

Exercise Sheet 1

The first exercise contains a repetition of the material covered in high school. In case you don't remember everything that's no problem, the theory will be covered in the lecture.

Hand in: Wednesday, **27.09.2017**, ahead of the lecture.

Exercise 1 (5 points)

Consider the function $y = f(x) = -x^2 + 2x + 8$.

- (1 point) Find the roots of $f(x)$.
- (1 point) Find the vertex S of the parabola.
- (2 points) Find the equation of the tangent at $x = 2$. If possible, avoid using differential calculus.
- (1 point) Sketch both the parabola and the tangent in a coordinate system.

Exercise 2 (6 points)

- (2 points) Find all solutions of the equation

$$3x^2 + 6x = \frac{4}{3} + \frac{8}{3x}$$

Hint: use factorization

- (2 points) Find the solution(s) of the equation $5^{4x} - 7 \cdot 5^{2x} + 10 = 0$.

Hint: use substitution

- (2 points) Find the roots of $x^3 - 2x^2 - 5x + 6$.

Hint: guess one root and then apply polynomial division

Exercise 3 (4 points)

- a) (2 points) Determine the unknown variables
- x
- ,
- y
- and
- z
- .

$$\begin{cases} 3x + 2y - z = 3 \\ 2x - y + z = 4 \\ 4x + 2y - 3z = -4 \end{cases}$$

- b) (2 points) Determine the unknown variables
- x
- ,
- y
- and
- z
- .

$$\begin{cases} 6x + y - 2z = 0 \\ 2x - y + z = 4 \\ 4x + 2y - 3z = -4 \end{cases}$$

Hint: One equation is a linear combination of the two others.

Exercise 4 (6.5 points)

This exercise is meant to review the rules for exponential and logarithmic functions.

- a) (1.5 points) Rewrite the following expressions using only one
- $\ln(\cdot)$
- expression:

1) $2 \ln(a) - \ln(3c)$

2) $3 \ln(y) + \frac{1}{3} \ln(y)$

3) $4 \ln(m) - \frac{1}{6} \ln(n)$

- b) (1 point) Likewise for the exponential function:
- $e^{5a} \cdot e^{x^3} \cdot (e^{1-t})^t$

- c) (2 points) Simplify whenever possible:

1) $e^{-\ln(2x)}$

2) $e^{3 \ln(w)}$

3) $e^{\frac{1}{2} \ln(2y)}$

4) $e^{-\frac{1}{3} \ln(d)}$

- d) (1 point) Rearrange the following expressions whenever possible:

1) $\ln(a^9 b)$

2) $\ln(e^3 + 5)$

3) $\ln\left(\frac{e^{-x}}{b}\right)$

e) (1 point) Solve $2^x - 2^{x-1} = 2$ for x .