Exercise Sheet 7

Derivative of a vector function, Storrer 8

Hand in: Wednesday, **08.11.2017**, ahead of the lecture.

MUST

Exercise 1

Consider the vector function $\vec{x}(t) = \begin{pmatrix} -\cos(t)+1 \\ \sin(t) \end{pmatrix}$

a) Compute $\dot{\vec{x}}(t) = \frac{d\vec{x}}{dt}$. What is the meaning of this vector function?

b) Compute $\vec{x}(t)$ und $\dot{\vec{x}}(t)$ für $t_1 = 0$, $t_2 = \frac{\pi}{4}$, $t_3 = \frac{\pi}{2}$, $t_4 = \frac{3\pi}{4}$ and $t_5 = \pi$.

c) Draw the graph of the function $\vec{x}(t) = \begin{pmatrix} -\cos(t)+1 \\ \sin(t) \end{pmatrix}$ for $0 \leq t \leq 2\pi$.

STANDARD

Exercise 2 (4 points)

Gegeben sei

$$\vec{x}(t) = \begin{pmatrix} 1+3t^2 \\ 2t-1 \\ t^3-t \end{pmatrix}$$

a) (1 point) Compute the velocity vector $\vec{v}(t) = \dot{\vec{x}}(t)$ and its magnitude $v(t) = \left|\dot{\vec{x}}(t)\right|$.

b) (1 point) Compute the acceleration vector $\vec{a}(t) = \ddot{\vec{x}}(t)$ and its magnitude $a(t) = \left|\ddot{\vec{x}}(t)\right|$.

c) (2 points) At what time t is the velocity $v(t)$ minimal? Compute its value. Particularly observe the boundary points.

Exercise 3 (6 points)

a) Draw a qualitative graph of the curve

$$\vec{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = \begin{pmatrix} t \\ t(t-1) \\ t^2 \end{pmatrix}$$

for $0 \leq t \leq 2$. Proceed the following way:

- (1 point) Draw the graph of the prjection of $\vec{x}(t)$ in the x_1x_2-plane (xy-plane).
- (1 point) Figure out how $x_3(t)$ looks for $0 \leq t \leq 2$.
- (2 points) Compute $\vec{x}(0)$, $\vec{x}(1)$, $\vec{x}(2)$ and draw the graph of the curve $\vec{x}(t)$.

b) (1 point) Determine the velocity vector $\dot{\vec{x}}(t) = \frac{d\vec{x}(t)}{dt}$. Compute $\dot{\vec{x}}(0)$, $\dot{\vec{x}}(1)$ and $\dot{\vec{x}}(2)$ as well as the magnitueds of these vectors.

c) (1 point) At what time t is the velocity maximal? When is it minimal?

Exercise 4 (2 points)

a) (1 point) For the vector function

$$\vec{x}(t) = \begin{pmatrix} 3t \\ t^2 - 2t \\ 4 - t^2 \end{pmatrix}$$

determine the tangent equation at $t = 2$.

b) (1 point) Are there two points in the interval $1 \leq t \leq 3$ with normal tangnet vectors?

HONOURS

Exercise 5 (7 points)

A two-dimensional closed noose has the following parameter representation

$$\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 3t^2 - 1 \\ 3t^3 - t \end{pmatrix} \qquad -1 \leq t \leq 1$$

a) (1 point) Determine $\dot{\vec{r}}(t)$.

b) (1 point) Determine the points where $\vec{r}(t)$ crosses the axes.

c) (1 point) Determine the points with vertical tangents.

d) (1 point) Determine the points with horizontal tangents.

e) (1 point) Consider the special point P(0/0) and determine the tangent vector in this point.

f) (1 point) Determine $\vec{r}(-1)$, $\vec{r}(0)$ and $\vec{r}(1)$.

g) (1 point) Draw a graph of the curve. Consider the points alredy computed and especially the point $P(0/0)$.