University of Zurich[UZH]

Institute of Mathematics

Lévy processes and their applications
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Titles and abstracts

Frank Aurzada

Small deviations and Chung type LIL for Lévy processes

The small deviation problem for a stochastic process $(X_t)_{t\in[0,1]}$ consists in determining the rate of the quantity

$$P(\sup_{t\in[0,1]} |X_t| \leq \varepsilon),$$

as $\varepsilon \to 0$. This problem has several relations to path and approximation properties of the process.

The talk presents results when X is a real-valued Lévy process. In this case, it is possible to quantify the small deviation rate explicitly in terms of the Lévy triplet. The results are closely connected to the so-called Chung type LIL, and the corresponding results are also given.

For multi-dimensional Lévy processes, the problem is open and I will discuss the arising difficulties.

This is joint work with Steffen Dereich, on the one hand, and Leif Döring and Mladen Savov, on the other hand.

Mathias Beiglböck

Mazur's lemma in stochastic analysis

By Mazur's lemma, any bounded sequence in a Hilbert space has a sequence of convex combinations of its members that converges strongly. This simple fact has a number of important consequences in stochastic analysis. In particular it can be used to give relatively short and elementary proofs to the Theorems of Doob-Meyer and Bichteler-Dellacherie-Mokobodski.

Loïc Chaumont

Coding multitype branching forests: application to the law of the total progeny

By extending the breadth first search algorithm to any d-type critical or subcritical irreducible branching tree, we show that such trees may be encoded through d independent, integer valued, d-dimensional random walks. An application of this coding together with a multivariate extension of the Ballot Theorem allow us to give an explicit form of the law of the total progeny, jointly with the number of subtrees of each type, in terms of the offspring distribution of the branching process.

Steffen Dereich

Multilevel Monte-Carlo algorithms for Lévy-driven SDE's

In this talk I report on the analysis of several multilevel Monte-Carlo algorithms for the computation of $\mathbf{E}f(Y)$, where $Y = (Y_t)_{t\in[0;1]}$ is the solution of a Lévy-driven SDE and f is a real-valued function on the path space. We consider numerical schemes that are based on shot noise representations that do or do not apply a Gaussian correction for the neglected small jumps. The approaches discussed are universal in the sense that we do not impose particular assumptions on the structure of the Lévy process (e.g. subordinated Lévy processes) except the existence of second moments.

Ron Doney

Local limit results for subordinators

The asymptotic behaviour of first passage times for subordinators is quite different from that for oscillating processes, typically exhibiting exponential decay rather than power-law decay. In this work we improve on the results of Jain and Pruitt (Ann. Probab., 1987) by finding uniform asymptotic estimates for the position of the process under weak assumptions, and then deduce the results for the passage times.

This is joint work with Victor Rivero, CIMAT, Mexico.

Leif Döring

Real-Valued Self-Similar Processes and Jump SDEs

The systematic study of self-similar Markov processes has seen a lot of development in the last decade. Extending Lamperti's characterization of positive self-similar Markov processes, several authors have characterized all non-negative self-similar Markov processes. In this talk we discuss a jump SDE characterization and show how to use it to attack real-valued self-similar processes.

Nathalie Eisenbaum

On positively correlated squared Gaussian processes

An infinitely divisible random vector with nonnegative components is always positively correlated. The converse implication is not true in general. We will show that in the case of a squared Gaussian vector (i.e. a vector $(\eta_1^2, \eta_2^2, \ldots, \eta_d^2)$ with $(\eta_1, \eta_2, \ldots, \eta_d)$ centered Gaussian vector) these two properties are equivalent. This result extends to permanental vectors.

Davar Khoshnevisan

Nonlinear noise excitation

We present a part of an ongoing effort that attempts to understand why solutions to many stochastic PDEs are intermittent. In particular, we show that there is a strong sense in which large families of SPDEs with intermittent solutions are extremely "excitable". More significantly, we show that this highly nonlinear level of noise excitation is, in a sense dichotomous: "Semidiscrete" equations are nearly always far less excitable than "continuous" equations. The reason for this dichotomy is also identified, and somewhat surprisingly has to do with the structure theory of certain topological groups.

This is based on on-going work with Kunwoo Kim.

Andreas Kyprianou

Law of the time to absorption at zero of a (not-necessarily) symmetric stable Lévy process

In this talk, we shall explain two methods for computing the law of the time to absorption of a stable process. In the case of symmetric stable processes, we use an explicit Wiener-Hopf factorisation for hypergeometric Lévy processes and the Lamperti transform for the radial part of the stable processes, which is a positive self-similar Markov process. We use these explicit details to consider, and indeed invert, the Mellin transform of the time to absorption at the origin which is equal in law to an integrated exponential hypergeometric Lévy process. In the non-symmetric case, the radial part of the process is no longer a positive self-similar Markov process and a different technique is needed. Here we describe the radial processes through a Markov additive Lévy process (more commonly found in queuing theory), which we can identify, again, in completely explicit form. The time to absorption at the origin can now be identified through the law of the integrated exponential Markov additive process. Remarkably, a peculiar, but nonetheless natural, two-dimensional recurrence equation, describing the Mellin transform of the latter, can be set up and solved explicitly. We discuss applications to the law of a stable process conditioned to avoid the origin. This is joint work with Alexey Kuznetsov (York, Canada), Juan Carlos Pardo (CIMAT, Mexico) and Alex Watson (Bath, UK).

Mladen Savov

Eigenvalue expansions of invariant Feller-Lamperti semigroups generated by non-local, non-selfadjoint operators via intertwining approach

The spectral theory for self-adjoint operators has been fully understood and extensively used to obtain eigenvalue expansions for semigroups of Markov processes and in particular diffusions. It seems there is no unified approach to the spectral theory for non-selfadjoint operators and eigenvalue expansions are usually difficult to obtain. In this talk we consider semigroups generated by non-local, non-selfadjoint operators. We discuss their spectrum, existence of eigenfunctions and coeigenfunctions and derive semi-group expansions for a large class of these semigroups. In more detail, if K_t is the semi-group of a positive self-similar Markov process (pssmp) of index 1 which can enter from zero then $P_t f(x) = K_{1-e^{-t}} f(e^{-t}x)$, for $x \geq 0$, defines a stationary Feller process on $\mathbb{R}^+$ with stationary measure ν, called IFLP. For the case when the IFLP jumps only downwards we present exhaustive results on the spectrum, the eigenfunctions and coeigenfunctions, and possible eigenvalue expansions. For the case with two sided jumps we present results which exclude spectral expansions and discuss the major problems in this setting. Our work is based on the development of general intertwinings for the semigroup K_t (resp. P_t) which reduce gradually the IFLP with two-sided jumps to IFLP with one-sided jumps then the latter to the squared radial norm of two-dimensional classical Ornstein-Ulenbeck process which is known to be generated by self-adjoint operator and possesses spectral expansion with Laguerre polynomials. The approach above expands P_t on a dense subset $L^2(\nu)$ but fails to do so for functions in natural spaces such as $L^2(\mathbb{R}^+)$ or $C_0(\mathbb{R}^+)$. Continuing with extensive study of the eigenfunctions and coeigenfunctions of P_t when the IFLP process jumps only downwards we obtain a spectral expansion $P_t f$ for a large class of semigroups for $f \in L^2(\mathbb{R}^+) \cup C_0(\mathbb{R}^+)$. This study is based on computing in Weirstrass products of the Mellin transform of general exponential functionals and its estimation along complex lines, special study of self-decomposable distributions and various other techniques. We have used as an inspiration and foundation of our works the papers of Carmona, Petit and Yor; Bertoin and Yor; Sato and Yamazato, Webster, Olver, Balkema et al., and others.

This is joint work with P. Patie (ULB Brussels).

Marc Yor

Peacocks and Lévy processes

Given certain processes increasing in the convex order, we find martingales with the same one-dimensional marginals.