathematical structure via a series of fundamental parameters, such as the minimum distance, weight distribution, and covering radius. I will then illustrate how these quantities interact with each other and how upper and lower bounds for them can be derived.

## Galois Geometry and Codes

John Sheekey

*Abstract:* Galois Geometry is the study of the geometry of subspaces of a vector space over a finite field. In the projective geometry  $\text{PG}(n - 1, q)$  we define points to be the 1-dimensional subspaces of an  $n$ -dimensional vector space over the field with  $q$  elements, lines to be the 2-dimensional subspaces, etc. Arrangements and intersection properties of sets of points, lines, hyperplanes etc have been studied for many years for a variety of reasons. From the generator matrix of a linear code, in either the hamming or rank metric, we can use the columns to define geometric objects in an appropriate projective geometry, and intersection properties of these objects with hyperplanes are then related to the weights of codewords. The correspondence between codes in the Hamming metric and (multi)sets of points in a projective geometry is well known and has been studied for many years; in particular the correspondence between MDS codes and arcs. The correspondence between (vector) codes in the rank metric and linear sets is more recent. In this lecture we will introduce these two correspondences, and illustrate how Galois Geometry and Coding Theory feed off each other, with results and techniques from one informing the other.