



## **Code-based digital signatures: state of the art and open challenges**

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# The NIST post-quantum standardization process

- In 2016 **NIST** has initiated a process for the development and standardization of one or more post-quantum public-key cryptographic algorithms.
  - 69 submissions in the 1° round
  - 26 admitted to the 2° round
  - 7 finalists and 8 alternates in the 3° round



## Selected for standardization

### Public-Key Encryption/KEMs:

- CRYSTALS-KYBER

### Digital Signatures:

- CRYSTALS-Dilithium
- Falcon
- SPHINCS+

- PKE/KEMs admitted to the 4° round: BIKE, Classic McEliece, HQC, SIKE
- Just a few days after its admission to the fourth round, SIKE has been attacked<sup>1</sup>.
- Rainbow has been the subject of a destructive key recovery attack although it had been admitted to the third round<sup>2</sup>.

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## Post-quantum digital signatures

- **Dustin Moody**, NIST PQC team, 5 July 2022:

...NIST also plans to issue a new Call for Proposals for public-key (quantum-resistant) digital signature algorithms by the end of summer 2022. NIST is primarily looking to diversify its signature portfolio, so signature schemes that are not based on structured lattices are of greatest interest. NIST would like submissions for signature schemes that have short signatures and fast verification (e.g., UOV). Submissions in response to this call will be due by June 1, 2023.

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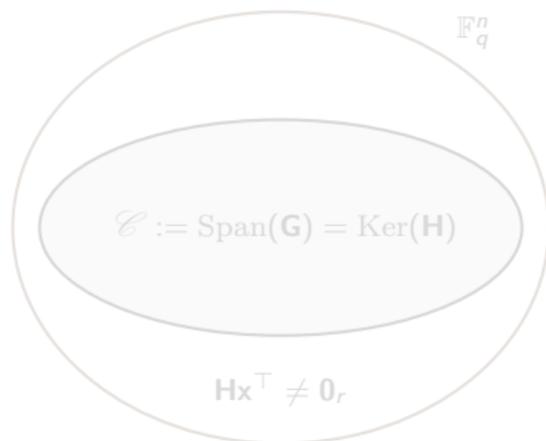
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# Linear codes

- A linear code  $\mathcal{C} \subseteq \mathbb{F}_q^n$  of length  $n$  and dimension  $k$  is a linear  $k$ -dimensional subspace of  $\mathbb{F}_q^n$ .

- Code parameters:

- $n$ : code length;
- $k$ : code dimension;
- $r = n - k$ : code redundancy;
- $R = k/n$ : code rate.



- Representations of a linear code:

- generator matrix  $\mathbf{G} \in \mathbb{F}_q^{k \times n}$ , s.t.  $\mathcal{C} = \{\mathbf{u}\mathbf{G} \mid \mathbf{u} \in \mathbb{F}_q^k\}$ ;
- parity-check matrix  $\mathbf{H} \in \mathbb{F}_q^{r \times n}$ , s.t.  $\mathcal{C} = \{\mathbf{c} \in \mathbb{F}_q^n \mid \mathbf{H}\mathbf{c}^T = \mathbf{0}_r\}$ .

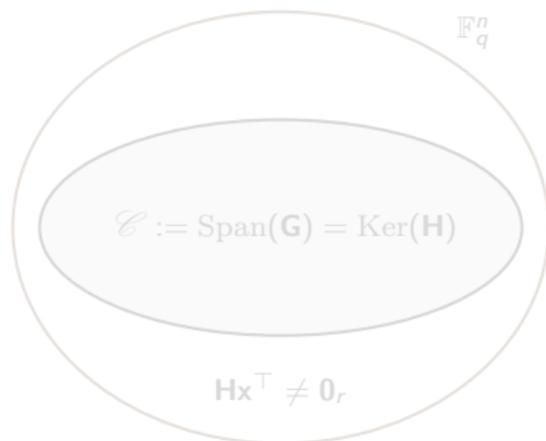
- Hamming weight:  $\text{wt}(\mathbf{a}) = |\{i \text{ s.t. } a_i \neq 0\}|$ .

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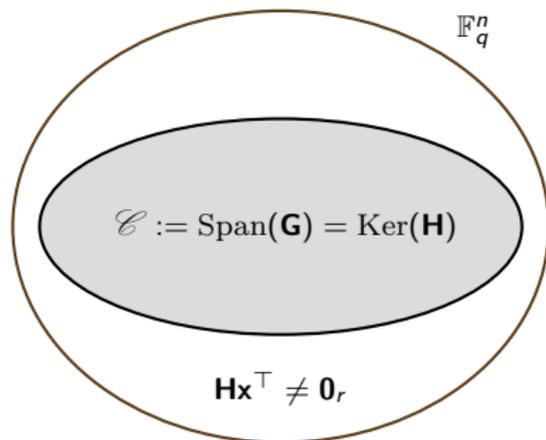
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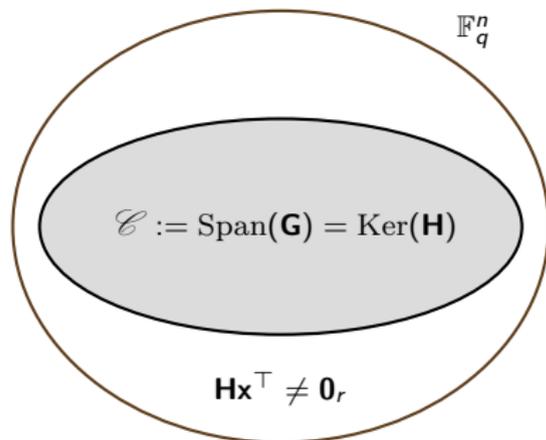
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## Syndrome Decoding Problem

- **Syndrome Decoding Problem (SDP)**: given an arbitrary parity-check matrix  $\mathbf{H} \in \mathbb{F}_q^{r \times n}$  and  $\mathbf{s} \in \mathbb{F}_q^r$ , find  $\mathbf{e}$  with weight  $\leq t$  such that  $\mathbf{s} = \mathbf{H}\mathbf{e}^\top$ .
- For the Hamming metric, SDP is NP-hard.
- For the binary case (i.e.,  $q = 2$ ), when  $t$  is sub-linear in  $n$  the best solver is Information Set Decoding (ISD), with running time

$$T_{ISD} = \mathcal{O}\left(2^{t \cdot \alpha(R)}\right), \quad \alpha(R) = -\log_2(1 - R)$$

- Quantum solver: Grover + ISD, complexity is still exponential in  $t$ :

$$\tilde{T}_{ISD} = \mathcal{O}\left(2^{t \cdot \frac{\alpha(R)}{2}}\right)$$

### Other choices are possible

SDP is hard also for different metrics (rank, Lee ...) and/or weight constraints (large weight ...).

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## Quantum vs signatures

- Quantum computers will endanger many widespread signature schemes (like DSA and RSA signatures).
- Only a few replacements are available up to now (lattice-based and hash-based signatures).
- Code-based digital signatures are quantum-safe...
- But finding efficient code-based solutions is still a challenge!

### Two main approaches to code-based signatures

- Hash-and-sign
- Derived from identification schemes

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  - **Secret key:** error correcting code  $\mathcal{C}$ ;
  - **Public key:** disguised parity-check matrix  $\mathbf{H}$  of  $\mathcal{C}$ .
  - **Signature generation:** compute  $\mathbf{s} = \text{Hash}(m)$  and decode  $\mathbf{s}$  into low weight vector  $\mathbf{e}$ ;
  - **Signature verification:** check that  $\mathbf{e}$  has low weight and  $\mathbf{H}\mathbf{e}^T = \text{Hash}(m)$ .
- However, finding a decodable syndrome is not easy!
  - Number of possible syndromes:  $N_s = q^r$ .
  - Every two vectors with weight  $\leq t$  have distinct syndromes.
  - Number of decodable syndromes:  $N_d = \sum_{i=1}^t \binom{n}{i} (q-1)^i$ .
  - Probability to pick a decodable syndrome is  $N_d/N_s$ : normally,  $N_s \gg N_d$ .
- Example with Goppa codes:  $q = 2$ ,  $r = mt$ ,  $n = 2^m \Rightarrow N_s = n^t$ .  
 $N_d \approx \frac{n^t}{t!}$ .  
Probability of picking a decodable syndrome =  $\frac{N_d}{N_s} \approx \frac{1}{t!}$ .
- However, small  $t$  requires large  $k$ : public key size increases.
- Small  $t$  and large  $k$  imply high code rate: Goppa code distinguishers become efficient.

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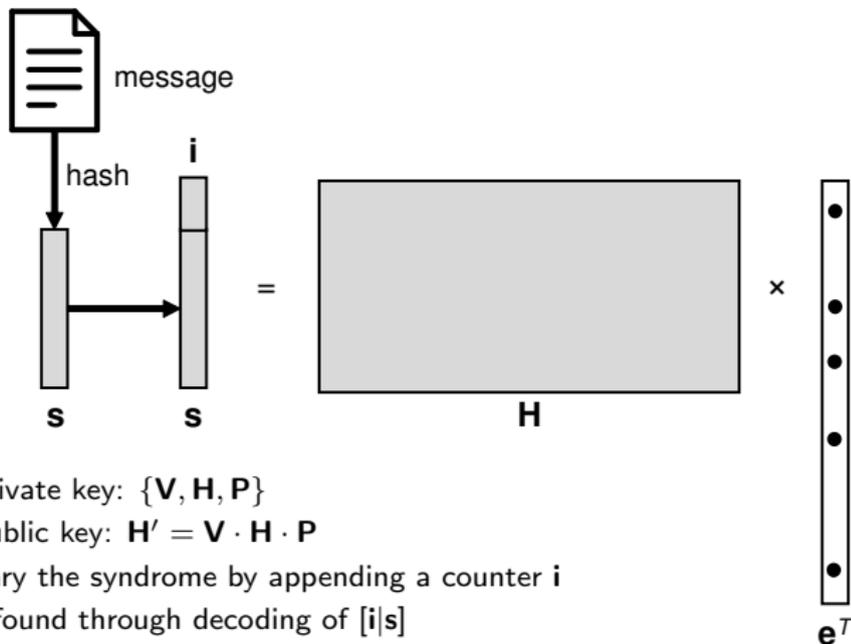
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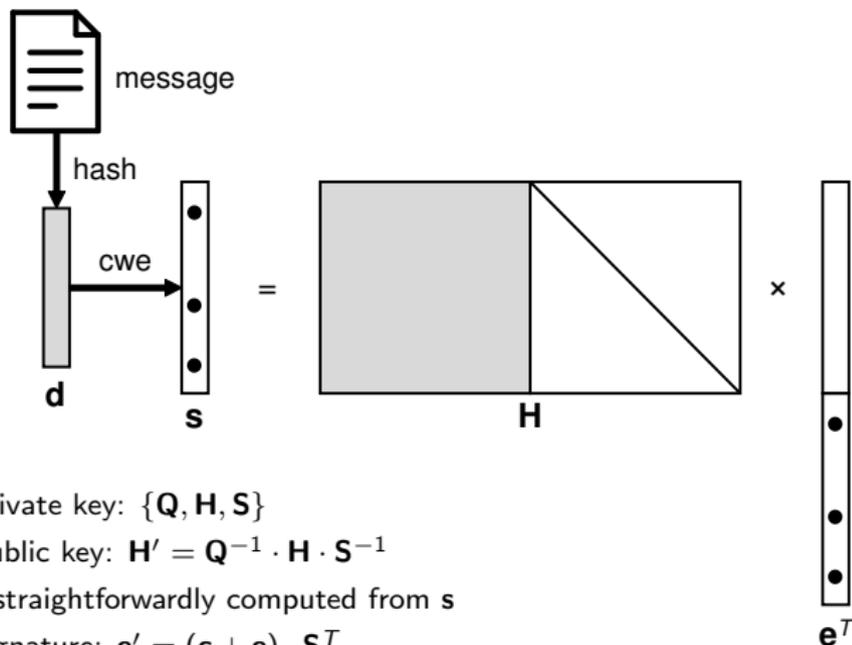
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- Private key:  $\{V, H, P\}$
- Public key:  $H' = V \cdot H \cdot P$
- Vary the syndrome by appending a counter  $i$
- $e$  found through decoding of  $[i|s]$
- Signature:  $e' = e \cdot P$
- Verification:  $[i|s] \stackrel{?}{=} H' \cdot e'^T$

## BBCRS (PQCrypto 2013)



- Private key:  $\{Q, H, S\}$
- Public key:  $H' = Q^{-1} \cdot H \cdot S^{-1}$
- $e$  straightforwardly computed from  $s$
- Signature:  $e' = (c + e) \cdot S^T$
- Verification:  $s \stackrel{?}{=} H' \cdot e'^T$

► M. Baldi, M. Bianchi, F. Chiaraluce, J. Rosenthal, D. Schipani, "Using LDGM Codes and Sparse Syndromes to Achieve Digital Signatures," Proc. PQCrypto 2013, vol. 7932 of Springer LNCS, pp. 1–15, 2013.

## BBCRS - rationale

- The sparsity of  $\mathbf{s}$  and the systematic form of  $\mathbf{H}$  allow trivially deriving a sparse  $\mathbf{e}$  without decoding
- The random codeword  $\mathbf{c}$  adds noise to  $\mathbf{e}$  and makes the signature stochastic
- If  $\mathbf{S}$  is sparse then the signature  $(\mathbf{c} + \mathbf{e}) \cdot \mathbf{S}^T$  is sparse too

### Pro

Very fast signature generation (no decoding)

### Con

To make the decoding problem hard for low-weight vectors we need sparse signatures and hence:

- a sparse codeword  $\mathbf{c}$  (easy to find using LDGM codes)
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- In the BBCRS scheme all the components of the signature  $(\mathbf{c} + \mathbf{e}) \cdot \mathbf{S}^T$  are sparse and binary
- The signature is obtained as the sum of a few rows of  $\mathbf{S}^T$
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- Collect many signatures (100'000)
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- A. Phesso, JP. Tillich, "An Efficient Attack on a Code-Based Signature Scheme," Proc. PQCrypto 2016, vol. 9606 of Springer LNCS, pp. 86–103, 2016.

# From BBCRS to SPANSE

## How to avoid the attack against BBCRS?

- Restrict to one-time use (at least for the moment ☺)
- Possibly avoid sparsity of signatures, but how?
  - Replace the sparse binary  $\mathbf{S}$  with a dense one over  $\mathbb{F}_q$
  - Rely on the hardness of large-weight vector decoding over  $\mathbb{F}_q$

## Zero-concentrated $\mathbf{S}$

$\mathbf{S}$  no longer sparse, but dense with the constraint of having small-valued entries taken from  $\mathbb{F}_q$ .

- $d_i$ : fraction of entries equal to  $i, i = 0, 1, 2, \dots, q - 1$ , in each row of  $\mathbf{S}$ .
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## Security level and key size

- Public key size:  $K_s = 2436.6$  kB
- Number of different signatures:  $N_s = 2^{263.9}$
- Number of different codewords:  $N_c = 2^{133.8}$
- ISD work factor:  $2^{132}$
- PGE+SS work factor:  $2^{131.6}$

## Another approach to code-based signatures

### Hash-and-sign code-based digital signatures

- Classic approach (McEliece)
- Rely on a trapdoor based on some hidden structure
- Key recovery attacks may target such a hidden structure

### Code-based signatures derived from ID schemes

- Alternative approach
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## Identification schemes

### Phase 1

The prover randomly generates a pair  $(sk, pk)$

### Phase 2

The prover exchanges messages with the verifier, who is only equipped with  $pk$ , with the goal of demonstrating knowledge of  $sk$

### Decision

The verifier decides whether to accept the prover or not

Properties:

- 1 zero-knowledge: no information about the secret key is revealed during the identification process
- 2 completeness: an honest prover always gets accepted
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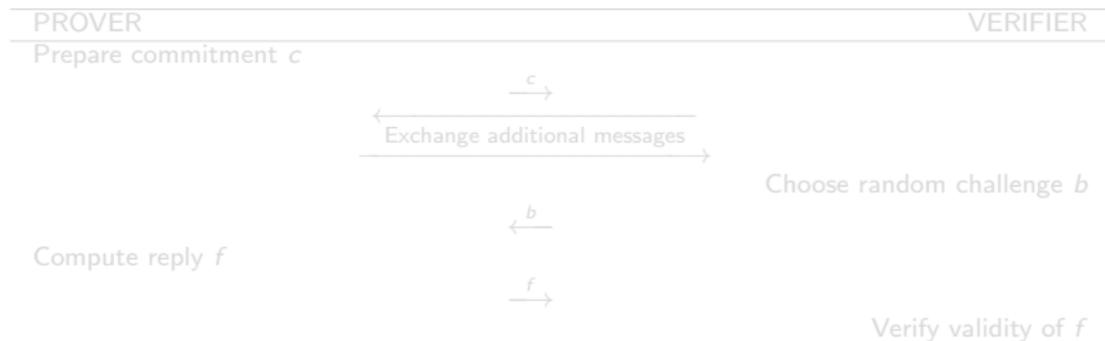
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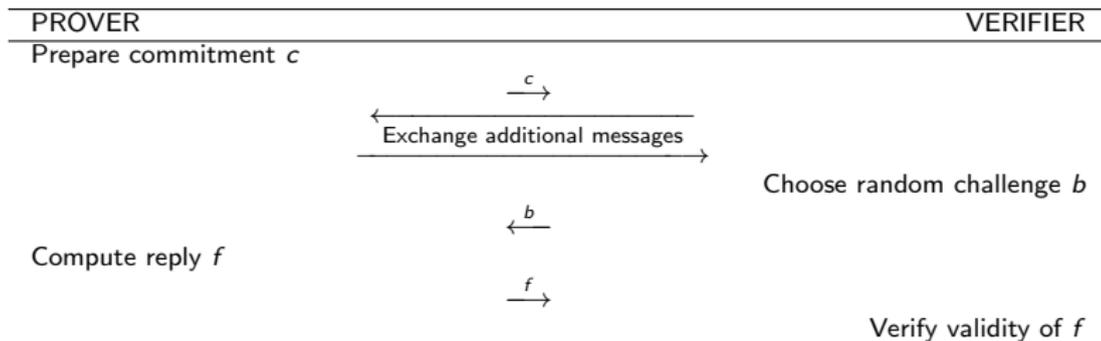
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- Single round interaction between prover and verifier:



- The honest prover can always reply correctly, an adversary is able to reply with some **cheating probability**  $\delta$ . With  $N$  rounds, the cheating probability gets reduced to  $\delta^N$ .

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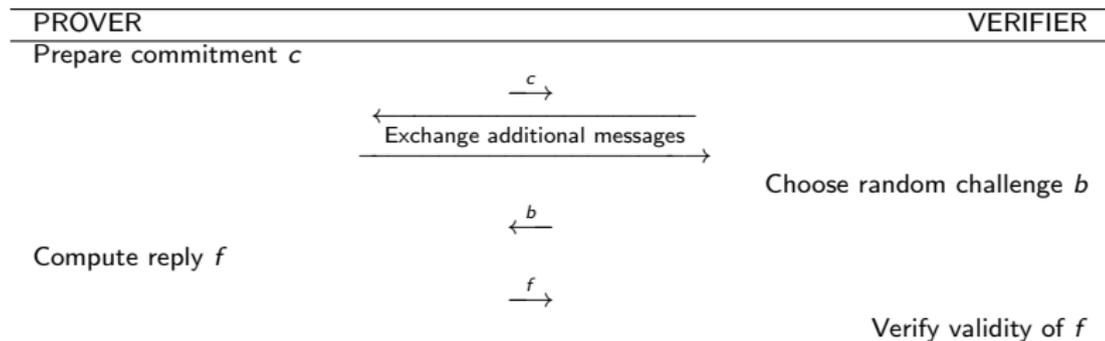
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# Code-based identification schemes

- **Stern's scheme:**
    - random binary codes
    - 3 passes
    - cheating probability =  $2/3$
  - **Veron's scheme:**
    - dual version of Stern's scheme
    - lower communication cost
  - **CVE scheme:**
    - random codes over  $\mathbb{F}_q$
    - 5 passes
    - cheating probability =  $\frac{q-1}{2q}$
  - **AGS scheme:**
    - random binary quasi-cyclic codes
    - 5 passes
    - cheating probability  $\approx 1/2$
    - compression of communications
- 
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## Signatures from identification schemes

- Identification schemes can be rendered non-interactive
  - The Fiat-Shamir transformation obtains a signature scheme as the transcript of one execution of a 3-pass identification scheme
  - This requires fixing the list of challenges by deriving them from some sort of seed
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## Examples of signatures from ID schemes

- Security level:  $\lambda = 128$  bits
- Cheating probability:  $2^{-128}$
- Signature size = communication cost of the underlying identification scheme

	CVE	AGS	R-CVE
Number of rounds	129	128	135
Public key size (bits)	832	1574	305
Average signature size (kB)	43.263	41.040	23.201
Max signature size (kB)	51.261	56.992	22.484

- These schemes follow the classical approach and can be optimized with several modern tricks (e.g., MPC-in-the-head, seed trees), which do not affect security.
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# Identification schemes based on code equivalence

- **Code Equivalence Problem (CEP):** given  $\mathcal{C}$  and  $\mathcal{C}'$ , find an isometry  $\tau$  such that  $\mathcal{C}' = \tau(\mathcal{C})$ .
  - Hard-to-solve instances exist:
    - permutations and self dual codes;
    - monomials and random codes over  $\mathbb{F}_q$  with  $q \geq 5$ .
  - LESS: first cryptographic scheme based on code equivalence:
    - 3-pass code-based ID scheme;
    - soundness error is  $1/2$ ;
  - LESS-FM: follow-up, improved version of LESS:
    - updated and secure parameters;
    - optimized underlying ID scheme;
    - tunable features (e.g., 5.3 kB signatures with 20.5 kB public keys, or 10.4 kB signatures with 11.6 kB public keys).
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## Conclusion

- New post-quantum digital signature schemes are needed
- Lattice-based solutions apparently are the first choice, but diversity is required
- Code-based signatures are post-quantum
- Two main approaches: hash-and-sign and identification-based
- Some promising schemes already exists
- Research is needed to make them (at least) as efficient as lattice-based ones

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