Exercise Sheet 8

Integral calculus, Storrer 9 - 12

Hand in: Wednesday, **15.11.2017**, ahead of the lecture.

MUST

Exercise 1

a)

$$\int (x^4 - 4x^3 + x - 1)dx$$

b)

$$\int \left(\frac{2}{x} - \frac{t}{x^2}\right)dx$$

c)

$$\int \left(\sqrt[4]{x} - \frac{1}{\sqrt[5]{x}}\right)dx$$

STANDARD

Exercise 2 (4 points)
Determine the antiderivatives of the following functions.

a) (1 point)

$$f(x) = 2x^6 - 2x^2 + 3x + 2$$

b) (1 point)

$$f(x) = \frac{4}{x^3} + \frac{2}{x}$$

c) (1 point)

$$f(x) = \frac{1}{\sqrt[5]{x}} - \sqrt[4]{x}$$

d) (1 point)

$$f(x) = \frac{1}{\tan(x)}$$

Exercise 3 (6 points)

a) Determine the antiderivative $F(x)$ of $f(x)$ such that $F(x_0) = y_0$.

1) (1 point) $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$ $\quad x_0 = 4 \; , \; y_0 = 2$

2) (1 point) $f(x) = x - \frac{1}{x}$ $\quad x_0 = \sqrt{e} \; , \; y_0 = \frac{e}{2}$

3) (1 point) $f(x) = 2\sin(x)\cos(x)$ $\quad x_0 = \frac{\pi}{2} \; , \; y_0 = 0$

b) Compute the values of the following definite integrals.

1) (1 point) $\displaystyle\int_{-2}^{2} \left(x^2 - x + 1\right) dx$

2) (1 point) $\displaystyle\int_{-2}^{2} \left(e^{2t} + e^{-2t}\right) dt$

3) (1 point) $\displaystyle\int_{0}^{\pi/3} (\tan(x) + x)\, dx$

Exercise 4 (6 points)
Consider the function $f(x) = x^3 - 2x^2 - x + 2$. Shift the function $f(x)$ by -1 along the x-axis to get the function $h(x)$. Compute the area between both functions. Proceed as follows:

a) (2 points) Draw the graph of $f(x)$. Compute zeros, extrema, points of inflection and the y-intercept.

b) (1 point) Draw the graph of $h(x)$.

c) (1 point) Compute the intersection point of $f(x)$ and $h(x)$.

d) (2 points) Compute the sought area.

HONOURS

Exercise 5 (4 points)
We want to compute the integral $\int x^2 dx$ using Riemann series.

a) (1 point) Consider the function $y = x^2$ on [0, s] and split it into n intervals of the same length. The length of each sub-interval is $\frac{s}{n}$. Draw a qualitative graph.

b) (1 point) Compute the sum U_n of all rectangles below the function $y = x^2$.

c) (1 point) Compute the sum O_n of all rectangles above the function $y = x^2$.

d) (1 point) Show that

$$\lim_{n\to\infty} U_n = \lim_{n\to\infty} O_n = \frac{s^3}{3}.$$

Hint: You will need the following formula.

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$