Workshop on Branching-Type Structures

September 3 - 5, 2018

Titles and abstracts

Louigi Addario-Berry (McGill University, Montreal)

Searching for extremes in the continuous random energy model

Write T_n for the rooted binary tree of depth n. Nodes at depth i are indexed by strings $v_1 v_2 \ldots v_i \in \{0,1\}^i$.

Let A be an arbitrary distribution function of a probability distribution on $[0,1]$. Consider a time-inhomogeneous binary branching random walk where the offspring of a generation-i particle have independent centred Gaussian increments with variance $nA(i+1)/n) - nA(i/n)$. The *continuous random energy model* (or CREM) with parameters A and n is the resulting centred Gaussian process $(X_v)_{v \in T_n}$. It is characterized by the covariance formula $\mathbb{E}[X_v X_{v'}] = nA(R_n(v,v'))$, where $R_n(v,v') = \frac{1}{n}\max\{i : v_i = v'_i\}$.

We describe the algorithmic threshold, under a natural computational model, for finding nodes $v \in T_n$ for which X_v is large. More precisely, we show that for quite general functions A, there is $c^* = c^*(A)$ such that, as $n \to \infty$, it is possible to find v with $X_v > (c^* - o(1))n$ in linear time, whereas it is statistically impossible to find v with $X_v > (c^* + o(1))n$ in subexponential time.

Joint work with Pascal Maillard.

Elie Aïdékon (Université Pierre et Marie Curie, Paris)

Points of infinite multiplicity of a planar Brownian motion

Abstract: Points of infinite multiplicity are particular points which the Brownian motion visits infinitely often. Following a work of Bass, Burdzy and Khoshnevisan, we construct and study a measure carried by these points.

Joint work with Yueyun Hu and Zhan Shi.

Gerold Alsmeyer (Universität Münster)

The smoothing transform in random environment

Given a supercritical Galton-Watson tree in i.i.d. random environment $\mathbf{e}_0 := (e_0, e_1, \ldots)$, there is a natural mean one martingale obtained by quenched normalization of the generation size. It converges almost surely to a limit the quenched law of which depends on $\mathbf{e}_0$. Due to self-similarity, for any individual v in the tree, one can obtain in the same way the limit of the corresponding subtree martingale, where the subtree is uniquely determined by having v as its root. If v belongs to generation n, then the quenched law of this limit depends on $\mathbf{e}_n = (e_n, e_{n+1}, \ldots)$. The family of all these limits is connected by a familiar object, called *smoothing transform*, which maps a law P on $[0,\infty)$ to the law of

$$\sum_{i\geq 1} T_i X_i$$

where $T=(T_1,T_2,\ldots)$ denotes a sequence of given nonnegative random weights and $X_1,X_2,\ldots$ are i.i.d. with law P and independent of T. However, in the present context of a random environment, this mapping is random in the quenched regime. The purpose of this talk is to provide further information on the specification of the model, followed by the definition of the notion of a fixed point which here turns out to be a stochastic kernel. In the second part I will show some results regarding the set of all such fixed points for a smoothing transform in random environment.

Julien Berestycki (University of Oxford)

Branching Brownian motion with decay of mass and the non-local Fisher-KPP equation

The non-local variant of the celebrated Fisher-KPP equation describes the growth and spread of population in which individuals diffuse, reproduce and - crucially - interact through a non-local competition mechanism. This type of equation is intrinsically harder to study than the classical Fisher-KPP equation because we lose such powerful tools as the comparison principle and the maximum principle. In this talk, I will show how this equation arises as the hydrodynamic limit of a particle system -the branching Brownian motion with decay of mass, and use this to study front propagation behaviours.

This is based on joint work with Louigi Addario-Berry and Sarah Penington.

Anton Bovier and Lisa Hartung (Universität Bonn)

From Poisson to the cascade in branching Brownian motion – a closer look

Branching Brownian motion is a classical process in probability and member of a wider class of "log-correlated fields". It is also a special and very particular case of larger class of Gaussian processes indexed by trees that can be parametrised by a covariance (or "speed") function $A;[0,1]\to[0,1]$ that allows to interpolate between the iid case ($A(x)=0, x\in[0,1).A(1)=1$) and constant process $A(x)\equiv 1$. BBM corresponds to the case $A(x)=x$ is the borderline case for the behaviour of the extremes of the process. If $A(x)<x$ on $(0,1)$, then the extreme are almost as in the iid case and the order of the maximum is $\sqrt{2}t-\frac{1}{2\sqrt{2}}\ln t$. For BBM, Bramson has shown that the logarithmic correction is three times bigger, and if $A(x)>x$ for some $x\in[0,1]$, then both the linear and the logarithmic term change. While the change of the linear term is continuous in A, the log-corrections change in a discontinuous way. In these talks we look more closely at how this transition happens by considering speed functions A_t depending on time were $A'_t(x)=\sigma_1^2(t)=1\pm t^{-\alpha}, x\leq 1/2$ and $A'_t(x)=\sigma_2^2(t)=1\mp t^{-\alpha}, x>1/2$. In both cases, the maximum behaves as in BBM as long as $\alpha\geq 1/2$. If $\sigma_1^2=1-t^{-\alpha}$, with $\alpha\leq 1/2$, the constant in front of the logarithm becomes $\frac{1+4\alpha}{2\sqrt{2}}$, while in the case $\sigma_1^2=1+t^{-\alpha}$ and $0<\alpha<1/2$, the order of the maximum is $\sqrt{2}\frac{\sigma_1(t)+\sigma_2(t)}{2}t-\frac{6(1-\alpha)}{2\sqrt{2}}\ln t$. We also establish in all cases the asymptotic law of the maximum and characterise the extremal process, which turns out to coincide essentially with that of standard BBM.

The talk will be presented in two parts by the two of us.

Nicolas Curien (Université Paris-Sud Orsay)

Critical parking on a random tree ...and random planar maps!

Imagine a plane tree together with a configuration of particles (cars) at each vertex. Each car tries to park on its node, and if the latter is occupied, it moves downward towards the root trying to find an empty slot. This model has been studied recently by Bruner and Panholzer as well as Goldschmidt and Przykicki where is it shown that parking of all cars obeys a phase transition ruled by the density of cars. We study the annealed critical model of random plane tree together with a parking configuration of cars. Surprisingly this object is connected to stable looptree of parameter 3/2 and to processes encountered in the theory of random planar maps!

The talk is based on ongoing work with Olivier Hénard.

Nina Gantert (Technische Universität München)

Large deviations for the maximum of a branching random walk

We consider real-valued branching random walks and prove a large deviation result for the position of the rightmost particle. The position of the rightmost particle is the maximum of a collection of a random number of dependent random walks. We characterise the rate function as the solution of a variational problem. We consider the same random number of independent random walks, and show that the maximum of the branching random walk is dominated by the maximum of the independent random walks. For the maximum of independent random walks, we derive a large deviation principle as well. It turns out that the rate functions for upper large deviations coincide, but in general the rate functions for lower large deviations do not. As time permits, we also give some results about branching random walks in random environment.

The talks is based on joint work with Thomas Höfelsauer.

Bénédicte Haas (Université Paris 13)

Stable graphs: distribution and line-breaking construction

For $\alpha \in (1, 2]$, the α-stable graph arises as the universal scaling limit of critical random graphs with i.i.d. degrees having a given α-dependent power-law tail behavior. It consists of a sequence of compact measured metric spaces (the limiting connected components), each of which is tree-like, in the sense that it consists of an R-tree with finitely many vertex-identifications (which create cycles). In this talk we will discuss the geometric properties of such a component with given mass and number of vertex-identifications.

Based on a joint work with Christina Goldschmidt (Oxford) and Delphin Sénizergues (Université Paris 13).

Alexander Iksanov (Taras Shevchenko National University, Ukraine)

Nested occupancy schemes in random environments

Let $(P_r)_{r\in\mathbb{N}}$ be a collection of positive random variables satisfying $\sum_{r\geq 1} P_r = 1$ a.s. Assume that, given $(P_r)_{r\in\mathbb{N}}$, 'balls' are allocated independently over an infinite collection of 'boxes' 1, 2, ... with probability P_r of hitting box r, $r \in \mathbb{N}$. The occupancy scheme arising in this way is called the *infinite occupancy scheme in the random environment* $(P_r)_{r\geq 1}$.

A popular model of the infinite occupancy scheme in the random environment assumes that the probabilities $(P_r)_{r\in\mathbb{N}}$ are formed by an enumeration of the a.s. positive points of

$$\{e^{-X(t-)}(1 - e^{-\Delta X(t)}) : t \geq 0\}, \tag{1}$$

where $X := (X(t))_{t\geq 0}$ is a subordinator (a nondecreasing Lévy process) with $X(0) = 0$, zero drift, no killing and a nonzero Lévy measure, and $\Delta X(t)$ is a jump of X at time t. Since the closed range of the process X is a regenerative subset of $[0, \infty)$ of zero Lebesgue measure, one has $\sum_{r\geq 1} P_r = 1$ a.s. When X is a compound Poisson process, collection (1) transforms into a *residual allocation model*

$$P_r := W_1 W_2 \cdot \ldots \cdot W_{r-1}(1 - W_r), \quad r \in \mathbb{N}, \tag{2}$$

where W_1, $W_2, \ldots$ are i.i.d. random variables taking values in $(0, 1)$.

Next, following [1] I define a nested infinite sequence of the infinite occupancy schemes in random environments. This means that I construct a nested sequence of environments (random probabilities) and the corresponding 'boxes' so that the same collection of 'balls' is thrown into all 'boxes'. To this end, I use a weighted branching process with positive weights which is nothing else but a multiplicative counterpart of a branching random walk.

The nested sequence of environments is formed by the weights $(R(u))_{|u|=1} = (P_r)_{r\in\mathbb{N}}$, $(R(u))_{|u|=2}, \ldots$, say, of the subsequent generations individuals in a weighted branching process. Further, I identify individuals with 'boxes'. At time $j = 0$, all 'balls' are collected in the box $\oslash$ which corresponds to the initial ancestor. At time $j = 1$, given $(R(u))_{|u|=1}$, 'balls' are allocated independently with probability $R(u)$ of hitting box u, $|u| = 1$. At time $j = k$, given $(R(u))_{|u|=1}, \ldots, (R(u))_{|u|=k}$, a ball located in the box u with $|u| = k - 1$ is placed independently of the others into the box ur, $r \in \mathbb{N}$ with probability $R(ur)/R(u)$.

Assume that there are n balls. For $r = 1, 2, \ldots, n$ and $j \in \mathbb{N}$, denote by $K_{n,j,r}$ the number of boxes in the jth generation which contain exactly r balls and set

$$K_{n,j}(s) := \sum_{r=\lceil n^{1-s} \rceil}^{n} K_{n,j,r}, \quad s \in [0, 1],$$

where $x \mapsto \lceil x \rceil = \min\{n \in \mathbb{Z} : n \geq x\}$ is the ceiling function. With probability one the random function $s \mapsto K_{n,j}(s)$ is right-continuous on $[0, 1)$ and has finite limits from the left on $(0, 1]$ and as such belongs to the Skorokhod space $D[0, 1]$. I am going to present sufficient conditions which ensure functional weak convergence of $(K_{n,j_1}(s), \ldots, K_{n,j_m}(s))$, properly normalized and centered, for any finite collection of indices $1 \leq j_1 < \ldots < j_m$ as the number n of balls tends to ∞. If time permits, I shall discuss specializations of the general result to $(P_r)_{r \in \mathbb{N}}$ given by (1) and (2).

The talk is based on a work in progress, joint with Sasha Gnedin (London).

References

[**1**] Bertoin J. *Asymptotic regimes for the occupancy scheme of multiplicative cascades.* Stoch. Proc. Appl. 2008, V. 118, P. 1586-1605.

Igor Kortchemski (CMAP, École polytechnique, Palaiseau)

The geometry of random minimal factorizations of a long cycle

We will be interested in the structure of random typical minimal factorizations of the n-cycle into transpositions, which are factorizations of (1,...,n) as a product of n1 transpositions. We shall establish a phase transition when a certain amount of transpositions have been read one after the other. One of the main tools is a limit theorem for two-type Bienaymé-Galton-Watson trees conditioned on having given numbers of vertices of both types, which is of independent interest.

This is joint work with Valentin Féray.

Andreas Kyprianou (University of Bath, Bath)

Stochastic Analysis of the Neutron Transport Equation

We introduce the neutron transport equation (NTE) as seen from the probabilistic perspective and discuss some of its analytical and stochastic properties. In particular we discuss a new adaptation of the NTE incorporating transmutation and non-transumutation radioactivity, we discuss the role of the importance map and introduce the the Many-to-one formula in the context of Monte-Carlo simulation with a view to nuclear regulation.

Oren Louidor (Technion, Haifa)

On the extreme and large-value landscape of the discrete Gaussian free field and friends

I will discuss some new results concerning extreme and large values of the 2D discrete Gaussian free field and related processes. These include finer structural properties of its extremal landscape, scaling limits for its high (but not extreme) level sets and the asymptotic growth of the infinite volume pinned DGFF.

Based on joint work (some in progress) with M. Biskup, A. Cortines, L. Hartung and D. Yeo.

Pascal Maillard (Université Paris Sud)

1-stable fluctuations of branching Brownian motion at critical temperature

In this talk, I will present recent results (obtained with Michel Pain) on fluctuations of certain functionals of branching Brownian motion including the additive martingale with critical parameter and the derivative martingale. We prove non-standard central limit theorems for these quantities, with the possible limits being 1-stable laws with and asymmetry parameter depending on the functional. In particular, the derivative martingale and the additive martingale satisfy such a non-standard central limit theorem with, respectively, a totally asymmetric and a Cauchy distribution.

Robin Stephenson (Department of Statistics, University of Oxford)

Scaling limits of multi-type Markov branching trees, and applications to recursive tree constructions and multi-type Galton-Watson trees

Consider a population where individuals have two characteristics: a size, which is a positive integer, and a type, which is a member of a finite set. This population reproduces in a Galton-Watson fashion, with one additional condition: given that an individual has size n, the sum of the sizes of its children is less than or equal to n. We call multi-type Markov branching tree the family tree of such a population.

As a generalisation of the results of Haas and Miermont on monotype Markov branching trees, we show that under some assumptions (mainly that macroscopic dislocation of a large fragment are rare), Markov branching trees have scaling limits in distributions which are self-similar fragmentation trees, monotype or multi-type.

We then give two applications: the scaling limits of sequences of random trees obtained by some algorithmic constructions, and new results on the scaling limits of multi-type Galton-Watson trees.

This is joint work with Bénédicte Haas.

Vincent Vargas (ENS, Paris)

Exact Formulas in the Theory Gaussian Multiplicative Chaos

The theory of Gaussian multiplicative chaos (GMC) enables to define random measures formally defined by the exponential of a log-correlated field. These measures have applications in numerous fields of probability theory and theoretical physics among which: Liouville field theory, turbulence, random matrix theory or the study of extremes of log-correlated fields. I will report on a series of recent works which aim at proving the first exact formulas for these measures: the DOZZ formula, the Fyodorov-Bouchaud formula, etc... The proofs are based on the introduction of certain (conformal field theoretic) "observables" which satisfy simple PDEs.

Based on works by: Kupiainen, Remy, Rhodes, Vargas, Zhu.

Alexander Watson (University of Manchester)

The growth-fragmentation equation: probabilistic and spectral aspects

The growth-fragmentation equation describes a system of growing and dividing particles, and arises in models of cell division, protein polymerisation and even telecommunications protocols. Several important questions about the equation concern the asymptotic behaviour of solutions at large times: at what rate do they converge to zero or infinity, and what does the asymptotic profile of the solutions look like? Does the rescaled solution converge to its asymptotic profile at an exponential speed? These questions have traditionally been studied using analytic techniques such as entropy methods or splitting of operators. In this talk, I discuss a probabilistic approach to the study of this asymptotic behaviour. The method is based on the Feynman-Kac formula and a related martingale technique.

This is joint work with Jean Bertoin.