Exercise Sheet 10

Integration of vector functions and improper integrals, Storrer 14 + 20

Hand in: Wednesday, **29.11.2017**, ahead of the lecture.

MUST

Aufgabe 1
Compute the improper integrals.

a)

$$\int_{-\infty}^{0} e^x dx$$

b)

$$\int_{0}^{9} \frac{1}{\sqrt{x}}\, dx$$

STANDARD

Aufgabe 2 (4 Punkte)
Compute the improper integrals.

a) (1 Punkt)

$$\int_{1}^{\infty} \frac{1}{\sqrt[3]{x^4}} dx$$

b) (1 Punkt)

$$\int_{0}^{\pi/2} \frac{\cos t}{\sqrt{\sin t}} dt$$

c) (1 Punkt)

$$\int_{0}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$$

d) (1 Punkt)

$$\int_{0}^{1} \ln(x) dx$$

Aufgabe 3 (6 Punkte)
Let the force field $\vec{F}$ be given by

$$\vec{F}(\vec{x}) = \frac{1}{x_1^2 + x_2^2} \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

Compute the integral of the force field along a circle of radius r centered at the origin.

a) (2 Punkte) First determine $\vec{x}(t)$ und $\dot{\vec{x}}(t)$.

b) (2 Punkte) Compute the integral along the curve from $E(r/0)$ to $W(-r/0)$ oriented counterclockwise.

c) (2 Punkte) Compute the integral along the curve from $N(0/r)$ to $S(0/-r)$ oriented clockwise.

Aufgabe 4 (4 Punkte)
Let the vector field

$$\vec{F}(\vec{x}) = \begin{pmatrix} x_1 + x_2 \\ x_3^2 \\ x_1 x_2 x_3 \end{pmatrix}.$$

Compute the integral

$$\int_C \vec{F} d\vec{x}$$

from the origin to the point $P(3/2/1)$ if

a) (2 Punkte) C is a straight line from the origin to P.

b) (2 Punkte) C is the curve

$$\vec{x}(t) = \begin{pmatrix} 3t \\ 2t^3 \\ t^2 \end{pmatrix}.$$

HONOURS

Aufgabe 5 (6 Punkte)
Consider the equation of a helix

$$\vec{x}(t) = \begin{pmatrix} \cos t \\ \sin t \\ t \end{pmatrix}.$$

a) (1 Punkt) Draw the graph of the helix where t lies between 0 and 4π.

b) (2 Punkte) Compute the path integral

$$\int_H \vec{F} d\vec{x} \qquad \text{with} \qquad \vec{F}(\vec{x}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix},$$

where H is the path along the helix for t ranging from 0 to 4π.

c) (3 Punkte) Compute the improper path integral

$$\int_{H_\infty} \vec{F} d\vec{x} \qquad \text{with} \qquad \vec{F}(\vec{x}) = \frac{1}{x^2 + y^2 + z^2} \begin{pmatrix} yz \\ -xz \\ z+1 \end{pmatrix},$$

where H_∞ is the infinite helix, that is t ranges from $-\infty$ to ∞. To this end consider first t between a and b and then compute the limit for $a \to -\infty$ and $b \to \infty$.