STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS TO TERM STRUCTURE PROBLEMS IN MATHEMATICAL FINANCE

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The four lectures of this course provide an introduction to aspects of the theory of SPDEs. Some of those aspects are quite unusual like geometric properties of solution processes of SPDEs, or like numerical implementations for SPDEs. Nevertheless those aspects appear to be useful in mathematical Finance and maybe beyond. Some ideas of this theory trace back more than one hundred years (e.g. the Frobenius Theorem), some ideas and concepts are quite new and refer to very recent results (e.g. the numerical properties).

I try to show inspiring ways towards existence and uniqueness of solutions for SPDEs, I demonstrate qualitative, geometric properties of solution processes, and I try to work on numerical properties, algorithms and concrete implementations for certain SPDEs. A substantial introduction to strongly continuous semigroups on Hilbert spaces, (local) foliation theory on Hilbert spaces and cubature methods for S(P)DEs is provided. As far as possible and useful I shall work with Lévy-driven SPDEs. Examples from mathematical Finance like the Heath-Jarrow-Morton equation from interest rate theory will be the guiding examples for the choice of topics of these four lectures.

The lectures will be accompanied by presentation slides and lecture notes (both available shortly before the summer school starts). I plan the following structure for the lectures:

- Solution concepts for SPDEs, existence and uniqueness of mild solutions through transformation methods, (regular) finite dimensional realizations.
- HJM-equation of interest rate theory, existence of finite dimensional realizations for SPDEs, Frobenius theorem, affine realizations.
- Malliavin Calculus for SPDEs, first variation processes, a Hörmander-type theorem.
- Numerical treatment of SPDEs, general results on weak and strong schemes for SPDEs, cubature methods, implementations for HJM-equations.

I would like to organize at least one open-problem session.

As preparatory material for the geometric theory of SPDEs and some applications to interest rate theory I warmly recommend T. Björk's and L. Svensson's article "On the existence of finite dimensional realizations for non-linear forward rate models" (Math. Finance 11, 2001), where all (except the numerical) aspects of the theory are thoroughly discussed in a framework without technical difficulties.

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