Exercise Sheet 11

Differential Equations, Storrer 15

Hand in: Wednesday, **06.12.2017**, ahead of the lecture.

MUST

Exercise 1

a) Compute λ for $N'(t) = \lambda \cdot N(t)$ when you know that $N(t) = e^{10 \cdot t}$ holds.

b) Solve the DE $y' = x$ with $y(0) = 1$.

STANDARD

Exercise 2 (5 points)

Consider the differential equation $y' = y - x^2 + 1$.

a) (2 points) Draw the direction field of the DE. Make a table according to Storrer, page 213. To this end consider all integer values for x and y with $-3 \leq x \leq 3$ and $-3 \leq y \leq 3$.

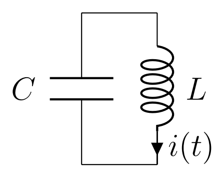
b) (1 point) Show that $y = x^2 + 2x + 1$ is a solution of the DE.

c) (2 points) Mark all the points in the table of subtask a) that are part of the soultion $y = x^2 + 2x + 1$. Mark this particular solution on the direction field of subtask a).

Exercise 3 (6 points)

a) (2 points) A free and undamped (eletric) circuit can be described by the differential equation

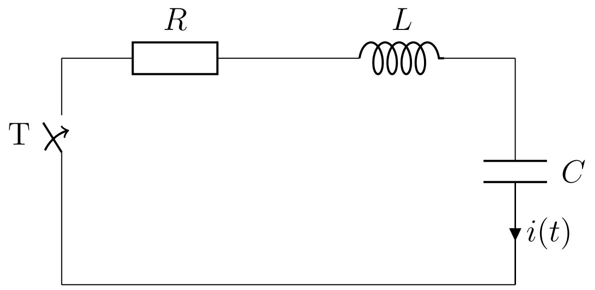
$$\ddot{i} + \omega_0^2 i = 0.$$



Here, ω_0 is the natural frequency that can be computed from $\omega_0 = \frac{1}{\sqrt{LC}}$.

Show that the function $i(t) = \widehat{I} \cdot \sin(\omega_0 t + \varphi)$ solves the DE!

b) (4 points) Consider a weak damping of an RLC circuit where R is a resistor, L an inductor and C a capacitor.



You do not need any electrotechnical knowledge to solve this exercise! We are mainly interested in the differencial equation.

The current $i(t)$ satisfies the DE $\quad \dfrac{d^2 i}{dt^2} + 2\sigma \dfrac{di}{dt} + \omega_0^2 i = 0 \quad \Leftrightarrow \quad \ddot{i} + 2\sigma \dot{i} + \omega_0^2 i = 0$

$$\text{where} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{and} \quad \sigma = \frac{R}{2L}.$$

Show that

$$i(t) = \widehat{I} \sin(\omega t + \varphi) e^{-\sigma t} \qquad \text{with} \quad \omega = \sqrt{\omega_0^2 - \sigma^2} \quad \text{and} \quad \varphi : \text{phase change}$$

satisfies the DE. Proceed as follows.

- (1 point) Determine $\dot{i} = \frac{di}{dt}$
- (1 point) Determine $\ddot{i} = \frac{d^2 i}{dt^2}$
- (2 points) Plug $\dot{i}$ and $\ddot{i}$ into the DE

Exercise 4 (8 points)

a) Show that the function y solves the DE.

- (3 points) $e^{-x} y'' + e^{-x} y' = 2x + 3 \quad$ mit $\quad y = x e^x$
- (3 points) $x y'' - 2 x y' + x y = -x e^x \sin(x) \quad$ mit $\quad y = e^x \sin(x)$

b) Determine the stationary solutions of the following differential equations.

- (1 point) $\dot{y} = y^2 + y - 2$
- (1 point) $\dot{y} = y^3 - 2y^2 - y + 2$

Hint: Stationary solutions are independent of the time.

HONOURS

Exercise 5 (2 points)
Let y be a differentiable function on $\mathbb{R}$ that can be written as $y = f(x) \cdot \sin(x)$ where $f(x)$ is a differentiable functions on $\mathbb{R}$.
Show that y solves the DE

$$y' = \left[\frac{f'(x)}{f(x)} + \cot(x)\right] \cdot y.$$