Exercise Sheet 2

Vectors I , Storrer 1 + 2

Hand in: Wednesday, **04.10.2017**, ahead of the lecture.

MUST

Exercise 1

Let the vectors $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$.

a) Compute

(1) $2 \cdot \vec{a}$ (2) $3 \cdot \vec{b}$ (3) $4\vec{a} + 3\vec{b}$ (4) $3\vec{a} - 2\vec{b}$

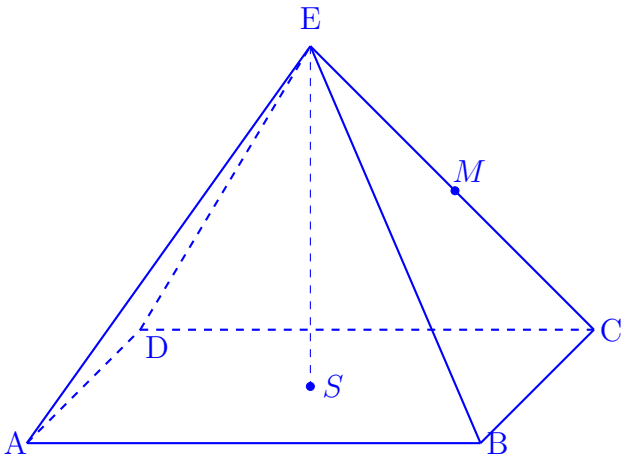
(5) $|\vec{a}|$ (6) $|\vec{b}|$ (7) $\vec{a} \cdot \vec{b}$ (8) $\vec{a} \times \vec{b}$

b) Sketch $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$. You may choose $\vec{a}$ and $\vec{b}$ arbitrarily but not parallel!

STANDARD

Exercise 2 (6 points)

Consider a pyramid with rectangular base $ABCD$ as shown in the following picture. The apex E is located above the centroid (geometric center) of the base.



Let M be the center of the line segment CE.
We fix the points $A(1/2/0)$, $B(3/0/0)$, $D(7/8/0)$ and $E(5/4/8)$.

a) (1 point) Determine the parametric equation of the plane ADE.

b) (2 points) Determine the coordinate equation of the plane ADE.

c) (1 point) Determine the coordinates of the point M.

d) (1 point) Determine the parametric equation of the line BM.

e) (1 point) Determine the point of intersection of the line BM and the plane ADE.

Exercise 3 (4 points)
Consider the same pyramide as in exercise 2.

a) (1 point) Compute the area of the lateral face ABE.

b) (1 point) Compute the angle between the edges AB and AE.

c) (2 points) Let the points $P(3/2/-2)$ and $Q(4/1/2)$ be given. Find the points R on the x-axis such that the angle between RP and RQ is exactky $90°$.

Exercise 4 (3 points)

a) (2 points) Decide if the equation below is valid for all vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

$$\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$$

If you think the equation is not true, a simple counterexample is enough.

b) (1 point) Show that the following equation is valid for all vectors $\vec{a}$ and $\vec{b}$.

$$\left(\vec{a} \times \vec{b}\right) \times \vec{a} = \vec{a} \times \left(\vec{b} \times \vec{a}\right)$$

HONOURS

Exercise 5 (2 points)
Show that

$$\cos^3(x) = \frac{3}{4}\cos(x) + \frac{1}{4}\cos(3x).$$