DEVELOPING INNOVATIVE FRAMEWORKS FOR EFFICIENT CODE-BASED SIGNATURES

Edoardo Persichetti

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In This Talk

- Introduction
- A Look into the Past
- New Frameworks
- Conclusions
Part I

INTRODUCTION
In a few years time large-scale quantum computers might be reality. But then (Shor, '94):

- RSA
- DSA
- ECC
- Diffie-Hellman key exchange
- and many others ... **not secure**!

→ NIST’s Post-Quantum Cryptography Standardization Call (2017).

Main areas of research:

- Lattice-based cryptography.
- Hash-based cryptography.
- **Code-based cryptography.**
- Multivariate cryptography.
- Isogeny-based cryptography.
Motivation

Code-based cryptography has been doing really well for encryption/key establishment.

3 finalists in NIST’s process:
- Classic McEliece (binary Goppa)
- BIKE (QC-MDPC)
- HQC (QC Random Codes)

The same cannot be said for code-based signatures.

Only 4 NIST submissions, all either broken or withdrawn.

Yet, signature schemes are a crucial component in cryptography.

Can we fix this?
In general, it is hard to decode random codes.

**Problem (General Decoding)**

*Given:* $G \in \mathbb{F}_q^{k \times n}$, $y \in \mathbb{F}_q^n$ and $w \in \mathbb{N}$.

*Goal:* find a word $e \in \mathbb{F}_q^n$ with $\text{wt}(e) \leq w$ such that $y - e = x \in C_G$.

Easy to see this is equivalent to the following.

**Problem (Syndrome Decoding)**

*Given:* $H \in \mathbb{F}_q^{(n-k) \times n}$, $y \in \mathbb{F}_q^{(n-k)}$ and $w \in \mathbb{N}$.

*Goal:* find a word $e \in \mathbb{F}_q^n$ with $\text{wt}(e) \leq w$ such that $He^T = y$.

NP-Complete (Berlekamp, McEliece and Van Tilborg, 1978; Barg, 1994).

Unique solution when $w$ is below a certain threshold.

Very well-studied, solid security understanding (ISD).
Choose a code family with efficient decoding algorithm associated to description $\Delta$ and hide the structure.

To get trapdoor, need one more ingredient.

**Assumption (Code Indistinguishability)**

*It is possible to describe an error-correcting code via a matrix $M$ which is indistinguishable from a randomly generated matrix of the same size.*

Example: use change of basis $S \in \text{GL}(k, q)$ and permutation $P \in S_n$ to obtain equivalent code.

Hardness of assumption depends on chosen code family.
Part II

A Look into the Past
Use the traditional SDP-based trapdoor within hash-and-sign framework as in e.g. Full Domain Hash (RSA).

Given message $msg$, trapdoor OW function $f$ and hash function $H$.

Create signature $\sigma = f^{-1}(H(msg))$. Verify if $f(\sigma) = H(msg)$.

For CBC, trapdoor is decoding: CFS scheme.
(Courtois, Finiasz, Sendrier, 2001)

...except, domain is not “full”.

Complex sampling leads to slow signing, large keys and potential weaknesses.
(Bleichenbacher, 2009; Faugère Gauthier-Umana, Otmani, Perret, Tillich, 2013; Landais, Sendrier, 2012; Bernstein, Chou, Schwabe, 2013)

Recent renditions still exhibit very similar features.
(Debris-Alazard, Sendrier, Tillich, 2018)
An interactive protocol to prove knowledge of a secret...

...without revealing anything about it.

Correctness: honest prover always gets accepted.

Soundness: dishonest prover (impersonator) has a bounded probability of succeeding.

Zero-Knowledge: no information about the secret is leaked.
ZKIDs can be turned into signature schemes using Fiat-Shamir transformation.

- Replace verifier’s challenge with $H(com, msg)$.
- Form signature as $\sigma = (com, rsp)$.
- Verify as in identification protocol.

This method for building signatures is very promising and usually leads to efficient schemes.

(Schnorr, 1989;...)

Strong security guarantees. No trapdoor is required!

For CBC, can avoid decoding: rely directly on SDP.

Use random codes and exploit hardness of finding low-weight words.

(Stern, 1993)
Stern’s ZKID Protocol

Select hash function $H$.

**Key Generation**
- Choose random binary code $C$, given by parity-check matrix $H$.
- SK: $e \in F_2^n$ of weight $w$.
- PK: the syndrome $s = He^T$.

**Prover**
Choose $y \in F_2^n$ and permutation $\pi$.
Set $c_1 = H(\pi, Hy^T)$, $c_2 = H(\pi(y))$
$c_3 = H(\pi(y + e))$

\[c_1, c_2, c_3 \xrightarrow{b} \]

If $b = 0$ set $rsp = (y, \pi)$
If $b = 1$ set $rsp = (y + e, \pi)$
If $b = 2$ set $rsp = (\pi(y), \pi(e))$

**Verifier**
Select random $b \in \{0, 1, 2\}$.
Verify $c_1, c_2$.
Verify $c_1, c_3$.
Verify $c_2, c_3$ and $wt(\pi(e)) = w$. 
High soundness error implies that adversary has non-trivial cheating probability; for Stern’s scheme, soundness error is 2/3.

This means several repetitions are necessary to amplify error and reach target authentication level.

Transmitting the entire transcript produces a very long signature (e.g. $\geq 100$ kB).

Several variants proposed over the years:

- Véron, 1996.
- Aguilar, Gaborit, Schrek, 2011.
- ...

Goal: decreasing soundness error. For example, CVE scheme achieves $\frac{q}{2(q - 1)} \approx 1/2$. Efficient for large finite fields.
Part III

New Frameworks
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We can use this in CBC! For example, apply this to CVE setting.

(Gueron, P., Santini, 2020)
GPS Protocol

KeyGen: as in CVE, usual syndrome \( s \), matrix \( H \).

**Helper**

- Generate random \( y, \tilde{e} \in \mathbb{F}_q^n \), with \( \tilde{e} \) of weight \( w \), from seed.
- Compute \( aux = \{ \text{Com}(y + c\tilde{e}) \}_{c \in \mathbb{F}_q} \).
- Send seed to prover and aux to verifier.

**Prover**

Regenerate \( y, \tilde{e} \) from seed.
Determine \( \mu \) s.t. \( e = \mu(\tilde{e}) \)
\[
\alpha = \text{Com}(\mu, H(\mu(y))^T)
\]

Select random \( c \in \mathbb{F}_q \).
\[
z = y + c\tilde{e}
\]

Verify \( \alpha = \text{Com}(\mu, H(\mu(z))^T - cs) \).
Verify \( \text{Com}(z) \) with corresponding value from \( aux \).

Here the soundness error is \( 1/q \).
Use “cut-and-choose” technique to remove preprocessing.
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Several optimizations are possible (e.g. Merkle trees, seed trees).
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Can potentially yield smaller signatures, at the cost of increased computation (signing/verification time).
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GPS scheme parameters ($\lambda = 128$, sizes in kB):

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<tr>
<td>512</td>
<td>23</td>
<td>128</td>
<td>220</td>
<td>101</td>
<td>90</td>
<td>0.10</td>
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<td>1024</td>
<td>19</td>
<td>256</td>
<td>207</td>
<td>93</td>
<td>90</td>
<td>0.11</td>
<td>23.98</td>
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<td>2048</td>
<td>16</td>
<td>512</td>
<td>196</td>
<td>92</td>
<td>84</td>
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Observation: if \( H = (H'|I_{n-k}) \) write \( e = (e_A, e_B) \), so \( s = H(e_A, e_B)^T \).

Then \( e_A \) uniquely determines \( e \) given \( s \) and \( H \).
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Let $\mathbb{F}_q \subset \mathbb{F}_{\text{poly}}$ such that $n \leq |\mathbb{F}_{\text{poly}}|$ and take distinct $\gamma_1, \ldots, \gamma_n \in \mathbb{F}_{\text{poly}}$. 
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Then $\deg(Q) = w$ and $\wt(e) \leq w$ is equivalent to

$$Q \cdot S - P \cdot F = 0$$

where $F = \prod_{i=1}^{n}(X - \gamma_i)$ and $\deg(P) \leq w - 1$. 

This transforms SDP into a polynomial problem and completely avoids the need for an isometry.

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Write $Q(X) = \sum_{j=1}^{M} Q^{(j)}(X)$ and then $(P \cdot F)(X) = \sum_{j=1}^{M} (P \cdot F)^{(j)}(X)$. 
Proof of Knowledge (in a nutshell)

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To verify that $Q(X)S(X) = (P \cdot F)(X)$, check $Q(r_l)S(r_l) = (P \cdot F)(r_l)$ for $1 \leq l \leq t$ and $r_l$ elements of an extension field $\mathbb{F}_{\text{points}}$ of $\mathbb{F}_{\text{poly}}$ (Schwartz-Zippel lemma).
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This is done directly on shares \( Q^{(j)}(r_l), S^{(j)}(r_l) \) and \( (P \cdot F)^{(j)}(r_l) \), via standard MPC techniques to verify multiplication triple.
Signature scheme obtained via usual means (cut-and-choose, repetition, Fiat-Shamir).

Performance is extremely competitive!

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Possible improvement using linear complexity to avoid interpolation. (P., Randrianarisoa, 2022)

$q$-ary parameters can be refined, leading to improved performance (e.g. $\text{Sig} \approx 7$ kB).

Optimized implementation underway.
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(P., Randrianarisoa, 2022)

$q$-ary parameters can be refined, leading to improved performance (e.g. Sig $\approx$ 7 kB).
Signature scheme obtained via usual means (cut-and-choose, repetition, Fiat-Shamir).

Performance is extremely competitive!

Scheme parameters ($\lambda = 128$, sizes in kB):

<table>
<thead>
<tr>
<th>$M$</th>
<th>$\tau$</th>
<th>$q$</th>
<th>$n$</th>
<th>$k$</th>
<th>$w$</th>
<th>$\mathbb{F}_{\text{poly}}$</th>
<th>$\mathbb{F}_{\text{points}}$</th>
<th>PK</th>
<th>Sig</th>
</tr>
</thead>
<tbody>
<tr>
<td>256</td>
<td>17</td>
<td>2</td>
<td>1280</td>
<td>640</td>
<td>132</td>
<td>$2^{11}$</td>
<td>$2^{22}$</td>
<td>0.96</td>
<td>11.2</td>
</tr>
<tr>
<td>256</td>
<td>17</td>
<td>$2^8$</td>
<td>256</td>
<td>128</td>
<td>80</td>
<td>$2^8$</td>
<td>$2^{24}$</td>
<td>0.15</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Possible improvement using linear complexity to avoid interpolation.

(P., Randrianarisoa, 2022)

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Optimized implementation underway.
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Group action structure allows to achieve advanced functionalities (e.g. identity-based, ring signatures). (Barenghi, Biasse, Ngo, P., Santini, 2022)
Public data: hash function $H$, code $C$ with generator $G$

**Key Generation**
- SK: invertible matrix $S$ and monomial matrix $Q$.
- PK: matrix $G' = SGQ$ (can be systematic form).

**Prover’s Computation**
- Choose random monomial matrix $\tilde{Q}$.
- Set $\tilde{G} = SystForm(G\tilde{Q})$ and $h = H(\tilde{G})$.
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- If $b = 0$ respond with $\tau = \tilde{Q}$.
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**Verifier’s Computation**
- If $b = 0$ verify that $H(SystForm(G\tau)) = h$.
- If $b = 1$ verify that $H(SystForm(G'\tau)) = h$. 
Part IV

Conclusions
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Explore the connection between codes and other post-quantum areas; isometry-based crypto?
Grazie, Danke, Merci, Grazcha, Thank you and Congratulations to Joachim!