Exercise Sheet 13

Inverse Function and Some Important Functions and Their Application, Storrer 17 + 18

Hand in: Wednesday, **20.12.2017**, ahead of the lecture.

MUST

Exercise 1

a) Which condition a function has to fulfill in order to be invertible?

b) How does an inverse function arise from the original function geometrically?

STANDARD

Exercise 2 (6 points)

a) Consider the function $y = f(x) = \frac{1}{2}(x+2)^3 - 1$ with $x \in \mathbb{R}$.

1) (1 point) Show that the function $f(x)$ is injective on the given domain, that is the assignment from x to $f(x)$ is unique.

2) (1 point) Determine the inverse function $f^{-1}(x)$.

3) (1 point) From domain $D(f)$ and image $W(f)$ of the original function determine directly $D(f^{-1})$ and $W(f^{-1})$.

4) (1 point) Draw the graphs of $f(x)$ and $f^{-1}(x)$.

b) (2 points) Determine the derivative of the function $y = \ln(x)$ for $x > 0$.
Hint: apply the formula for the derivative of the inverse function.

Exercise 3 (4 points)
Compute

a) (2 points) the derivative of $\arctan\left(\sqrt{\dfrac{x-1}{x+1}}\right)$.

b) (2 points) the integral of $f(x) = \dfrac{1}{4x^2 + 16x + 17}$.
Hint:
Reduce the integral to one of the form $f(x) = \dfrac{1}{1+x^2}$.

Exercise 4 (4 points)

a) (1 point) Draw a qualitative graph of the function $y = e^{-x^2}$. It is enough to show how the function behaves for $x \to \pm\infty$ and where the maximum is.

b) (1 point) Does the function $y = x \cdot e^{-x^2}$ arise from the one given in a) by application of simple geometric operations (such as shifting or stretching)?
An exact explanation is necessary!

c) (1 point) How about the function $y = e^{-(x+1)^2}$? Draw a qualitative graph!

d) (1 point) How about the function $y = 2 \cdot e^{-(x-1)^2}$? Draw a qualitative graph!

HONOURS

Exercise 5 (2 points)
Compute the following integrals. At first, rearrange the integrals until you find a known function. Do not use the table otherwise.

a) (1 point) $\displaystyle\int \frac{dx}{\sqrt{4-x^2}}$

b) (1 point) $\displaystyle\int \frac{dx}{\sqrt{x^2-4}}$

Also state the conditions on x.