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# Open problems in the area of pole placement

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## 1 Introduction

The static and the dynamic output pole placement problem belong to the prominent design problems of modern control theory and we refer to the survey articles [4, 10, 19, 21] where also more references to the literature are provided. Various facets of the pole placement problem attracted many researchers over the years. It was been recognized right at the beginning of the problem that the output pole placement problem is nonlinear in nature and a simple solution based on techniques from linear algebra cannot be expected. In recent years significant progress has been achieved. This progress is due in a major part to a better understanding of the system theoretic ingredients and its relation to algebraic geometry. Helpful in this regard is the behavioral approach which comes in its formulation closest to the algebraic geometric nature of the pole placement problem.

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Classically, the static output feedback problem is formulated in the following way: Let $A, B, C$ be real matrices of size $n \times n$, $n \times m$, and $p \times n$, respectively. These 3 matrices describe the linear system

$$\frac{d}{dt}x = Ax + Bu, \; y = Cx. \tag{37.1}$$

Let $\phi \in \mathbb{R}[\xi]$ be a monic polynomial of degree $n$. Then the static pole placement problem asks for conditions which guarantee the existence of a $m \times p$ matrix $K$ such that

$$\det(\xi I - A - BKC) = \phi(\xi).$$

After elimination of the latent variable $x$, equation (37.1) describes a linear differential system. This elimination can be done as follows in the controllable case. If $D(\xi)^{-1}N(\xi)$ is a left coprime factorization of the transfer function $C(\xi I - A)^{-1}B$ then the manifest behavior is described by

$$\begin{pmatrix} D\left(\frac{d}{dt}\right) & -N\left(\frac{d}{dt}\right) \end{pmatrix} \begin{pmatrix} y \\ u \end{pmatrix} = 0. \tag{37.2}$$

It is easy to verify that the design of a static compensator is equivalent to the construction of matrices $K_1, K_2$ such that

$$\det \begin{bmatrix} D(\xi) & -N(\xi) \\ K_1 & K_2 \end{bmatrix} = \phi(\xi). \tag{37.3}$$

In the next section we summarize some of the major known pole placement results. The essence of the problems and their solutions are most transparent in the behavioral language. For the connection between the behavioral point of view and the classical state space formulation we refer to [24, 25]. The following development of the theory follows closely the description given in [18].

## 2 A summary of known pole placement results

Recall from [24] that a *dynamical system* $\Sigma$ is a triple $\Sigma = (T, W, \mathfrak{B})$, where $T \subset \mathbb{R}$ is the time axis, $W$ is the signal space and $\mathfrak{B} \subset W^T$ is called the behavior. We restrict attention sequel to dynamical systems $\Sigma$ with time axis $T = \mathbb{R}$, signal space $W = \mathbb{R}^{m+p}$, and behavior $\mathfrak{B} \subset C^\infty(\mathbb{R}, \mathbb{R}^{m+p})$ specified by a so called *kernel representation*, i.e. there exist a polynomial matrix $P$ such that

$$\mathfrak{B} = \{w \in C^\infty(\mathbb{R}, \mathbb{R}^{m+p}) \mid P\left(\frac{d}{dt}\right) w = 0\}. \tag{37.4}$$

Systems in this class are called *linear differential systems*. Such a behavior $\mathfrak{B}$ admits many kernel representations (37.4) (see [24]). Two important invariants are the *rank* $r(\Sigma)$ and the *McMillan degree* $n(\Sigma)$, which are defined as follows: $r(\Sigma)$ is equal to the rank of a polynomial matrix $P$ which describes the behavior $\mathfrak{B}$ by a representation of the form (37.4). We will call a representation $P$ *(row) minimal* if the matrix $P$ has full row rank. If the polynomial matrix $P$ is row minimal then we define the McMillan degree $n(\Sigma)$ as the maximal degree of the full size minors in one (and therefore any) minimal representation.

A system $\Sigma = (\mathbb{R}, \mathbb{R}^{m+p}, \mathfrak{B})$ is *autonomous* if the rank $r(\Sigma) = m + p$. If the $(m+p) \times (m+p)$ matrix $P$ describes an autonomous behavior then the nonzero polynomial $\det P$ is called the *characteristic polynomial* of $\Sigma$ and will be abbreviated with $\chi_\Sigma$; $\chi_\Sigma$ is a projective invariant of the autonomous behavior, i.e. if

$$\det P(\xi) = a_0 + a_1\xi + \cdots + a_n\xi^n$$

then $(a_0, \ldots, a_n)$ defines a unique one dimensional subspace of polynomials, i.e. a point in the projective space $\mathbb{P}^n$. This point then only depends on the autonomous system $\Sigma$ and not on the particular representation. The roots of $\det P$ are by definition the *poles* of $\Sigma$.

Control in the behavioral theory is defined in the following way: Assume that $\Sigma_1 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathfrak{B}_1)$ and $\Sigma_2 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathfrak{B}_2)$ are two linear differential systems. Then the *interconnected system* $\Sigma_1 \wedge \Sigma_2$ is defined as:

$$\Sigma_1 \wedge \Sigma_2 := (\mathbb{R}, \mathbb{R}^{m+p}, \mathfrak{B}_1 \cap \mathfrak{B}_2).$$

We say $\Sigma_1 \wedge \Sigma_2$ is a *regular* or *independent* interconnection (see [25]) if the ranks "add up", i.e. if

$$r(\Sigma_1 \wedge \Sigma_2) = r(\Sigma_1) + r(\Sigma_2)$$

and we speak of a *singular* or *dependent* interconnection otherwise. As it is immediate from the definition the interconnected system $\Sigma_1 \wedge \Sigma_2$ is represented by

$$\begin{pmatrix} P_1\left(\frac{d}{dt}\right) \\ P_2\left(\frac{d}{dt}\right) \end{pmatrix} w = 0,$$

where $P_1, P_2$ are polynomial matrices representing $\Sigma_1$ respectively $\Sigma_2$. If $P_1, P_2$ are in addition row minimal representations then one verifies that $\Sigma_1 \wedge \Sigma_2$ is a regular interconnection if and only if $\binom{P_1}{P_2}$ is row minimal.

The fundamental question, which is in fact a generalized pole placement problem and which implies many of the "traditional" pole placement questions, is now as follows:

**Problem 1** Let $m, p, n, q$ be fixed positive integers. Under what condition is it true that for a generic set of linear differential systems $\Sigma_1 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathfrak{B}_1)$ having rank $r(\Sigma_1) = p$ and McMillan degree $n(\Sigma_1) = n$ the following holds: For every polynomial $\phi \in \mathbb{R}[\xi]$ of degree $n+q$ there exists a system $\Sigma_2 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathfrak{B}_2)$ having rank $r(\Sigma_2) = m$ and McMillan degree $n(\Sigma_2) = q$ such that $\Sigma_1 \wedge \Sigma_2$ forms a regular interconnection having characteristic polynomial $\chi_{\Sigma_1 \wedge \Sigma_2} = \phi$.

Problem 1 is the behavioral formulation of the dynamic pole placement problem. Closely related is the following formulation, which can be done for an arbitrary base field $\mathbb{F}$:

**Problem 2** Let $\mathbb{F}$ be a field and let $m, p, n, q$ be fixed positive integers. Let $P_1$ be a $p \times (m+p)$ matrix with entries in $\mathbb{F}[\xi]$ whose $p \times p$ minors have degree at most $n$. Let $\phi(\xi) \in \mathbb{F}[\xi]$ be a polynomial of degree $n+q$. Under what conditions does there exist a $m \times (m+p)$ matrix $P_2$ with entries in $\mathbb{F}[\xi]$ such that the $m \times m$ minors of $P_2$ have degree at most $q$ and

$$\det \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \phi?$$

If Problem 2 has a positive answer for a 'generic set' of $p \times (m+p)$ matrices of degree $n$ then one says that the generic rank $p$ system of McMillan degree $n$ is *arbitrary pole assignable* in the class of feedback compensators of McMillan degree $q$. If $q = 0$ one says that the generic rank $p$ system of McMillan degree $n$ is arbitrary pole assignable by static feedback compensators. Of course it is a major difficulty to make the notion of genericity precise in this formulation and we refer to [18] for details on this question.

## 2.1 Static pole placement results

The major results in the area of static pole placement are as follows:

**Theorem 3 (Brockett and Byrnes [3])** *If the base field $\mathbb{F}$ is algebraically closed and if $mp \geq n$ then the generic rank $p$ system of McMillan degree $n$ is arbitrary pole assignable by static feedback compensators. Moreover if $mp = n$ then (counting multiplicity) the number of non-equivalent feedback compensators assigning a particular closed loop characteristic polynomial is independent of the closed loop polynomial $\phi(\xi) \in \mathbb{F}[\xi]$ and is equal to*

$$d(m,p) = \frac{1!2! \cdots (p-1)!(mp)!}{m!(m+1)! \cdots (m+p-1)!} \tag{37.5}$$

Since $mp \geq n$ is easily seen to be a necessary condition, Theorem 3 gives the best possible bound when the base field $\mathbb{F}$ is algebraically closed.

The number $d(m,p)$ is the degree of the Grassmann variety, which was computed in the last century by Schubert [20]. Although Theorem 3 assumes that the base field $\mathbb{F}$ is algebraically closed it does also provide some results for real pole assignment:

**Corollary 4** *If $\mathbb{F} = \mathbb{R}$, $mp = n$, and $d(m,p)$ is odd, then the generic rank $p$ system of McMillan degree $n$ is arbitrary pole assignable by static real feedback compensators.*

Berstein determined when $d(m,p)$ is odd.

**Proposition 5 (Berstein [1])** *The number $d(m,p)$ is odd if and only if* $\min(m,p) = 1$ *or*
$\min(m,p) = 2$ *and* $\max(m,p) = 2^t - 1$, *where $t$ is a positive integer.*

When $d(m,p)$ is even, the best known sufficiency result over the reals is due to Wang:

**Theorem 6 (Wang [22])** *If $\mathbb{F} = \mathbb{R}$ and $mp > n$, then the generic rank $p$ system of McMillan degree $n$ is arbitrary pole assignable by static feedback compensators.*

In the last 3 years several elementary proofs of Wang's theorem were given [9, 15, 16, 26].

When the Schubert number $d(m,p)$ is even, there is a difference of one degree of freedom between the sufficiency condition of Wang ($mp > n$) and the general necessary condition ($mp \geq n$). The following lemma states that in general $mp \geq n$ is not a sufficient condition for generic pole placement with real static compensators:

**Lemma 7 (Willems and Hesselink [27])** *If $\mathbb{F} = \mathbb{R}$ and if $m = p = 2$ and $n = 4$ then the static pole placement problem is not generically solvable over the reals $\mathbb{R}$.*

## 2.2 Dynamic pole placement results

Also for the dynamic problem there are some general necessary conditions. A simple dimension argument first carried out in [27] reveals that

$$q(m + p - 1) + mp \geq n \tag{37.6}$$

is a necessary condition for the generic rank $p$ system of McMillan degree $n$ to be arbitrary pole assignable in the class of feedback compensators of McMillan degree $q$. In [13, 14] the sufficiency result of Theorem 3 was extended:

**Theorem 8 (Rosenthal-Ravi-Wang [13, 14])** *If the base field $\mathbb{F}$ is algebraically closed and if $q(m + p - 1) + mp \geq n$ then the generic rank $p$ system of McMillan degree $n$ is arbitrary pole assignable in the class of feedback compensators of degree at most $q$. Moreover if there is equality in* (37.6) *then the number of non-equivalent feedback compensators assigning a particular closed loop characteristic polynomial is independent of the* *closed loop polynomial $\phi(\xi) \in \mathbb{F}[\xi]$ and equals (counting multiplicity) to*

$$d(m,p,q) = (-1)^{q(m+1)}(mp+q(m+p))! \sum_{n_1+\cdots+n_m=q} \frac{\prod_{k<j}(j-k+(n_j-n_k)(m+p))}{\prod_{j=1}^{m}(p+j+n_j(m+p)-1)!} \tag{37.7}$$

Note that the number $d(m,p,0)$ is equal to $d(m,p)$ as introduced in (37.5). As in the case of the static pole placement problem condition (37.6) is also sufficient if $d(m,p,q)$ is odd. If $d(m,p,q)$ is even the strongest known sufficiency result over the reals is:

**Theorem 9 (Rosenthal-Wang [18])** *Let $\mathbb{F} = \mathbb{R}$ and assume that*

$$q(m+p-1) + mp - \min(r_m(p-1), r_p(m-1)) > n \tag{37.8}$$

*where $r_m = q - m[q/m]$ and $r_p = q - p[q/p]$ are the remainders of $q$ divided by $m$ and $p$, respectively. Then the generic rank $p$ system of McMillan degree $n$ is arbitrary pole assignable in the class of real feedback compensators of degree at most $q$.*

# 3 A list of open problems

## 3.1 Necessary and sufficient conditions for generic pole assignment over the reals

It has been conjectured by S.-W. Kim that $m = p = 2$, $n = 4$ is the only case where $mp = n$ is not a sufficient condition for generic pole assignment with static compensators over the reals. This conjecture was disproved by Rosenthal and Sottile in [17] by exhibiting a concrete counterexample in the situation where $m = 2$ and $p = 4$. This lead to the following conjecture:

**Conjecture 10 (Rosenthal-Sottile [17])** *If $d(m,p)$ is even and $n = mp$, then the static pole placement problem is not generically solvable over the reals $\mathbb{R}$.*

This conjecture is open. For the general dynamic problem much less is known and the gap between the general necessary condition (37.6) and the best known sufficiency condition (37.8) is in general quite wide and it can be as large as $(m-1)(p-1)$. It is a challenging task to further narrow this gap, and obtain a sharp bound as Theorem 6. In addition, it is of interest to have exact algebraic criteria which allow to determine the minimal order of a real compensator needed, over the reals. For the static pole placement problem such algebraic conditions have been recently derived in [17].

### 3.2 Numerical algorithms for pole placement

The sufficiency conditions in Theorems 3,6,8 and 9 are mainly theoretical in nature and there are no good numerical algorithms available in many situations where we know the existence of a feedback compensator. Theorem 6 and 9 were derived by a technique called linearization around a 'dependent compensator'. In principle this technique can be used to actually compute feedback compensators (see [18, 23] for more details). Geometrically dependent compensators form the *base locus* (see [14]) of the associated pole placement map and the numerical paradigm should probably be to construct solutions which are as far as possible away from the base locus. In principle it is also possible to tackle any pole placement problem directly through the defining polynomial equations and applying methods like e.g. Gröbner basis computations. Such an approach was taken in [17] but it is clear that computations will be limited to small dimensions.

In summary we consider it a challenge to derive stable numerical algorithms to solve pole placement problems.

### 3.3 The invariant polynomial assignment problem

The characteristic polynomial forms in general not the only invariant of an autonomous behavior. A finer set of invariants are the invariant factors. Two autonomous behaviors are isomorphic if and only if the polynomial matrices in a row minimal representation have the same invariant factors. This follows readily from the Smith forms of these polynomial matrices. It is therefore reasonable to ask for necessary and sufficient conditions which guarantee the assignment of a set of invariant factors for a system $\Sigma_1 = (\mathbb{R}, \mathbb{R}^{m+p}, \mathfrak{B}_1)$ having rank $r(\Sigma_1) = p$. We refer to [25] for details.

### 3.4 Feedback stabilization versus pole placement

It has been shown in [5, 11] that if a system is not generically pole assignable then there exist an open set of systems (open in the Euclidean topology) which cannot be stabilized. Based on those results one might think that the study of pole assignability does cover the question of stabilizability. This is only partially true. We believe that it is worthwhile to have results which allows one to describe the set of systems of a fixed McMillan degree that cannot be stabilized by regular feedback interconnections of compensators of degree at most $q$.

### 3.5 The problem of simultaneous pole placement and simultaneous stabilization

Simultaneous pole placement tries to answer the following question: Given a set of plants $\Sigma_i = (\mathbb{R}, \mathbb{R}^{m+p}, \mathfrak{B}_i)$, $i = 1, \ldots, r$ degree $n(\Sigma_i) = n_i$ and having constant rank $r(\Sigma_1) = p$. Let $\phi_i$ be a set of polynomials of degree $n_i + q$. We are seeking a feedback compensator $\Sigma = (\mathbb{R}, \mathbb{R}^{m+p}, \mathfrak{B})$ of degree $q$ and rank $r(\Sigma) = m$ such that all feedback interconnections $\Sigma_i \wedge \Sigma$ are regular and the characteristic polynomial $\chi_{\Sigma_i \wedge \Sigma} = \phi_i$.

In the work of Ghosh [6] some sufficiency results were derived. Those results are however far from being optimal. We would like to note that the question is an algebraic problem and this question is fully decidable. This is different from the simultaneous stabilization problem studied by Blondel [2].

## 3.6 The problem of decentralized pole placement

Let $P$ describe a linear differential system of rank $p$ and McMillan degree $n$. The decentralized pole placement problem asks for the construction of a set of compensators $Q_1, \ldots, Q_r$ such that a desired closed loop characteristic polynomial of the form

$$\phi(\xi) = \det \left[ \begin{array}{cccc} \multicolumn{4}{c}{P(\xi)} \\ \hline Q_1(\xi) & 0 & \cdots & 0 \\ 0 & Q_2(\xi) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & Q_r(\xi) \end{array} \right] \tag{37.9}$$

can be achieved. In [12] necessary and sufficient conditions over the complex numbers were derived. There are however no strong results for real pole assignment known.

## 3.7 General matrix extension problems

It has been explained in [19] that many pole placement problems can be expressed as matrix extension problems. It would be desirable to have strong necessary and strong sufficient conditions available which cover a wide range of matrix extension problems. A first general result in this direction has been derived in [7]. For example, consider an $n \times n$ matrix of the form

$$\begin{bmatrix} * & \cdots & * & * & \cdots & * & * & \cdots & * \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & \cdots & * & * & \cdots & * & * & \cdots & * \\ * & \cdots & * & ? & \cdots & ? & * & \cdots & * \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & \cdots & * & ? & \cdots & ? & * & \cdots & * \\ * & \cdots & * & * & \cdots & * & * & \cdots & * \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & \cdots & * & * & \cdots & * & * & \cdots & * \end{bmatrix} \tag{37.10}$$

where the $*$'s signify known elements, and the ?'s are elements that can be chosen freely. Is it possible to choose, for any monic $n$–th order $\phi \in \mathbb{F}[\xi]$, and for generic numbers $*$'s, the numbers ?'s such that the matrix has characteristic polynomial $\phi$, provided the number of ?'s exceeds $n$? If $\mathbb{F} = \mathbb{C}$ this has been answered in [7]. Over the reals the question is open. Note that the problem is almost, but not quite, a consequence of Theorem 6.

## References

[1] I. Berstein. On the Lusternik-Šnirel'mann category of real Grassmannians. *Proc. Camb. Phil. Soc.*, 79:129–239, 1976.

[2] V. Blondel. *Simultaneous Stabilization of Linear Systems*, volume 191 of *Lecture Notes in Control and Information Sciences*. Springer-Verlag London Ltd., London, 1994.

[3] R. W. Brockett and C. I. Byrnes. Multivariable Nyquist criteria, root loci and pole placement: A geometric viewpoint. *IEEE Trans. Automat. Control*, AC-26:271–284, 1981.

[4] C. I. Byrnes. Pole assignment by output feedback. In H. Nijmeijer and J. M. Schumacher, editors, *Three Decades of Mathematical System Theory*, Lecture Notes in Control and Information Sciences, volume 135, pages 31–78. Springer Verlag, 1989.

[5] C. I. Byrnes and B. D. O. Anderson. Output feedback and generic stabilizability. *SIAM J. Control Optim.*, 22(3):362–380, 1984.

[6] B. K. Ghosh. An approach to simultaneous system design. part II: Nonswitching gain & dynamic feedback compensation by algebraic geometric methods. *SIAM J. Control Optim.*, 26(4):919–963, 1988.

[7] W. Helton, J. Rosenthal, and X. Wang. Matrix extensions and eigenvalue completions, the generic case. *Trans. Amer. Math. Soc.*, 349(8):3401–3408, 1997.

[8] N. Karcanias and C. Giannakopoulos. Grassmann invariants, almost zeros and the determinantal zero, pole assignment problems of linear multivariable systems. *Internat. J. Control*, 40(4):673–698, 1984.

[9] J. Leventides and N. Karcanias. Global asymptotic linearization of the pole placement map: A closed form solution for the constant output feedback problem. *Automatica*, 31(9):1303–1309, 1995.

[10] N. Munro. Pole assignment: A review of methods. In M. G. Singh, editor, *Systems and Control Encyclopedia*, pages 3710–3717. Pergamon Press, 1990.

[11] M. S. Ravi, J. Rosenthal, and X. Wang. On generic stabilizability and pole assignability. *Systems & Control Letters*, 23(2):79–84, 1994.

[12] M. S. Ravi, J. Rosenthal, and X. Wang. On decentralized dynamic pole placement and feedback stabilization. *IEEE Trans. Automat. Contr.*, 40(9):1603–1614, 1995.

[13] M. S. Ravi, J. Rosenthal, and X. Wang. Dynamic pole assignment and Schubert calculus. *SIAM J. Control Optim.*, 34(3):813–832, 1996.

[14] J. Rosenthal. On dynamic feedback compensation and compactification of systems. *SIAM J. Control Optim.*, 32(1):279–296, 1994.

[15] J. Rosenthal, J. M. Schumacher, X. Wang, and J. C. Willems. Generic eigenvalue assignment for generalized linear first order systems using memoryless real output feedback. In *Proc. of the 34th IEEE Conference on Decision and Control*, pages 492–497, New Orleans, Louisiana, 1995.

[16] J. Rosenthal, J. M. Schumacher, and J. C. Willems. Generic eigenvalue assignment by memoryless real output feedback. *Systems & Control Letters*, 26:253–260, 1995.

[17] J. Rosenthal and F. Sottile. Some remarks on real and complex output feedback. *Systems & Control Letters*, 33(2):73–80, 1998.

[18] J. Rosenthal and X. Wang. Output feedback pole placement with dynamic compensators. *IEEE Trans. Automat. Contr.*, 41(6):830–843, 1996.

[19] J. Rosenthal and X. Wang. Inverse eigenvalue problems for multivariable linear systems. In C. I. Byrnes, B. N. Datta, D. Gilliam, and C. F. Martin, editors, *Systems and Control in the Twenty-First Century*, pages 289–311. Birkäuser, 1997.

[20] H. Schubert. Beziehungen zwischen den linearen Räumen auferlegbaren charakteristischen Bedingungen. *Math. Ann.*, 38:598–602, 1891.

[21] V. L. Syrmos, C. T. Abdallah, P. Dorato, and K. Grigoriadis. Static output feedback—a survey. *Automatica*, 33(2):125–137, 1997.

[22] X. Wang. Pole placement by static output feedback. *Math. Systems, Estimation, and Control*, 2(2):205–218, 1992.

[23] X. Wang. Grassmannian, central projection and output feedback pole assignment of linear systems. *IEEE Trans. Automat. Contr.*, 41(6):786–794, 1996.

[24] J. C. Willems. Paradigms and puzzles in the theory of dynamical systems. *IEEE Trans. Automat. Control*, AC-36(3):259–294, 1991.

[25] J. C. Willems. On interconnections, control, and feedback. *IEEE Trans. Automat. Control*, 42(3):326–339, 1997.

[26] J. C. Willems. Generic eigenvalue assignability by real memoryless output feedback made simple. A. Paulraj, V. Roychowdhury, and C.D. Schaper, editors, In *Communications, Computation, Control and Signal Processing*, Kluwer, pages 343-354, 1997.

[27] J. C. Willems and W. H. Hesselink. Generic properties of the pole placement problem. In *Proc. of the 7th IFAC Congress*, pages 1725–1729, 1978.