Exercise Sheet 4

Limits and derivative, Storrer 3 + 4

Hand in: Wednesday, **18.10.2017**, ahead of the lecture.

MUST

Exercise 1
In order to compute the slope of $f(x) = x^3$ at the point $x_0 = 1$, fill the following table. Note that for decreasing Δx the slope approaches the limit

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}.$$

See also Storrer page 42 and 43.

x	x_0	$\Delta x = x - x_0$	$f(x) = x^3$	$f(x_0) = x_0^3$	$\Delta f = f(x) - f(x_0)$	$\frac{\Delta f}{\Delta x}$
2	1			1		
1.5	1			1		
1.3	1			1		
1.1	1			1		
1.05	1			1		
1.01	1			1		
1.001	1			1		

STANDARD

Exercise 2 (4 points)

a) (2 points) Compute the derivative of the function $y = f(x) = x^4$ by using the difference quotient

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}.$$

Apply polynomial division!

b) (2 points) Compute the derivative of the function $y = f(x) = \sqrt{x}$ by using the differential quotient.
Hint: apply the 3rd binomial theorem $a^2 - b^2 = (a + b)(a - b)$

Exercise 3 (4 points)
Consider the function $y = f(x) = |x - 2|$ and investigate its differentiability.

a) (1 point) Can we say that $|x-2|' = 1$ holds?

b) (1 point) Redefine the same function piecewise without using the absolute value.

c) (2 points) Compute - where possible - the differential quotient on the corresponding regions. What happens at $x = 2$?

Exercise 4 (5 points)
Is the function $f(x)$ differentiable at x_0? If so, compute the derivative at x_0. You are allowed to use the rules of derivation.

a) $f(x) = \begin{cases} \frac{x^3}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}, \ x_0 = 0$

b) $f(x) = |e^{3x-1}(x^2 - 4x + 4)|, \ x_0 = 3$

c) $f(x) = \begin{cases} e^{-x}, & x \leq 0 \\ e^x, & x > 0 \end{cases}, \ x_0 = 0$

d) $f(x) = \begin{cases} \sin(x), & x \leq 0 \\ x^3 + 2x^2 + x, & x > 0 \end{cases}, \ x_0 = 0$

e) $f(x) = \begin{cases} \frac{1}{x}, & 0 < x < 1 \\ 1 - x, & x \geq 1 \end{cases}, \ x_0 = 1$

You get 1 point for each correct subtask.

Hint: If the function is not continous it is for sure not differentiable.

HONOURS

Exercise 5 (3 points)
We want to determine the derivative of the function $f(x) = \dfrac{x}{1-x}$, where $x \neq 1$.
Deduce this derivative without applying the quotient rule for derivative.

a) (1 point) Describe shortly in words how you proceed.

b) (1 point) Write down the difference quotient and simplify the expression.

c) (1 point) Deduce the derivative by computing the limit $\lim\limits_{\Delta x \to 0} \dfrac{\Delta f}{\Delta x}$.