Exam MAT 183

Friday, 23.06.17 14:00-16:00

- Put your **student ID card** on the desk in front of you during the exam.
- Permitted material: calculator and all lecture material, i.e. the script, exercise sheets, exercise solutions, your own notes.
- NOT permitted in the exam: all communication devices (mobile phones, smartphones, notebooks, tablets, etc)
- For each examination problem, there is place to write your solution on the **GERMAN** exam paper (**front and back**). Each paper is labelled with your name, your matriculation number and the examination number. Should you require additional writing paper, use a new piece of paper for each problem which is labelled with **problem number, name and matriculation number**.
- If you write out detailed arguments leading to a solution, you should submit these along with your solution - points may be awarded for correct reasoning.
- Do not use pencil or green or red pens.
- Access to the toilets must be reported to an invigilator who will record your name and the time.
- **At the end of the exam**, all papers must be put in the correct order (1-8) and submitted in the envelope. **Sign but do not seal** the envelope.

One-minute-exercises

Problem 1

a) (*1 point*) Let X be a $\mathcal{N}(2,4)$ random variable. Calculate $P[X \in [-1,0]]$. We want the Z-transformed to be explicit.

b) (*1 point*) Which is the most common sum of eyes for a toss of two independent dices? How big is the probability that this sum is achieved?

c) (*1 point*) Let X be $U[0,1]$ distributed. Calculate $P[1/X > 3]$.

d) (*1 point*) Let X be $U[0,1]$ distributed. Let $b > 1$, such that $P[1/X > b] = 1/c$. What is the relation between b and c?

e) (*1 point*) Let X be a $\mathrm{Be}(p)$ random variable. What is the distribution of X^2?

f) (*1 point*) Let X be a Be(p) random variable and Y a exp(λ) random variable independent from X. Calculate $E[aX + bY]$ and $V[aX + bY]$.

g) (*1 point*) Towards what value does (written in R) "`sum(rchisq(n, 3))/n`" converge to if $n \to \infty$ and why?

h) (*1 point*) Let X be a exp(2) random variable. Calculate $P[2^X + 2 \leq 3]$.

Probability theory

Problem 2

Let X be a exp(2) random variable.

a) (*2 points*) Calculate $P[2^X + 1 \leq 3]$.

b) (*2 points*) What is the median of $X^2 + 3$?

Probability theory

Problem 3

Let X be a random variable with the following distribution: $P[X = 0] = 0.3, P[X = 1] = 0.3, P[X = 2] = 0.3, P[X = 3] = 0.1$.

a) (*1 Point*) Calculate $E[X]$.

b) (*1 Point*) Calculate $V[X]$ and $sd[X]$.

c) (*1 Point*) Calculate the expected value and the variance of $X + 1$.

d) (*1 Point*) Calculate the variance of X^2.

Probability theory

Problem 4

Let $X_1, \ldots, X_{900}$ be iid U[2,4] random variables.

a) (*2 Points*) Compute with the CLT

$$P\left[\sum_{i=1}^{900} X_i > 2670\right].$$

b) (*2 Points*) Let $Y := \sum_{i=1}^{900} X_i$. Compute with the CLT the 10 % quantile of Y.

Statistik

Problem 5

Barbara wants to test if a normally distributed random variable with variance 1 has mean 0 or not (two-sided). She gets for this two data points x_1, x_2, which were generated independently. Now she takes the sum of the two data points. If the absolute value of the sum is bigger than 2.2, she will discard the $\mathcal{H}_0$ hypothesis. Else she will accept it. [Hint: What is the distribution of the sum of independent normal random variables?]

a) (*2 Points*) How big is the α?

b) (*1 Point*) In the following test she gets the value 2.5. What is the P-value of this event?

Statistics

Problem 6

You do the test below in R. Respond explicitly to the following questions (1 point each):
a) What kind of test is this (exact name and hypotheses)?
b) Compute b.
c) How can you compute the estimated standard error with the information below, without using the six numbers again? Compute in this manner the estimated standard error.
d) Assume you want to test, if $\mu = 2$ or not (two-sided). Which value does the test statistic have?
e) Compute the P-value for the one-sided test $\mathcal{H}_0 : \mu \leq 0$ vs $\mathcal{H}_0 : \mu > 0$?

```
> a <- c(-2.2, -0.7, 3, 1.5, 0.2, b)
> t.test(a)
data:  a
t = 0.93891, df = 5, p-value = 0.3909
alternative hypothesis:  true mean is not equal to 0
95 percent confidence interval:
-1.390281 2.990281
sample estimates:
mean of x
0.8
```

Statistics

Problem 7

You are interested in knowing if for a certain illness there is a relation to one's gender. There are 400 women and 400 men randomly chosen. Within these people 160 men and 134 women have the illness.

a) Use the χ^2-test on the 5 % - level to find out, if gender and having the illness are independent or not. (3 points)

b) Does this test need the same amount of tested women and men (yes/no)? (1 point)

Statistics

Problem 8

Below you have an expression in R. Respond to the following questions.

a) (*1 Point*) Which is the predictor variable and which is the "response"-variable?

b) (*1 Point*) Enter the points and draw the regression line in a sketch as exact as possible. Especially all points must be on the right side of the regression line, else no point is given. Choose one of the six points and explain for this point with what computation you can find out on which side it lies, if you can not see graphics on your calculator.

c) (*1 Point*) Compute the correlation coefficient between a and b.

d) (*1 Point*) What value would the test statistic for the slope have, if you would want to test if the slope is -0.5 or not?

```
> a <- c(2,3,4,5,6,7)
> b <- c(4.2, 4.0, 2.8, 3.1, 2.6, 1.3)
> d <- lm(b ~ a)
> summary(d)
...
Coefficients:
             Estimate     Std.  Error     t value     Pr(>|t|)
(Intercept)  5.3657       0.4825          11.121     0.000372 ***
a            -0.5257       0.1002         -5.244       0.006322 **
...
```