

Exercise Sheet 14

Functions of Several Variables, Storrer 22 + 23

MUST

Exercise 1

Compute the partial derivatives $f_x = \frac{\partial f}{\partial x}$ and $f_y = \frac{\partial f}{\partial y}$ of the function $f(x, y) = \sqrt{2x + 3xy + 4y}$. Also show that $f_{xy} = f_{yx}$.

STANDARD

Exercise 2

Let

$$f(x, y) = 5xe^{-xy} + \ln(\sqrt{x^2 + y^2 - 1}) + \cos(\pi x + y) + \frac{x^2}{2}.$$

- Describe the maximal domain of f both mathematically and in a few words.
- Compute $f_x(2; 0)$, $f_y(0; 2)$, $f_{xy}(-2; 0)$, $f_{yy}(5; 0)$.

Exercise 3

Is the function

$$h : \mathbb{R} \setminus \{0\} \times \mathbb{R} \rightarrow \mathbb{R} \quad \text{with} \quad h(x, y) = x \cdot e^{-\frac{y}{x}}$$

a solution to the difference equation

$$x \cdot h_{xy} + 2(h_x + h_y) = y \cdot h_{yy}?$$

Exercise 4

Find all local extrema of the functions

- $f(x, y) = 3x^2 - 4x + 2xy + y^2$
- $f(x, y) = (x^2 + y^2)^2 - 2(x^2 - y^2)$
- $f(x, y) = \frac{1}{3}x^3 - x^2 + y^3 - 12y$

HONOURS

Exercise 5

Show: Among all triangles with constant perimeter U the one with maximal area F is the equilateral triangle.