1. Invited Talks

Some local and global properties of viscosity solutions of elliptic pde's

Italo Capuzzo Dolcetta, University "Roma La Sapienza", Italy

Abstract. The aim of the talk is to present some recent research on viscosity solutions of fully nonlinear elliptic equations of the type

$$F(x, u(x), Du(x), D^2u(x)) = 0$$

posed in a general, perhaps unbounded, open set.

Some of the issues touched in this talk are

- the Alexandrov-Bakelman-Pucci estimate,
- the boundary weak Harnack inequality,
- the Krylov-Safonov Growth Lemma,
- the Weak Maximum Principle for bounded solutions in unbounded domains,
- qualitative Phragmen-Lindelof type theorems,
- gradient estimates of Glaeser type.

On a conjecture by De Giorgi in large dimensions

Manuel del Pino, Universidad de Chile, Chile

Abstract. Associated to a large dilation of minimal surface which is the graph of an entire function of 8 variables found by Bombieri De Giorgi and Giusti, we construct a solution to the Allen Cahn equation in dimension 9 whose zero set is close to this surface and it is monotone in one direction. This provides a negative answer to a well-known question raised by De Giorgi in 1978, for dimension 9 or higher.

Parabolic Systems with Polynomial Growth

Frank Duzaar, University of Erlangen-Nuremberg, Germany

Abstract. We present a unified treatment of various interconnected aspects of the regularity theory for parabolic systems with polynomial growth: optimal partial regularity results, the first estimates on the Hausdorff dimension of the singular set, and the first Calderón–Zygmund estimates for related non-homogeneous problems are here achieved. A main point of the talk is to show a central bulk of techniques that simultaneously apply to the three issues. The presented results are obtained in a recent joint work with G. R. Mingione (Parma) and K. Steffen (Düsseldorf).

Existence and blow-up results for some evolution equations with nonlinear sources

Daniela Giachetti, Università degli Studi di Roma "La Sapienza", Italy

Abstract. We deal with blow-up and existence results for solutions of some evolution equations presenting a reaction term $g(u)$. More precisely, as far as the blow-up phenomenon is concerned, we consider the following problem

$$\begin{cases} u_t - \triangle(u^m) = \lambda g(u) & \text{in} \quad \Omega \times (0,\infty), \\ u = 0 & \text{on} \quad \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in} \quad \Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is an open bounded set, $0 < m < 1$, $\lambda > 0$, $u_0 \in L^\infty(\Omega)$ is non-negative and the function $g(u)$ satisfies the following conditions

$$g : [0,+\infty) \to \mathbb{R}^+, \quad C^1([0,+\infty)), \quad g(0) > 0,$$

$$g \text{ is nondecreasing and convex}$$

and

$$\int_0^\infty \frac{d\sigma}{g(\sigma)} < +\infty,$$

and we prove that for λ not too small (i.e. $\lambda > \lambda^*$) Fujita property occurs i.e. whatever is the initial datum u_0, all the solutions blow-up in finite time in the sense that

$$\mathrm{limsup}_{t \to T^-} \|u(x,t)\|_{L^\infty(\Omega)} = +\infty.$$

On the other hand, as far as the existence is concerned, we will prove existence of global solutions to

$$\begin{cases} u_t - \triangle(|u|^{m-1}u) = g(u) + \mu & \text{in} \quad \Omega \times (0,\infty), \\ u = 0 & \text{on} \quad \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in} \quad \Omega \end{cases}$$

essentially covering the complementary situation i.e.

$$\int_0^\infty \frac{d\sigma}{g(\sigma)} = +\infty,$$

for general data μ and u_0, i.e. μ finite Radon measure and $u_0 \in L^1(\Omega)$, and for $\frac{N-1}{N} < m < 1$. Here we mean $|u|^{m-1}u = 0$ in the set where $u = 0$.
For simplicity, we consider continuous functions $g : \mathbb{R} \to [0,+\infty)$ such that

$$g(s) \le C(1 + |s| \log^* |s|),$$

where $\log^* s = \max\{1, \log s\}$. We find also finite energy solutions for more regular data.

A selfimproving property for solutions of degenerate elliptic equations

Luigi Greco, University of Naples, Italy

Abstract. We consider the familiar A-harmonic equation

$$\operatorname{div} A(x, Du) = 0 \tag{1}$$

in a domain of $\mathbb{R}^n$. In the case of uniformly ellipticity, we have

$$|A(x, Du)|^q \approx |Du|^p \,,$$

$p > 1$, $1/p + 1/q = 1$, and therefore it is natural to require for a solution u a priori that $u \in W^{1,p}_{\mathrm{loc}}$. As is well-known, under this assumption the gradient of solutions is locally integrable with an exponent greater than p. On the other hand, Iwaniec-Sbordone and Lewis first considered the so-called very weak solutions, for which the natural a priori integrability assumption is relaxed. Their works show that, if the gradient of a solution u is assumed to be integrable with an exponent smaller than p, but sufficiently close to p, then actually $u \in W^{1,p}_{\mathrm{loc}}$. Hence, we have the following property. There exist exponents $1 < r_1 < p < r_2$ such that, if u solves equation (1), then

$$u \in W^{1,r_1}_{\mathrm{loc}} \Longrightarrow u \in W^{1,r_2}_{\mathrm{loc}} \,.$$

The exponents depend on the ellipticity constant, in the sense that when it becomes larger and larger they must be taken closer and closer to p.

We establish a similar result in the case of degenerate elliptic equations. Following a common use in the framework of the mappings with finite distortion, we write the ellipticity condition by the inequality

$$\frac{|\xi|^p}{p} + \frac{|A(x,\xi)|^q}{q} \le \mathcal{K}(x)\langle A(x,\xi), \xi\rangle \,.$$

The function $\mathcal{K}$ which measures the degree of degeneracy of the ellipticity bounds is assumed to satisfy

$$\exp(P(\mathcal{K})) \in L^1_{\mathrm{loc}} \,,$$

for an increasing function P verifying the so-called divergence condition

$$\int_1^\infty \frac{P(t)}{t^2}\, dt = \infty \,. \tag{2}$$

We also show optimality of our results. In particular, no improvement takes place in general without the divergence condition (2).

The results presented are contained in a joint work with Flavia Giannetti and Antonia Passarelli di Napoli.

Global regularity in variational problems

Jan Kristensen, University of Oxford, United Kingdom

Abstract. We discuss global higher differentiability results for minimizers of general integral functionals in the multi-dimensional vectorial case. A particular consequence of our results is the extension of the well-known interior partial regularity results for minimizers to a corresponding version that holds up to the boundary: almost all boundary points (in the sense of surface measure) admit a relative neighbourhood in which the minimizer is smooth.

Finite dimensional reduction of dissipative systems

Alain Miranville, Université de Poitiers, France

Abstract. Our aim in this talk is to discuss objects which allow to reduce, in some proper sense, the study of dissipative infinite dimensional dynamical systems associated with PDEs arising, e.g., from physics to the study of a finite dimensional object. In particular, we will focus on the notion of an exponential attractor and will show that it can also be applied to nonautonomous systems in a satisfactory way.

Sobolev mappings and PDE's

Gioconda Moscariello, Università degli Studi di Napoli "Federico II", Italy

Abstract. Let Ω and Ω' be bounded domains in $\mathbb{R}^n$. A homeomorphism $f : \Omega \xrightarrow{onto} \Omega'$ of Sobolev class $W^{1,1}_{loc}(\Omega;\Omega')$ is said to be a bisobolev map if it belongs to $W^{1,1}_{loc}(\Omega;\Omega')$ and its inverse $g = f^{-1}$ belongs to $W^{1,1}_{loc}(\Omega';\Omega)$.
An interesting property of a bisobolev planar map $f = (u, v)$ is that its components u and v have the same critical points and they both satisfy, in the sense of distributions, a "genuine" not uniformly elliptic equation of the type

$$\operatorname{div} \mathcal{A}(x)\nabla u = 0 \qquad \text{in } \Omega$$

where $\mathcal{A}(x)$ is a symmetric matrix with $\det\mathcal{A}(x) = 1$ for a.e. $x \in \Omega$.
An example shows that the situation is different in higher dimension. Anyway, in the general case a bisobolev map has finite inner distortion, i.e. the Jacobian J_f is strictly positive almost everywhere on the set where the transpose of the cofactor matrix of Df is not zero. Conversely, the inverse of a homeomorphism $f \in W^{1,n-1}_{loc}(\Omega;\Omega')$ having finite inner distortion, belongs to the Sobolev class $W^{1,1}_{loc}(\Omega;\Omega')$ and has finite outer distortion, i.e. the Jacobian J_g is strictly positive almost everywhere on the set where the differential matrix $Dg(x)$ is not zero..

Blowup in reaction-diffusion equations

Hirokazu Ninomiya, Meiji University, Japan

Abstract. In this talk we consider the relation of the dynamics of the ordinary differential equations to that of the corresponding reaction-diffusion equations when diffusion terms are added. We are mainly interested in the influence of diffusion on the global existence of solutions. We present examples of systems where diffusion induces or inhibits blow-up. More precisely, there is a certain kind of nonlinearity such that the blow-up of solutions of a two-component reaction-diffusion system on a bounded domain with Neumann boundary conditions can occur, while all the solutions of the ordinary differential equations converge to the stationary solution.

On the other hand, for a single reaction-diffusion equation we present the example where the blow-up of solutions can be inhibited by adding the diffusion. We also propose the sufficient condition for the blowup of solutions to a single reaction-diffusion equation.

Boundary conditions for the high order homogenized equation

Grigory Panasenko, University of Jean Monnet Saint Etienne, France

Abstract. The high order homogenization technique generates the so-called infinite order homogenized equation. Its coefficients were widely discussed in composite mechanics literature because they are closely related to the so-called high order strain gradients theories. However, it was not clear, what is the correct mathematical setting for this equation and what are the asymptotically exact boundary conditions. In the present paper we give a variational formulation for the high order homogenized equation by the projection of the initial problem on the änsatz subspace*:* This formulation generates the appropriate bondary conditions for the high order homogenized equation. The error estimates for the solution of the original problem and the homogenized one are obtained.

Parabolic Liouville theorems and their applications

Peter Polácik, University of Minnesota, School of Mathematics, USA

Abstract. Parabolic Liouville theorems state that if u is an entire solution of a specific parabolic equation (semilinear heat equation with power nonlinearity) and u is contained in an admissible class of functions, then u is necessarily the trivial solution. After a discussion of available Liouville theorems for nonnegative solutions, we shall focus on a Liouville theorem for radial sign-changing solutions. Among applications, we shall mention universal a priori estimates of solutions and a construction of infinitely many periodic solutions of periodic-parabolic boundary value problems.

Quasilinear Elliptic Equations: regularity and existence results[1]

Patrizia Pucci, Università degli Studi di Perugia, Italy

Abstract. In this talk we first present some regularity results given in [6] for the solutions of the general quasilinear elliptic equation

$$\mathrm{div}\boldsymbol{A}(x,u(x),Du(x)) = B(x,u(x),Du(x)) \quad \text{in} \quad \Omega, \tag{1}$$

where Ω is a domain of $\mathbb{R}^n$, not necessarily bounded, while $\boldsymbol{A} : \Omega \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ and $B : \Omega \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ are Carathéodory functions, satisfying the condition

there exist $a_1, a_4 > 0$ *and* $a_2,\ a_3 \geq 0$ *such that for a.a.* $x \in \Omega$ *and for all* $(z,\xi) \in \mathbb{R}\times\mathbb{R}^n$

$$(A1)\quad \begin{aligned} &(a) \quad \langle \mathbf{A}(x,z,\xi),\xi\rangle \geq a_1|\xi|^p - a_2|z|^p - a_3;\\ &(b) \qquad |\mathbf{A}(x,z,\xi)| \leq a_4|\xi|^{p-1}. \end{aligned}$$

Hence we consider also the case when $a_3 > 0$ in (a) which corresponds to the inhomogeneous version of problem (1), as first treated widely in [2, 3]. The nonlinear term B can be possibly singular at some point, as the special nonlinearities considered in [1, 5]. Finally, we briefly show the existence and non–existence results obtained in [1, 5].

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A deterministic control based approach to second order fully nonlinear PDEs

Sylvia Serfaty, Université Paris 6 and Courant Institute, France

Abstract. In joint work with Robert V. Kohn, we exhibit some discrete deterministic two player games which approximate the viscosity solutions to 1) curvature flows, 2) elliptic and parabolic fully nonlinear second order equations. With Cyril Imbert, we extend this to nonlocal equations.

Bellman function technique and mutual estimates of L^p-norms

Vasily Vasyunin, Petersburg Department of Steklov Institute of Mathematics, Russia

Abstract. In the talk I shall try to explain how the Bellman function technique can help in obtaining sharp estimates in different problems of analysis. As an example of application of this technique I have chosen estimates of the L^p-norm of a function under assumptions that two norms with other p are fixed, together with some multiplicative relation of these two norms. Such well-known inequalities as the reverse Hoelder inequality for Mackenhoupt weights or Gehring inequality are partial cases of these estimates. I would like to emphasize that in all cases sharp constants in the corresponding inequalities are found.

A kinetic approach to coagulation equations and coarsening models

Juan Velázquez, Instituto de Ciencias Matematicas (CSIC-UAM-UC3M-UCM), Spain

Abstract. In this talk I will review several mathematical aspects of several of the mathematical models that are used to describe the dynamics of coarsening in crystal growth. Aspects like the derivation of these models starting from particle models, as well as the mathematical analysis of these problems will be discussed.

Fractal Singular Homogenization

Maria Agostina Vivaldi, Università di Roma Sapienza

Abstract. We consider a two-dimensional elastic membrane which has been reinforced with the inclusion of a lower-dimensional elastic strip. The strip is thin, but is conductivity is large. We study the asymptotic behavior of such a composite medium, when the thickness of the strip vanishes and the conductivity of the strip becomes infinite, the product of the thickness and the conductivity remaining bounded. A peculiar feature of the model is that the strips develop, in the limit, infinite length and fractal geometry.

On a certain class of population models with nonlinear diffusion

Yoshio Yamada, Waseda University, Japan

Abstract. My talk is concerned with a population model with nonlinear diffusion terms. This model was first introduced by Shigesada, Kawasaki and Teramoto in order to describe habitat segregation between two competitive species. Our system consists of two reaction-diffusion equations with nonlinear diffusion. From the mathematical point of view, it is very important to get complete information on the following two topics; (i) the existence of global solutions for any nonnegative initial data and any spatial dimension, (ii) the structure of positive steady-state solutions. I will discuss recent results on these topics and related problems.

2. Short communications & Minisymposia

A free boundary problem for a concrete carbonation process

Toyohiko Aiki, Gifu University, Japan

Abstract. In this talk we consider the one-dimensional free boundary problem describing a concrete carbonation process. The problem is to find a curve separating the carbonized zone, the mass concentrations of CO_2 in water and in air. In our previous work we established the existence and the uniqueness of a solution to the free boundary in case the initial functions are bounded on the domain. Here, we show some results for initial functions belonging to L^2-class.

Maximum Principles for Homogeneous Elliptic Systems of Partial Differential Equations

Mohammad Almahameed, Irbid National University, Jordan

Abstract. In this paper we discuss a classical maximum principle for weakly coupled second order homogeneous elliptic systems. We find a sufficient condition for the classical maximum principle which extends the result of Winter and Wong for negative semidefinite matrix to a more general form.

Homogenization techniques and asymptotic stability for the electrical conduction in biological tissues

Micol Amar, Daniele Andreucci, Paolo Bisegna, Roberto Gianni, La Sapienza, Italy

Abstract. It is well known that electric potentials can be used in diagnostic devices to investigate the properties of biological tissues. Such techniques are essentially based on the possibility of determining the physiological properties of a living body by means of the knowledge of its electrical resistance. This leads to an inverse problem for the Laplace equation, which is the standard equation, when only a resistive behavior of the body is assumed. However, it has been observed that, applying high frequency alternating potentials to the body, a capacitive behavior takes place. This effect (known in physics as Maxwell-Wagner effect) is due to the electric polarization at the interface of the cell membranes, which act as capacitors, and it has been studied, among others, by the authors, who assume that the biological tissue is modelled as a composite media with a periodic microscopic structure composed by two finely mixed phases (intra and extra cellular) separated by an imperfect interface (cellular membrane).

The mathematical description of the electrical current conduction through the tissue is given by a system of decoupled equations in the two phases, whose solutions are coupled because of the interface conditions, since they have to satisfy the property of flux-continuity and a transmission condition of dynamic type. The microscopic model depends on the frequency at which the external potential acts and the authors consider different situations. The homogenization theory is used in order to pass from this microscopic description to the macroscopic one, which is given by new equations, replacing the elliptic one used up to now.

The authors study also the time exponential asymptotic stability of the solution, providing in this way a theoretical justification to the complex elliptic problem currently used in electrical impedance tomography.

Modeling phototransduction in rods through homogenization and concentration

Daniele Andreucci, Università la Sapienza Roma, Italy

Abstract. We discuss a model for the phototransduction cascade in vertebrate rods, which is a quite complex phenomenon, both from the geometrical and from the functional point of view. A good level of simplification, amenable to fast numerical solution, is achieved by means of the mathematical theories of homogenization and concentration of capacity.

We also touch on the following points of interest: variability of the response to a single photon event; well stirredness assumptions; the problem of adaptation to different light intensities.

Large solutions to semilinear elliptic equations with Hardy potential and exponential nonlinearity

Catherine Bandle, University of Basel, Switzerland

Abstract. On a bounded smooth domain $\Omega \subset \mathbb{R}^N$ we study solutions of a semilinear elliptic equation with an exponential nonlinearity and a Hardy potential depending on the distance to $\partial\Omega$. We derive global a priori bounds of the Keller–Osserman type. Using a Phragmen–Lindelöf alternative for generalized sub and super-harmonic functions we discuss existence, nonexistence and uniqueness of so-called *large* solutions, i.e., solution which tend to infinity at $\partial\Omega$. The approach develops the one used by the same authors for a problem with a power nonlinearity instead of the exponential nonlinearity.

On the Motion around Floating Bodies

Josef Bemelmans, RWTH Aachen University, Germany

Abstract. We investigate the motion of a viscous incompressible fluid around a floating body. In particular we consider a circular cylinder, partly filled with liquid, and an axially symmetric body that has smaller density than the fluid. If the body is positioned on the axis of the cylinder and rotates with small angular velocity there exists an equilibrium configuration: the weight of the body and the force exerted on it by the fluid determine its position. The free surface of the fluid is governed by surface tension, and we assume that this capillary surface meets the body at a right angle.

We show existence of a solution (v, p) to the Navier-Stokes equations of class $C^{2,\alpha} \times C^{1,\alpha}$; the corresponding free boundary is of class $C^{3,\alpha}$. The existence proof uses a method that is due to D.H. Sattinger who investigated the flow in a cylinder when a rod is inserted in the center. The velocity is regular up to the ridge where the capillary surface meets the rigid body; this is due to the fact that Dirichlet conditions are prescribed on the fixed boundary and a Neumann-type condition on the free surface; necessary and sufficient conditions for the solutions to be regular are due to E.A. Volkov.

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Reconstruction of interface changes of a conductivity inclusion from modal measurements

Elena Beretta, University "La Sapienza", Italy

Abstract. In this talk we propose an optimization approach for reconstructing interface changes of a conductivity inclusion from measurements of eigenvalues and eigenfunctions associated with the transmission problem for the Laplacian. Based on a rigorous asymptotic analysis, we derive an asymptotic formula for the perturbations in the modal measurements that are due to small changes in the interface of the inclusion. We then provide a key dual identity which naturally yields to the formulation of the proposed optimization problem.

Wiener regularity of certain fractal boundaries

Marco Biroli, Politecnico di Milano, Italy

Abstract. We prove the Wiener regularity of certain fractal boundaries. The domains in consideration are not ϵ-δ domains. The result is obtained in collaboration with N. Tchou.

Boundary regularity of non-linear parabolic systems

Verena Bögelein, University of Parma, Italy

Abstract. We consider Cauchy-Dirichlet problems involving a non-linear parabolic system of the type

$$\begin{cases} \partial_t u - \operatorname{div} a(x,t,Du) = 0 & \text{in } \Omega \times (0,T), \\ u = g & \text{on } (\partial\Omega \times (0,T)) \cup (\Omega \times \{0\}), \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^n$ and $u\colon \Omega \times (0,T) \to \mathbb{R}^N$. We present a necessary and sufficient criterion for a boundary point to be regular, i.e. to be a Hölder continuity point for the spatial gradient of solutions. Moreover, we shall establish that indeed almost every boundary point, with respect to the usual surface measure of the parabolic boundary, is regular. Note that due to counterexamples everywhere regularity fails to hold in general. The results presented are obtained in collaboration with F. Duzaar from the University of Erlangen and G. Mingione from the University of Parma.

The transmission problem for elliptic second order equations in a domain with conical boundary points

Mikhal Borsuk, University of Warmia and Mazury in Olsztyn, Poland

Abstract. We investigate the behavior of weak solutions to the transmission problem for linear and quasi-linear elliptic divergence second order equations in a neighborhood of the boundary conical point. We obtain best possible estimates of the solution modulus to the transmission problem near conical boundary point.

On lubrication problem in a thin domain with a rough boundary and Tresca fluid-solid interface law

Mahdi Boukrouche, Saint-Etienne University, France

Abstract. We study the asymptotic behavior of the solution of a lubrication problem in a thin domain, with a thickness of order ε, and a rough surface. The roughness is defined by a quasi-periodic function with period ε. We suppose that the flow is subject to a Tresca fluid-solid interface conditions. We prove a new result on the lower-semicontinuity for the two-scale convergence, which allow us to obtain rigorously the limit problem, and to establish the uniqueness of its solution.

Differential inclusions in Banach spaces

Messaoud Bounkhel, King Saud University, Saudi Arabia

Abstract. In this work we introduce and study a new type of differential inclusions in reflexive Banach spaces. We prove the existence of solutions for such type of differential inclusions. Our results extend various results from Hilbert spaces setting to reflexive Banach space setting.

Asymptotic analysis of a transport model of organic material through a stratified porous medium: concentration effect

Alain Brillard, University of Haute-Alsace, France

Abstract. We consider the transport through capillarity of an organic material inside a porous medium, using Leverett's model. We first prove an existence result for a weak solution of this nonlinear evolution problem, using a regularization process. We then describe the asymptotic behavior of the solution, when the permeability k_ε of the porous medium is associated to a scalar function which only depends on the third variable, assuming that k_ε (resp. the inverse of k_ε) converges to some measure λ_* (resp. λ^*). We use Γ-convergence arguments in order to describe this asymptotic behavior. We finally characterize the asymptotic behavior of the problem, considering special choices of the permeability k_ε, which correspond to stratified porous media, and give a numerical test for a 1D model.

On the Lawrence–Doniach Model of Superconductivity with Magnetic Fields Parallel to the Axes

Lia Bronsard, McMaster University, Canada

Abstract. The Lawrence–Doniach (LD) energy models highly anisotropic superconductors having a layered structure. Unlike the Ginzburg–Landau (GL) model, which represents a superconductor as a continuous three-dimensional solid, in the LD model the superconductor is idealized as a network of equally spaced, parallel superconducting planes. We will consider minimizers of the LD energy under periodic boundary conditions in three dimensions, with applied magnetic fields which are oriented perpendicular or parallel to these superconducting planes. We study various asymptotic limits, with the distance between the planes and the radius of vortices both tending to zero, for applied magnetic fields which depend on these two parameters. By introducing periodicity we eliminate boundary pinning effects and concentrate on the lower critical field and how the nature of the vortex lattice is determined by the orientation of the applied field and the relationships between the physical parameters. This talk represents joint work with S. Alama and E. Sandier.

Enhanced Resolution for Structured Media

Yves Capdeboscq, University of Oxford, United Kingdom

Abstract. This talk will discuss recent advances in detection of small inhomogeneities, and will particularly discuss the "Resolution beyond the diffraction limit" recently introduced by Fink et Al. First, the physical notions of resolution and focal spot will be revisited revisited. Then we shall prove that we one achieve a resolution enhancement in detecting a target inclusion if it is surrounded by an appropriate structured medium.

Some elliptic problems on irregular domains

Raffaela Capitanelli, University "La Sapienza", Italy

Abstract. We present regularity results for the solutions of some elliptic problems in domains which have an irregular boundary.

Materials with Memory: Exponential Decay of solutions in Evolution Problems

Sandra Carillo, University "La Sapienza" Italy

Abstract. A linear integro-differential equation which models the temperature's evolution within a rigid heat conductor with memory is here considered. The thermodynamical model proposed by Fabrizio, Gentili and Reynolds [2] is adopted. In this framework, suitable expressions of the minimum free energy [1] are introduced to construct functional spaces which are both meaningful under the analytic as well as the physical viewpoint. Indeed, in such spaces, on one side, existence and uniqueness results can be proved, on the other one, their meaningfulness under the physical viewpoint can be also shown. In particular, those conditions which guarantee solution's exponential decay at infinity are studied. Finally, connections with a nonlinear wave-type equations are mentioned [3].

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Phase-field systems with nonlinear coupling and dynamic boundary conditions

Cecilia Cavaterra, Università degli Studi di Milano, Italy

Abstract. We consider a phase-field system of Caginalp type describing the evolution of the order parameter and the relative temperature in a material occupying a three-dimensional bounded domain. The two equations are nonlinearly coupled through a quadratic growth function. Moreover, the order parameter fulfills a dynamic boundary condition while the temperature is subject to a boundary condition of Dirichlet, Neumann or Robin type. First, we prove the existence and uniqueness of global solutions. Then we analyze the associated dynamical system and we establish the existence of global as well as exponential attractors.

A singular perturbation result for multiscale financial models with stochastic volatility

Annalisa Cesaroni, University of Padova, Italy
(joint work with Martino Bardi and Luigi Manca)

Abstract. We consider singular perturbations for a class of stochastic control problems. Our motivations are the models of pricing and trading derivative securities in financial markets with stochastic volatility. We do not assume any boundedness condition on the fast variables, but we require that the fast subsystem admits a Lyapunov-like function, at least in the complement of a compact set. This a sufficient condition for ergodicity. Under this assumption, we prove the convergence of the value functions associated to the optimal control problems to the solutions of limit (eective) Cauchy problems for parabolic equations of Hamilton–Jacobi-Bellman type. Our methods are based on the theory of viscosity solutions and of the homogenization of fully nonlinear PDEs.

Spectral asymptotics in networks of thin domains

Mikhail Cherdantsev, Cardiff University, United Kingdom

Abstract. We consider an asymptotics of eigenvalue problem for the Dirichlet Laplacian in a network of thin domains. The thickness of the domains h is a small parameter. The main goal is to construct the asymptotics of the problem in terms of the limiting spectral problem on graph, which serves as a skeleton for the network.

Finite Dimensional Attractors for a Doubly Nonlinear Parabolic Equation with Singular Potential

Laurence Cherfils, University of La Rochelle, France

Abstract. We will consider a doubly nonlinear parabolic equation with singular potential. We first show that the solutions are a priori strictly separated from the singularities of the potential. This turns out to be our main argument in the proof of the existence and uniqueness of solutions. We then prove the existence of finite dimensional attractors via the so-called l-trajectories method.

Asymptotic variational equivalence

Marco Cicalese, University "Federico II", Italy

Abstract. In the framework of Γ-convergence we will introduce a notion of asymptotic variational equivalence between families of functionals and discuss its implications. As an application, we will show that the Ginzburg-Landau functional modelling superconductivity, the XY spin-system energy functional and the elastic energy functional induced by screw dislocations are variationally equivalent. These results are part of a joint work with R. Alicandro and M. Ponsiglione.

Symmetric semiclassical bound states to a nonlinear magnetic Schrödinger equation

Silvia Cingolani, Politecnico di Bari, Italy

Abstract. We consider the NLS equation

$$(-\varepsilon i\nabla + A(x))^2 u + V(x)u = |u|^{p-2}u, \qquad x \in \mathbb{R}^N,$$

where $N \geq 3$, $2 < p < 2N/(N-2)$, $A : \mathbb{R}^N \to \mathbb{R}^N$ is a magnetic potential and $V : \mathbb{R}^N \to \mathbb{R}$ is a bounded electric potential.

We consider a group G of orthogonal transformations of $\mathbb{R}^N$, and we assume that $A(gx) = gA(x)$ and $V(gx) = V(x)$ for any $g \in G$, $x \in \mathbb{R}^N$.

We present a multiplicity result of semiclassical bound states to the NLS equation, having specific symmetries. In particular we show that there is a combined effect of the symmetries and the electric potential V on the number of solutions of this type. This result is contained in a recent joint paper with Monica Clapp.

Periodic homogenization of some nonlinear problems by the unfolding method

Doina Cioranescu, University Paris 6, France

Abstract. We consider nonlinear integral energies with a periodic Carathéodory density, highly oscillating. The periodic unfolding method gives a direct proof of the homogenization results by making use of simple weak convergence arguments in L^p spaces. The method applies to convex and quasi-convex densities, as well as to constrained integral type energies. The result we present here were obtained in collaboration with Alain Damlamian and Riccardo De Arcangelis.

Differential calculus and conformal geometry on fractals

Fabio Cipriani, Politecnico Milano, Italy

Abstract. We shall provide the representation of self-similar Dirichlet forms on p.c.f. fractals by derivations. The associated differential calculus will allow to construct Fredholm modules in the sense of Atiyah representing the core of conformal geometries.

On the regularity of global attractors

Monica Conti, Politecnico di Milano, Italy
(joint paper with Vittorino Pata)

Abstract. This talk is focused on a novel technique to establish the boundedness in more regular spaces for global attractors of dissipative dynamical systems, without appealing to uniform-in-time estimates. As an application, we consider the semigroup generated by the strongly damped wave equation with critical nonlinearity, whose attractor is shown to possess the optimal regularity.

Existence and asymptotic behavior of solution to a quasilinear singular elliptic problems

Dragos-Patru Covei, Constantin Brancusi University of Tg-Jiu, Romania

Abstract. This paper deals with the existence and asymptotic behavior of entire positive solutions of the quasilinear elliptic equation for the Lane, Emden and Fowler Type.

Existence and uniqueness results for a class of PDEs of Monge–Kantorovich type

Graziano Crasta, Roma La Sapienza, Italy

Abstract. We consider a minimization problem with constraints on the gradient, and we show that the associated Euler–Lagrange equation is a system of PDEs of Monge–Kantorovich type.

We describe some new existence and uniqueness results for this system of PDEs.

The flow associated to weakly differentiable vector fields

Gianluca Crippa, Università di Parma, Italy

Abstract. I will review some of the main points of the well-posedness theory for transport and continuity equations with velocity fields whose regularity does not fit the classical Cauchy-Lipschitz framework. There are many motivations for this study, for instance applications to fluido-dynamics and to conservation laws. I will also mention the connection with the well-posedness of the ordinary differential equation, through a kind of extension of the classical methods of characteristics.

On the existence of the minima of degenerate variational integrals

Luigi D'Onofrio, Università degli Studi di Napoli "Parthenope", Italy

Abstract. The central theme running through this talk is the embedding of Sobolev weighted space into Sobolev-Orlicz spaces under minimal assumptions on the weight. As application we prove the existence and regularity of the minimum of functionals of the Calculus of Variations with growth governed by the weight we consider.

Bifurcations of weak solutions of systems of elliptic differential equations

Konrad Dabrowski, Nicolaus Copernicus University, Poland

Abstract. We study global bifurcations of weak solutions of systems of elliptic differential equations with Dirichlet boundary conditions considered on SO(2)-invariant domains. For that purpose we define the degree for SO(2)-equivariant maps, satisfying some transversality condition. We study the Rabinowitz alternative for these systems of equations.

The Dirichlet problem for Willmore surfaces of revolution

Anna Dall'Acqua, Otto-von-Guericke Universität Magdeburg, Germany

Abstract. The Willmore functional is the integral of the square of the mean curvature over the unknown surface. We consider the minimisation problem among all surfaces which obey suitable boundary conditions. The Willmore equation as the corresponding Euler-Lagrange equation may be considered as frame invariant counterpart of the clamped plate equation. This equation is of interest not only in mechanics and membrane physics but also in differential geometry.

We consider the Willmore boundary value problem for surfaces of revolution with arbitrary symmetric Dirichlet boundary condition. Using direct methods of the calculus of variations, we prove existence and regularity of minimizing solutions.

Nonexistence of positive solutions to nonlinear nonlocal elliptic systems

Zoubir Dahmani, Mostagaenm, Algeria

Abstract. In our work, we consider the question of nonexistence of positive solutions for nonlinear elliptic systems involving fractional diffusion operators. Using a weak formulation approach, we derive sufficient conditions.

Global well-posedness for a nonlocal Gross–Pitaevskii equation with non-zero condition at infinity

André De Laire, Pierre et Marie Curie – Paris 6, France

Abstract. We study the global well-posedness of the Cauchy problem for a nonlocal Gross–Pitaevskii equation in any dimension, with non-zero boundary condition at infinity. We consider nonlocal even potentials that are positive definite or positive tempered distributions.

Dynamics of the viscous Cahn–Hilliard equation

Tomasz Dlotko, Silesian University, Poland

Abstract. We discuss global solvability and existence of the global attractor for the semigroup generated on $H_0^1(\Omega)$ by the *viscous Cahn-Hilliard equation*

$$\begin{aligned} &(1-\nu)u_t = -\Delta(\Delta u + f(u) - \nu u_t) \text{ in } \Omega,\\ &u(t,x) = \Delta u(t,x) = 0 \text{ in } \partial\Omega,\\ &u(0,x) = u_0(x), \end{aligned} \tag{1}$$

where $\nu \in [0,1]$, $f \in C^1(R,R)$ satisfies suitable growth and dissipation conditions and Ω is a bounded smooth domain in $R^n, n \geq 3$.

Imposing growth restriction on the nonlinear term f ('critical' in the phase space $H_0^1(\Omega)$) the above problem is *gradient* in the sense of J.K. Hale with the global attractor given as a union of the unstable manifolds of the set of equilibria (all the equilibria are assumed to be *hyperbolic*). The family (parameter $\nu \in [0,1]$) of the global attractors to (1) is shown to be continuous (in the sense of Hausdorff distance), in particular when $n = 3, 4$ the attractor of the Cahn-Hilliard problem ($\nu = 0$) coincides, in a sense to be specified, with the attractor of the semilinear heat equation ($\nu = 1$).

Universal estimates for parabolic equations and some applications

Nikolai Dokuchaev, Trent University, Canada

Abstract. We suggest a modification of estimates for Sobolev norms of L_2-type for solutions of parabolic equations. In particular, we found an "universal" upper limit estimate that can be achieved by variations of the zero order coefficient. As an example of applications, an asymptotic estimate was obtained for the gradient at initial time. As an another example of application, existence and regularity results are obtained for quasi-linear parabolic equations and equations with time delay for the gradient.

Homogenization of a parabolic problem with an imperfect interface

Patrizia Donato, University of Rouen, France
joint work with E. Jose (University of Philippines)

Abstract. We study the heat diffusion in a two-component composite conductor with a periodic interface. Due to an imperfect contact on the interface, the heat flow through the interface is proportional to the jump of the temperature field. We study the limit behaviour of this parabolic problem when the period tends to zero. In the most interesting case, a memory effect appears on the limit problem.

Some results on the homogenization of a second order evolution problem in domains with imperfect inclusions. Part II

Luisa Faella, Università degli Studi di Cassino, Italy

Abstract. We study the homogenization of a second order linear evolution problem in a domain Ω of $\mathbb{R}^n$ with imperfect interface. Namely, we assume that $\Omega = \Omega_{1\varepsilon} \cup \overline{\Omega_{2\varepsilon}}$ with $\Omega_{1\varepsilon}$ connected and $\Omega_{2\varepsilon}$, union of ε-periodic connected inclusions of size ε, and we prescribe on $\Gamma_\varepsilon = \partial\Omega_{2\varepsilon}$, the interface separating $\Omega_{1\varepsilon}$ and $\Omega_{2\varepsilon}$, the continuity of the conormal derivatives and a jump of the solution proportional to the conormal derivatives through a function of order ε^γ.

More precisely, we describe, for different values of the parameter $\gamma \in \mathbb{R}$, the asymptotic behavior, as $\varepsilon \to 0$, of the solution of the following problem:

$$\begin{cases} u_\varepsilon'' - \operatorname{div}(A^\varepsilon \nabla u_\varepsilon) = f_\varepsilon \quad \text{in } (\Omega_{1\varepsilon} \cup \Omega_{2\varepsilon}) \times]0,T[, \\ [A^\varepsilon \nabla u_\varepsilon] \cdot n_{1\varepsilon} = 0 \quad \text{on } \Gamma_\varepsilon \times]0,T[, \\ A^\varepsilon \nabla u_{1\varepsilon} \cdot n_{1\varepsilon} = -\varepsilon^\gamma h^\varepsilon [u_\varepsilon] \quad \text{on } \Gamma_\varepsilon \times]0,T[, \\ u_\varepsilon = 0 \quad \text{on } \partial\Omega \times]0,T[, \\ u_\varepsilon(0) = U_\varepsilon^0 \quad \text{in } \Omega, \\ u_\varepsilon'(0) = U_\varepsilon^1 \quad \text{in } \Omega, \end{cases}$$

where $A^\varepsilon(x) := A(x/\varepsilon)$, A being a periodic, symmetric, bounded and positive definite matrix field, $h^\varepsilon(x) := h(x/\varepsilon)$, with h positive, bounded and periodic and $u_\varepsilon = (u_{1\varepsilon}, u_{2\varepsilon})$, $u_{i\varepsilon}$ being defined in Ω_i^ε, $i = 1, 2$. We denoted by $n_{i\varepsilon}$ the unitary outward normal to $\Omega_{i\varepsilon}$ and by $[\ ]$ the jump trough Γ_ε.

This problem models the wave propagation in a medium made by two components with very different coefficients of propagation. This leads to the jump boundary condition on the interface.

According to γ, different limit behaviors are obtained. In particular for $\gamma = 1$ a linear memory effect appears.

Some non-standard corrector results are also given.

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On the behavior of solutions to Schrödinger equations near an isolated singularity of the electromagnetic potential

Veronica Felli, Alberto Ferrero, Susanna Terracini, Milano Bicocca, Italy

Abstract. We will present the results obtained in collaboration with Alberto Ferrero and Susanna Terracini about the asymptotics of solutions to Schrödinger equations with singular magnetic and electric potentials. By using a Almgren type monotonicity formula, separation of variables, and an iterative Brezis–Kato type procedure, we describe the exact behavior near the singularity of solutions to linear and semilinear (critical and subcritical) elliptic equations with an inverse square electric potential and a singular magnetic potential with a homogeneity of order -1.

PDE's on random fractals

Uta Freiberg, University of Jena, Germany

Abstract. We discuss the definition of classical objects from potential theory as Dirichlet form, Laplacian and Brownian motion on certain classes of random fractals. Moreover, we present results on spectral asymptotics of such objects.

Cosserat Theory of Ginzburg–Landau for Thin Films

Bernardo Galvão-Sousa, McMaster University, Canada

Abstract. In this work, we derive some Ginzburg–Landau models for thin-films obtained by means of Γ–convergence. The different models are determined by the asymptotic ratio between the magnitude of the parallel applied magnetic field and the thickness of the film. Depending on the regime, we show that a decrease in the density of Cooper pairs holds or not. We also show that in the case of variable thickness of the film, its geometry will affect the effective applied magnetic field, thus influencing the position of vortices.

In the critical case, when the phase transitions are created at the same rate as the thin film, we obtain a phase transition model where the energy sees that the model came from a three-dimensional one. In the case where the thin film is created at a higher rate than the phase transitions, we derive a fully two-dimensional model that does not include its three-dimensional origin. The last regime, when the phase transition is created at a faster rate than the thin film, is more complex. Depending on the growth condition and the compatibility of the two phases, we deduce a model without phase transitions or a phase transitions' model with scale separation, i.e., a model equivalent to creating the phase transition first and then the thin film.

Continuous Families of Exponential Attractors for Singularly Perturbed Equations with Memory

Stefania Gatti, Università di Modena e Reggio Emilia, Italy

Abstract. This talk deals with a one-parameter family of perturbed abstract evolution problems, which, for each fixed parameter $\varepsilon \in [0,1]$, generates a semigroup $S_\varepsilon(t) : \mathcal{H}_\varepsilon \to \mathcal{H}_\varepsilon$ on some Banach space $\mathcal{H}_\varepsilon$. In particular, the perturbation is allowed to become singular at $\varepsilon = 0$. The main result is a general theorem on the existence of exponential attractors $\mathcal{E}_\varepsilon$ satisfying a suitable Hölder continuity property with respect to the symmetric Hausdorff distance at every $\varepsilon \in [0,1]$. Our hypotheses allow to apply the result to the abstract evolution equations with memory

$$\partial_t x(t) + \int_0^\infty k_\varepsilon(s) B_0(x(t-s))\, ds + B_1(x(t)) = 0, \quad \varepsilon \in (0,1],$$

where $k_\varepsilon(s) = (1/\varepsilon)k(s/\varepsilon)$ is the rescaling of a convex summable kernel k with unit mass. Such a family can be viewed as a memory perturbation of the equation

$$\partial_t x(t) + B_0(x(t)) + B_1(x(t)) = 0,$$

formally obtained in the singular limit $\varepsilon \to 0$. These results have been obtained in a joint paper with Alain Miranville, Vittorino Pata and Sergey Zelik.

Junction of ferromagnetic thin films

Antonio Gaudiello, University of Cassino, Italy
(joint work with Rejeb Hadiji)

Abstract. In this paper, starting from the classical 3D micromagnetic energy, we determine, via an asymptotic analysis, the free energy of two joined ferromagnetic thin films. We distinguish different regimes depending on the limit of the ratio between the small thickness of the two films.

Rigid body exhibiting stochastic dynamics

Mohamed Amine Ghezzar, CFPT Sonatrach, Algeria

Abstract. Based on as methods derived from nonlinear control theory and the modelization of split boundary problems, we present a novel technical approach for synchronizing the dynamics of a rigid body exhibiting stochastic motion. In this framework, the active control technique is modified and employed to design control functions based on Lyapunov stability theory and Routh-Hurwitz criteria, so that a drive-response system of a rigid body achieves anti-synchronism in the chaotic state. Global asymptotic stability and convergence of the sum of the dynamical variables representing the Eulerian state space of the two rigid bodies was verified by numerical simulations.

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Singularities of harmonic flows from the disk to the sphere

Lorenzo Giacomelli, Sapienza University of Rome, Italy

Abstract. Let the p-energy of a vector field be the the p-th power of the L^p norm of its gradient. The p-harmonic flow from the disk to the sphere is the (formal) gradient flow of the p-energy of an unknown vector field which is defined on the unit disk, constrained to take values into the unit sphere, and subjected to constant (in time) Dirichlet boundary conditions. I'll briefly summarize the state of the art, focusing on what is (not) known about the formation of singularities. Then I'll concentrate on the case $p = 1$, which corresponds to a constrained gradient flow for the total variation functional and has therefore BV as natural ambient space. Confining myself to the rotationally symmetric case, I'll discuss the characterization of smooth steady states (i.e. 1-harmonic maps), the local well-posedness of the problem, and (sharp) sufficient conditions for the formation of singularities in finite time.

On an Allen–Cahn type problem in phase segregation

Gianni Gilardi, University of Pavia, Italy
(joint work with P. Colli, P. Podio–Guidugli and J. Sprekels)

Abstract. We consider a two phase system where the physical variables (order parameter, chemical potential, and microentropy) satisfy suitable equations and relations.

As the strong formulation of the problem is ill-posed, we consider a weaker version of it and trasform the latter into an equivalent single equation for the order parameter, only.

We essentially obtain an integrodifferential Allen-Cahn type equation and complement it with boundary and initial conditions.

The present talk regards the results on well-posedness and long time behavior we have proved in a joint work with P. Colli, P. Podio–Guidugli, and J. Sprekels.

Classification of the solutions to the two-dimensional Ginzburg–Landau system for small superconducting domains

Tiziana Giorgi, New Mexico State University, USA

Abstract. We classify the solutions to the Ginzburg-Landau system for an infinite cylindrical superconductor, placed in a vacuum and subject to a constant applied field parallel to the cylinder axis, with small, smooth and simply connected cross section. We show that if the cross section is small enough, there is a critical value of the applied field below which the only solutions are up-to gauge equivalence classes the normal solution and the vortex-less solution, while above the critical value the system admits only the normal solution. Our results confirm that for this geometry, superconducting materials exhibit type I behavior for any choice of the Ginzburg–Landau parameter.

Detecting points in 2-D biological images

Daniele Graziani, University Nice Sophia Antipolis, France

Abstract. Detecting fine structures, like points or curves in two or three dimensional images respectively, is an important issue in image analysis. In biological images a point may represent a viral particle whose visibility is compromised by the presence of other structures like cell membranes or some noise. From a variational point of view the problem of isolating points is a difficult task, since it is not clear how these singularities must be classified in term of a some differential operator. In this talk we propose a new variational formulation involving the notions of p-capacity and divergence measure. This approach allows to give a rigorous mathematical definition of discontinuity without jump and to isolate the points via minimization of a suitable functional F. In the last part of the talk we suggest a possible approximation via Γ-convergence of the energy F.

On positive solutions to a second order elliptic equation with a singular nonlinearity

Galina Grishina, Bauman Moscow State Technical University, Russia

Abstract. We consider a second order elliptic equation with measurable bounded coefficients

$$(a_{ij}(x)u_{x_i})_{x_j} + p(x)|x|^s u^{-\sigma} = 0 \qquad x \in \Omega \setminus \{O\},$$

where $\sigma > 0$, s is any real number, $\Omega \subset \mathbb{R}^n$ is a bounded domain, which contains the origin O, and $p(x) > 0$.

The existence, nonexistence and behaviour of positive weak solutions near the isolated singular point O will be discussed.

On an anisotropic singular perturbation method

Senoussi Guesmia, University of Zurich, Switzerland
(joint work with M. Chipot)

Abstract. The purpose of my talk is to discuss the existence of the solution of some integro-differential problems arising in neutron transport theory. We show that the solution of our problem is a limit of solutions of some anisotropic singular perturbed problems.

Degenerate Structures and Fractals: Line integrals of 1-forms on fractals

Daniele Guido, University Roma "Tor Vergata", Italy

Abstract. Motivated by the attempt of giving a functional (i.e. non-commutative) description of the geometry of fractals, we introduce the notion of 1-forms on self-similar p.c.f. fractals with a harmonic structure, and the corresponding notion of line integrals. Issues of regularity, both for 1-forms and for paths, are treated. For topologically 1-dimensional fractals, such as the gasket, we show that 1-forms have a potential on a suitably defined covering, which allows an intrinsic notion of line-integral.

Partial Hölder continuity for elliptic systems with non standard growth

Jens Habermann, University Erlangen-Nuremberg, Germany

Abstract. The talk is concerned with partial $C^{0,\alpha}$ regularity for solutions $u \in W^{1,p(x)}(\Omega;\mathbb{R}^N)$ of elliptic systems with non standard $p(x)$ growth conditions. I.e. we consider elliptic systems

$$-\mathrm{div} a(x,u(x),Du(x)) = 0,$$

on an open bounded set $\Omega \subset \mathbb{R}^n$, satisfying nonstandard growth conditions of the form

$$L^{-1}\left(1+|z|\right)^{p(x)-2}|\lambda|^2 \leq \langle D_z a(x,\xi,z)\lambda,\lambda\rangle \leq L\left(1+|z|\right)^{p(x)-2}|\lambda|^2,$$

for any $x \in \Omega, \xi \in \mathbb{R}^n, z \in \mathbb{R}^{nN}$ and $\lambda \in \mathbb{R}^{nN}$, with $L \geq 1$. Under additional continuity and ellipticity assumptions, we prove that for any given $\alpha > 0$ there exists an open set $\Omega_0 \subset \Omega$ of full Lebesgue measure such that $u \in C^{0,\alpha}(\Omega_0)$, provided that the modulus of continuity of the exponent function p satisfies

$$\limsup_{\rho \to 0} \omega(\rho) \log \frac{1}{\rho} < +\infty. \tag{1}$$

V. Zhikov [2] proved, that assumption (1) is essential for any type of regularity, i.e. higher integrability can be shown under condition (1), whereas irregularities may occur in the case that (1) fails. In this sense the presented regularity result is optimal. The proof is based on the strategy of M. Foss & G. Mingione [1], using the method of A harmonic approximation with a new type of excess functional.

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Asymptotic Analysis, in a Thin Multidomain of Minimizing Maps with values in S^2

Rejeb Hadiji, Universtiy Paris 12, France
joint work with Antonio Gaudiello (Cassino, Italy)

Abstract. We consider a thin multidomain of R^3 consisting of two vertical cylinders, one placed upon the other: the first one with given height and small cross section, the second one with small thickness and given cross section. We first analyze, in this thin multidomain, the classical model for stationary micromagnetism due to Landau and Lifshitz, when the volumes of the two cylinders vanish. We derive the limit problem, which decomposes into two uncoupled problems, well posed on the limit cylinders (with dimensions 1 and 2, respectively). We precise how the limit problem depends on limit of the ratio between the volumes of the two cylinders. We also study the asymptotic behavior of the two limit problems, when the exterior limit fields increase. We show that in some cases, contrary to the initial problem, the energies of the limit problems diverge and we find the order of these energies.

Global attractor for the Davey–Stewartson system on $\mathbb{R}^2$

Manal Hussein, Université des Sciences et Technologies de Lille, France

Abstract. Our aim is to study the long time behavior of the solutions to the Davey-Stewartson system (DS), in the framework of the attractor theory which partakes of the theory of infinite-dimensional dynamical systems. So we prove that the weakly damped Davey–Stewartson system (DS), considered as an infinite-dimensional dynamical system in $H^1(\mathbb{R}^2)$, has a compact global attractor that is actually a compact subset of $H^2(\mathbb{R}^2)$.

Spectral geometry of self-similar CW-complexes

Tommaso Isola, University Roma "Tor Vergata", Italy

Abstract. We introduce a class of self-similar CW-complexes, as higher dimensional analogues of prefractal graphs.

We study two types of L^2-invariants - the Betti numbers, and the Novikov-Shubin numbers - which are sensitive to the large scale geometry.

We also consider a problem, originated in number theory, on the class of self-similar graphs, which consists in computing the number of equivalence classes of prime cycles in the graph, modulo local isomorphisms.

Both the above questions involve the Laplacian operator on the CW-complex.

Bifurcation Diagrams on an nonlinear elliptic problem on a spherical caps

Yoshitsugu Kabeya, Osaka Prefecture University, Japan

Abstract. We consider the scalar-field type nonlinear elliptic equation on a spherical cap under the homogeneous Dirichlet condition. We show the structure of bifurcation diagrams when the cap covers almost the whole sphere. We also show that some profiles of solutions on each branch. This talk is based on the joint work with Professor Catherine Bandle (University of Basel) and Professor Hirokazu Ninomiya (Meiji University).

Lamé operator in Hölder domains: asymptotics, regularity of solutions and spectral properties

Ilya Kamotskiy, University of Bath, United Kingdom

Abstract. We investigate the spectrum of a Lamé operator in bounded Hölder domains. We demonstrate that in the presence of sufficiently singular outward cusps this problem has properties typical for unbounded domains (with the singular point at the boundary playing a role somewhat similar to infinity): the loss of compactness, the existence of continuous spectrum, of incoming and outgoing waves, of a scattering matrix, limiting absorption principle etc. We discuss all these and further related issues. The talk will be based on the joint work with Dr. N.Babych.

Asymptotic of Capacity of a System of Closely placed Bodies

A. Kolpakov, A. G. Kolpakov, N. A. Filimonova, SibGUTI University, Russia

Abstract. It is demonstrated that the continuum problem of electroststic (a boundary value problem for Laplace equation) for a system of many closely placed bodies can be approximated by a finite dimensional (so called network) problem. The criterion of approximarion is the closeness of the energies (capacities) of the continuum and finite dimensional system.

Uniqueness and asymptotic behaviour for a class of parabolic partial differential equations with hysteresis and convection

Jana Kopfová, Siliesian University, Czech Republic

Abstract. We will deal with the question of existence, uniqueness and asymptotic behaviour of solutions of a parabolic equation with Preisach hysteresis and convection. This equation is part of a model system which describes the magnetohydrodynamic (MHD) flow of a conducting fluid between two ferromagnetic plates. The key point in the uniqueness proof is to get enough regularity for the gradient of the hysteresis term and then apply a discrete version of Moser iteration lemma.

An existence result for the Philip–Richards equation with hysteresis

Petra Kordulova, Silesian University in Opava, Czech Republic

Abstract. We present results about a class of PDEs whose model equation is represented by the Philip-Richards equation with soil-moisture hysteresis term. We assume that the hysteresis is represented by the Preisach hysteresis operator. We introduce a weak formulation of our problem in Sobolev spaces. An existence result is proved by a method based on an implicit time discretization scheme, a-priori estimates and passage to the limit in the convexity domain of the Preisach operator.

Linear elliptic and parabolic model problems in weighted spaces

Adam Kubica, Warsaw University of Technology, Poland

Abstract. We will present an elementary approach which allows us to deal with elliptic and parabolic problems in Sobolev–Kondratev spaces. We assume that the weight is a power of the distance from the origin or a distinguished axis. We define weak weighted solutions. We prove their existence and regularity under some restrictions on the weight. We show that these restrictions are essential.

Existence and non-existence of global solutions to initial boundary value problems for nonlinear evolution equations with the strong dissipation

Akisato Kubo, Fujita Health University, Japan

Abstract. The main purpose in the talk is to investigate existence and non-existence of global solutions of the initial Dirichlet-boundary value problem for evolution equations with the strong dissipation. Many authors studied classes consisting of such type of equations for which initial boundary value problems possess global solutions. For this purpose we consider a related problem and seek global solutions and blow-up solutions of it depending on whether it belongs to such classes or not.

Vortex motion in type II superconductors

Matthias Kurzke, University of Bonn, Germany
(joint work with D. Spirn, University of Minnesota)

Abstract. We study the time-dependent Ginzburg–Landau equations with a complex relaxation rate, a model that has been used to explain the Hall effect in type II superconductors.

By means of a Gamma-stability result that gives a quantitative relationship between the Ginzburg–Landau energy and the renormalized energy of the vortices, we derive the vortex motion law in the regime of low magnetic fields.

Free Steady Motions of Elastic Bodies in Liquids: Coupling of the Exterior Domain Navier-Stokes Problem with the Free Traction Problem of Nonlinear Elasticity

Mads Kyed, RWTH Aachen University, Germany

Abstract. We consider the unconstrained (free) steady-state motion of an elastic body in an incompressible viscous fluid. The motion is described by two coupled elliptic systems. The motion of the fluid, occupying the exterior domain with respect to the body, is governed by the Navier-Stokes equations, and the motion of the elastic body by the nonlinear St. Venant–Kirchhoff material laws. The two systems are coupled by assuming continuity of the normal stresses on the fluid-structure boundary. We impose no other boundary conditions on the body. Consequently, we consider a so-called free traction problem. Since the deformation of the body is an unknown, the problem is a free boundary problem. Moreover, since the motion of the body is unconstrained, also the orientation as well as the linear and angular velocity of the body are unknowns. Our main result is a proof of existence of solutions for sufficiently small data. The result is part of a joint work with Josef Bemelmans and Giovanni P. Galdi.

Heat flow problems in varying Hilbert spaces

Maria Rosaria Lancia, University "La Sapienza", Italy

Abstract. We present some recent results on parabolic trasmission problems across fractal layers that have asymptotically the thermal capacity comparable with the conductivity.

Partial Regulatory for Minimizers of Polyconvex Functionals depending on the Hessian Determinant

Chiara Leone, "Federico II" of Naples, Italy

Abstract. We consider a polyconvex functional depending on the second order derivatives of a function u in $W^{2,2}(\Omega)$ where Ω is a bounded open subset of R^2. We prove a $C^{2,\alpha}$-partial regularity result for minimizers of this functional.

Decay estimates and Liouville property for viscosity solutions of fully nonlinear inequalities

Fabiana Leoni, Sapienza Università di Roma, Italy

Abstract. For viscosity sub and supersolutions of fully nonlinear uniformly elliptic equations posed in unbounded domains, we address the issue of the behaviour at infinity with application to the existence of nonnegative global solutions.

A discrete model for Laplacian growth

Jean-Pierre Loheac, Ecole centrale de Lyon, France

Abstract. We here consider bi-dimensional Hele-Shaw flows with a punctual source. In order to get theoretical results, one can use Helmholtz-Kirchhoff method. This allows to transform the initial free boundary problem into an integro-differential equation. When we restrict this method to a particular class of free boundaries, we obtain a discrete model described by a non-linear differential equation. We shall detail this construction of this model and present examples of numerical experiments.

Rate of convergence for the homogenization of some fully nonlinear elliptic equations

Claudio Marchi, Università di Padova, Italy

Abstract. We consider the homogenization problem for some classes of fully nonlinear equations of uniformly elliptic type. It is well known that, under some periodicity assumptions, the solution u^ϵ to the starting problems converge locally uniformly to the solution u of an effective equation suitably defined.

We shall provide an estimate for the rate of convergence, namely for $\|u^\epsilon - u\|_\infty$. Finally, some examples arising in stochastic optimal control theory will be discussed.

This talk is based on joint work with F. Camilli.

Initial-Boundary Value Problem with a Nonlocal Constraint for a Higher Dimension Boussinesq Equation

Said Mesloub, King Saud University, Saudi Arabia

Abstract. In this talk, we consider a mixed initial boundary value problem for a higher dimension Boussinesq equation. We prove the unique solvability of the given problem in Sobolev space W_2^1. Galerkin's method was the main used tool for proving the unique solvability of the given nonlocal problem.

Uniform Decay in a Timoshenko-type System with Past History

Salim A. Messaoudi, KFUPM, Saudi Arabia and
Belkacem Said-Houari, Université Paul Sabatier, France

Abstract. Fernández Sare and Rivera [1] considered the following Timoshenko-type System

$$\rho_1\varphi_u - K(\varphi_x+\psi)_x = 0$$
$$\rho_2\psi_u - b\psi_{xx} + \int_0^\infty g(t)\psi_{xx}(t-s,\cdot)\,ds + K(\varphi_x+\psi) = 0$$

where ρ_1, ρ_2, b, K are positive constants in a positive differentiable exponentially decaying function. They established an exponential decay result in the case of equal wave-speed propagation $\left(\frac{\rho_1}{K}=\frac{\rho_2}{b}\right)$ and a polynomial decay result in the case of non equal wave-speed propagation $\left(\frac{\rho_1}{K}\neq\frac{\rho_2}{b}\right)$.

In this work, we study the same system, for g decaying polynomially, and prove polynomial stability results for the equal and non equal wave-speed propagation.

Our results are established under conditions on the relaxation function weaker than those imposed by Fernández Sare and Rivera.

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On the multiplicity of solutions of singularly perturbed elliptic problems on Riemannian manifolds

Anna Maria Micheletti, University of Pisa, Italy

Abstract. Given (M,g) a smooth compact Riemannian N-manifold, $N\geq 2$, we look for functions $u\in H^1_g(M)$ satisfying the following singularly perturbed elliptic problems

$$-\epsilon^2\Delta_g u + u = |u|^{p-2}u \text{ in } M \qquad (\mathrm{P})$$

Here $p>2$ if $N=2$, $2<p<2^*=(2N)/(N-2)$ if $N\geq 3$. We show that the number of solutions of the problem (P) depends on the topological properties of the manifold. In particular we consider the Lusternik–Schnirelmann category and Poincaré map polynomial. Also the geometry of M has effect on the number of solutions and we prove the stable critical points of the scalar curvature of (M,g) generate solutions of the problem (P).

Homogenization in Sobolev and BV spaces for manifold valued maps

Vincent Millot, University Paris Diderot - Paris 7, France
(joint work with J. F. Babadjian)

Abstract. We will present homogenization results for integral functionals defined for Sobolev maps taking values in a given smooth manifold. The results involve the notion of tangential homogenization by analogy with the tangential quasiconvexity introduced by Dacorogna, Fonseca, Maly & Trivisa. For energies with superlinear or linear growth, a Γ-convergence analysis will be presented in Sobolev and BV spaces respectively. This is a joint work with J. F. Babadjian.

Schroedinger equations with partially periodic potentials

Riccardo Molle, University Roma "Tor Vergata", Italy

Abstract. A class of nonlinear elliptic equations is considered, both on R^N and on unbounded domains with boundary periodic in some directions. Existence and multiplicity of solutions is discussed, according to the shape of the (partially periodic) potential, and a description of the lack of compactness in the problem is given. The results presented in this talk are a joint work with G. Cerami.

Some results on the homogenization of a second order evolution problem in domains with imperfect inclusions Part I

Sara Monsurrò, Università degli Studi di Salerno, Italy

Abstract. We study the homogenization of a second order linear evolution problem in a domain Ω of $\mathbb{R}^n$ with imperfect interface. Namely, we assume that $\Omega = \Omega_{1\varepsilon} \cup \overline{\Omega_{2\varepsilon}}$ with $\Omega_{1\varepsilon}$ connected and $\Omega_{2\varepsilon}$, union of ε-periodic connected inclusions of size ε, and we prescribe on $\Gamma_\varepsilon = \partial\Omega_{2\varepsilon}$, the interface separating $\Omega_{1\varepsilon}$ and $\Omega_{2\varepsilon}$, the continuity of the conormal derivatives and a jump of the solution proportional to the conormal derivatives through a function of order ε^γ.

More precisely, we describe, for different values of the parameter $\gamma \in \mathbb{R}$, the asymptotic behavior, as $\varepsilon \to 0$, of the solution of the following problem:

$$\begin{cases} u_\varepsilon'' - \operatorname{div}(A^\varepsilon \nabla u_\varepsilon) = f_\varepsilon \quad \text{in } (\Omega_{1\varepsilon} \cup \Omega_{2\varepsilon}) \times]0,T[, \\ [A^\varepsilon \nabla u_\varepsilon] \cdot n_{1\varepsilon} = 0 \quad \text{on } \Gamma_\varepsilon \times]0,T[, \\ A^\varepsilon \nabla u_{1\varepsilon} \cdot n_{1\varepsilon} = -\varepsilon^\gamma h^\varepsilon [u_\varepsilon] \quad \text{on } \Gamma_\varepsilon \times]0,T[, \\ u_\varepsilon = 0 \quad \text{on } \partial\Omega \times]0,T[, \\ u_\varepsilon(0) = U_\varepsilon^0 \quad \text{in } \Omega, \\ u_\varepsilon'(0) = U_\varepsilon^1 \quad \text{in } \Omega. \end{cases}$$

where $A^\varepsilon(x) := A(x/\varepsilon)$, A being a periodic, symmetric, bounded and positive definite matrix field, $h^\varepsilon(x) := h(x/\varepsilon)$, with h positive, bounded and periodic and $u_\varepsilon = (u_{1\varepsilon}, u_{2\varepsilon})$, $u_{i\varepsilon}$ being defined in Ω_i^ε, $i = 1, 2$. We denoted by $n_{i\varepsilon}$ the unitary outward normal to $\Omega_{i\varepsilon}$ and by $[\ ]$ the jump trough Γ_ε.

This problem models the wave propagation in a medium made by two components with very different coefficients of propagation. This leads to the jump boundary condition on the interface.

According to γ, different limit behaviors are obtained. In particular for $\gamma = 1$ a linear memory effect appears.

Some corrector results are also given.

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An iterative method for Saddle shaped solutions for some semilinear equations

Piero Montecchiari, Università Politecnica delle Marche, Italy

Abstract. We present a variational procedure to study by a global approach the problem of existence of saddle shaped solutions in $\mathbf{R}^N$ for semilinear elliptic equations $-\Delta u + W'(u) = 0$ where $W : \mathbf{R} \to \mathbf{R}$ is modeled on the classical double well potential $W(s) = (s^2 - 1)^2$.

Some remarks on solutions of some nonlinear plane elliptic Neumann problems concentrating on curves

Eugenio Montefusco, "Sapienza" Università di Roma, Italy

Abstract. It is well known that some nonlinear elliptic Neumannn problems (singularly perturbed) possess positive solutions concentrating on one or more points on the boundary of the domain (these points have to be critical points of the mean curvature of the boundary). More recently, it has also been proved that, for this class of problems, there exist solutions concentrating on some non-degenerated higher dimensional manifolds. We want to present some new examples of concentration on curves in two-dimensional domains.

Global Harnack inequalities for parabolic equations on bounded domains

Luisa Moschini, Università di Roma "La Sapienza", Italy

Abstract. We derive parabolic Harnack inequalities up to the boundary for various degenerate operators, we deduce from them sharp two-sided estimates on the fundamental solution of the corresponding Cauchy-Dirichlet problem. In all the cases we consider, we assume that the elliptic part, possibly degenerate, has a generalized first eigenfunction of which we know the asymptotic behaviour.

Estimates for the domain wall energy in ferromagnets

Roger Moser, University of Bath, United Kingdom

Abstract. The energy associated to the magnetization vector field of a ferromagnetic sample consists of several terms and involves a parameter that is typically small; thus it makes sense to study the limit as it tends to 0. The interplay of two of the energy terms gives rise to a version of the Modica-Mortola theory for maps into the 2-sphere. In particular we expect domains of constant magnetization in the limit, separated by sharp transitions, which are called domain walls in this context. But there is a third contribution to the energy, the so-called magnetostatic energy, that is more difficult to understand. We examine its role in the limit.

The magnetostatic energy involves a pseudo-differential operator, which gives rise to considerable technical difficulties. Nevertheless, it is possible to prove two interesting facts. First, the magnetostatic energy gives a positive contribution to the limiting energy except in very special situations. Second, under certain conditions the optimal transition across the walls is no longer given by a one-dimensional profile. In fact, a construction using microscopic zig-zag patterns reduces the energy.

Uniqueness of solution for a variable exponent Dirichlet problem

Viorica Motreanu, University of Zurich, Switzerland

Abstract. We provide uniqueness results for a Dirichlet problem driven by a differential operator in the form of the divergence of a monotone map by means of a comparison result and approximation.

A pointwise estimate of the solution to the p-Laplace evolution equation

Yuliya Namlyeyeva, NAS of Ukraine, Ukraine
(joint work with Leonid Pankratov and Igor Skrypnik)

Abstract. In the cylinder $(\mathcal{B}_R \backslash \overline{\mathcal{F}}) \times (0,T)$ we consider the Dirichlet problem for the p-Laplace evolution equation. Here $\mathcal{F}$ is an open set of diameter d, $\mathcal{B}_R \subset \mathbb{R}^n$ is an open ball of radius $R = R(T,d,p)$, $d \ll R$, and $p \in (\frac{2n}{n+1}, 2]$. Using Moserś iterative procedure, we derive the poitnwise estimates for the solution of this problem in terms of the diameter of the set $\mathcal{F}$.

This problem is closely related to Skrypnikś method of homogenization of the Dirichlet non–linear parabolic problems in non–periodic strongly perforated domains in the case when the perforations are small disjoint components.

Variational methods in the theory of perfectly plastic fluids

Joachim Naumann, Humboldt University Berlin, Germany

Abstract. The stationary motion of an incompressible fluid is governed by the system of PDEs

$$\nabla \cdot \mathbf{u} = 0, \qquad -\nabla \cdot S + \nabla p = \mathbf{f} \tag{1}$$

where $\mathbf{u} = (u_1, \dots, u_n)$ velocity, $S = \{S_{ij}\}$ deviatoric stress, p pressure, $\mathbf{f}$ external force. We consider the following constitutive law (R. von Mises (1913)):

$$D = 0 \Longrightarrow |S| \le g, \quad D \neq 0 \Longrightarrow S = \frac{g}{|D|} D \tag{2}$$

($D = D(\mathbf{u}) = \{D_{ij}(\mathbf{u})\}$, $D_{ij}(\mathbf{u}) = \dfrac{1}{2}\,(\partial_j u_i + \partial_i u_j)$ rate of strain, $g = \text{const} > 0$ yield value). The relations (2) model perfect plastic behavior of an incompressible fluid ("von Mises solid").

The weak formulation of (1), (2) in a bounded domain $\Omega \subset \mathbb{R}^n$ under Dirichlet boundary conditions on $\mathbf{u}$ leads to the problem

$$\text{minimize} \qquad \mathcal{F}(\mathbf{u}) := g \int_\Omega |D(\mathbf{u})| - \int_\Omega \mathbf{f} \cdot \mathbf{u}.$$

We solve this problem in the space $\mathrm{BD}(\Omega)$ under a physically motivated smallness assumption on $\mathbf{f}$. Our method of proof consists in approximating (2) by the power law

$$S_\varepsilon = g|D|^{\varepsilon - 1} D \qquad (\varepsilon > 0)$$

and carrying out the passage to the limit $\varepsilon \to 0$.

Point-particle limit of the nonlinear Hartree equation

Margherita Nolasco, University of L'Aquila, Italy

Abstract. We consider the non-linear Hartree equation with a slowly varying external potential and a short range, attractive two-body interaction. We prove the existence of stationary solutions which are approximatively given by a superposition of several Hartree solitons with their center of mass positions behaving, at the leading order, as classical particles at rest in the background potential.

Stabilization of Nonlinear Schrodinger Equation with Inhomogeneous Dirichlet Boundary Data

Turker Ozsari, University of Virginia, USA

Abstract. In this talk, we will talk about the global existence and stabilization of solutions of a weakly-damped nonlinear Schrodinger equation with inhomogeneous Dirichlet boundary data. We show that a physically reasonable decaying condition on the boundary data is enough to get stabilization of solutions. Moreover, our result shows that the decay rate of the boundary data controls the decay rate of solutions.

Partial regularity for a priori bounded minimizers of quasi-convex integrals

Antonia Passarelli di Napoli, University of Naples "Federico II", Italy

Abstract. We present $C^{1,\alpha}$ partial regularity results for a priori bounded minimizers of quasiconvex integrals verifying non standard growth conditions. A key ingredient is a gluing Lemma that allows us to construct suitable test functions.

Soliton Dynamics for CNLS systems and Related Topics

Benedetta Pellacci, University Napoli "Parthenope", Italy

Abstract. We will focus the attention on the dynamic, in the semi-classical limit, of families of solutions-not necessarily standing waves-of a subcritical focussing nonlinear Schrodinger systems generating from ground states under the action of external forces. Our intention is to recover the full dynamic of a solution as a point particle subjected to newtonian motion. This investigation will lead us to study some interesting properties of a class of ground state solution, such as being isolated, non-degenerated and being orbitally stable.

Long time approximations for solutions of wave equations in homogenization problems

M. Eugenia Perez, University Cantabria, Spain

Abstract. As is well known, quasimodes for positive, symmetric and compact operators on Hilbert spaces often arise in the literature when describing behaviors for high frequency vibrations (cf. [1] , [2] and [4]). Roughly speaking, a quasimode u can be defined as a function approaching a certain linear combination of eigenfunctions associated with eigenvalues in a "small interval" $[\lambda - r, \lambda + r]$. Its value in describing asymptotics for low and high frequency vibrations in certain singularly perturbed spectral problems, which depend on a small parameter ε, has been made clear recently in many papers (cf. [3] and [5] for more references). Considering ε-dependent second order evolution problems, we provide estimates for the time t in which *standing waves* of the type $e^{\mathrm{i}\sqrt{\lambda}t}u$ approach their solutions $\mathbf{u}^\varepsilon(t)$ when the initial data are quasimodes of the associated operators (cf. [6]). Here u, λ and r can also depend on ε. A general abstract framework and its applications to a spectral boundary homogenization problem are provided.

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Nonlinear Instability for the Primitive Equations

Madalina Petcu, Université de Poitiers, France

Abstract. We are interested in the study of the stability/instability for a flow of viscous incompressible fluid governed by the Primitive Equations. The Primitive Equations, equations that describe the evolution of the geophysical fluids, are introduced. It is proved that the linear instability implies non-linear instability for the three-dimensional Primitive Equations considered in a periodic domain.

A separation property and convergence to equilibria for a nonlocal phase-field system

Hana Petzeltova, Institute of Mathematics AS CR, Czech Republic

Abstract. We show that solutions of two-phase model involving a non-local interactive term and a singular potential separate from the pure phases even if it is not the case at the outset. This result allows to apply a generalized version of the Lojasiewicz–Simon theorem and to establish the convergence of any solution to a single equilibrium.

Bifurcation analysis of the phase field crystal model

Morgan Pierre, University of Poitiers, France

Abstract. K. R. Elder has recently derived a phase field model for the dynamics of crystal growth which includes elastic and plastic deformations. In contrast with other phase field approaches to elasticity, the model is constructed to produce phase fields that are periodic. The dimensionless version reads

$$u_t = \Delta\big(u + 2\Delta u + \Delta^2 u + f'(u)\big) \text{ in } \Omega \times \mathbf{R}_+,$$

where u is the order parameter, f is typically a double-well potential, and Ω is a bounded domain of $\mathbf{R}^d$ $(1 \leq d \leq 3)$. In this talk, we start a mathematical and numerical analysis of the steady states associated to this "phase field crystal model".

On the Gibbs–Thompson relation for anisotropic surface tension

Giovanni Pisante, Seconda Università di Napoli, Italy

Abstract. We consider the following family of energies

$$E_\varepsilon(u) = \int_\Omega \varepsilon f\left(Du\right) + \frac{W(u)}{\varepsilon}\, dx,$$

where f is a two homogeneous function and W is a double well potential. Under suitable hypotheses on f, as ε goes to 0, these energies approach the surface tension of a transition layer between two phases of a fluid that can be identified as the jump set of a function $u_0 \in \mathrm{BV}(\Omega; \{-1, 1\})$. More precisely we have that the functionals $E_\varepsilon(u)$ Γ-converge to the anisotropic perimeter of the limit interface , i.e.

$$\Gamma - \lim_{\varepsilon\to 0} E_\varepsilon(u_\varepsilon) = \int_{\Omega\cap\partial\{u_0=-1\}} \sqrt{f(\nu)}\, d\mathcal{H}^{n-1}$$

where ν is unit normal to $\partial\{u_0 = -1\}$.

The stable configurations of this fluid subjected to a given volume constraint correspond to solutions to the following variational problem

$$\min\left\{ E_\varepsilon(u) \ : \ \int_\Omega u\, dx = m \right\}.$$

which satisfy the Euler–Lagrange equations

$$\lambda_\varepsilon = \varepsilon \mathrm{div} D_p f\left(Du_\varepsilon\right) - \frac{1}{\varepsilon} W'(u_\varepsilon).$$

The aim of the talk is to discuss the asymptotic behaviour of the chemical potential λ_ε in terms of the geometry of the interface and in particular to underline its relation with the anisotropic curvature.

Tolksdorf 83 revisited

Alessio Porretta, Università di Roma Tor Vergata, Italy

Abstract. If (r, s) denote spherical coordinates in N dimensions, and if S is a subset of the N-dimensional unit sphere, we look for functions $u = r^b g(s)$ which are p-harmonic (i.e. with zero p-Laplacian) in the cone C_S with vertex in the origin and opening S. In 1983, P. Tolksdorf proved that, for any subdomain S of the sphere, there is a unique positive exponent b_S and a unique (up to a multiplicative constant) function $g(s)$ such that $u = r^b g(s)$ is p-harmonic in the cone. In a joint work with L. Veron, we give a totally different proof of Tolksdorf 's result and new interpretation and construction of the couple (b, g) in terms of ergodic quasilinear problems on the sphere.

Attractors for nonlinear parabolic problems in unbounded domains

Dalibor Prazak, Charles University, Czech Republic
(joint work with M. Grasselli and G. Schimperna)

Abstract. We study the long time behavior of solutions to nonlinear parabolic equations in unbounded domains. We propose to show the existence of a global attractor and estimate its Kolmogorov entropy, using the method of trajectories and suitable estimates with Bochner spaces with spatial weights.

Phragmen–Lindelof Principles for fully nonlinear equations with unbounded coefficients

Fabio Punzo, University "La Sapienza", Italy

Abstract. We address the following question: when a viscosity subsolution u to a degenerate elliptic equation

$$F(x, u, Du, D^2u) = 0 \text{ in } D,$$

which is assumed to be nonnegative on a subset of the boundary of D, is nonnegative in D. We deal with operators F satisfying natural structural conditions and possibly containing unbounded coefficients.

Weinberger method for a class of non constant boundary elliptic problems

Lakhdar Ragoub, Alyamamah University, Saudi Arabia

Abstract. We investigate a class of elliptic overdetermined problems using Weinberger method.

Boundedness and segregation of solutions for a Bose–Einstein type system

Miguel Ramos, University of Lisbon, Portugal

Abstract. We prove the existence of positive solutions for a system of the form $-\Delta u = u + u^3 - \beta u v^2$, $-\Delta v = v + v^3 - \beta u^2 v$, in a smooth bounded domain of R^3, with Dirichlet boundary conditions, as well as their segregation as the parameter β tends to infinity.

Repulsive Knot Energies and Bilinear Fourier Multipliers

Philipp Reiter, RWTH Aachen, Germany

Abstract. Imagine the motion of a knotted charged fiber within a viscous liquid. Will it reach a stationary point minimizing its kinetic energy? If so, will the resulting shape help to determine its knot type?

These questions led to the definition of a family of knot functionals modeling self-avoidance by Jun O'Hara in 1992. Later on, one particular functional among them, the so-called Möbius Energy, was extensively studied by Freedman, He, and Wang.

In this talk, we will briefly outline the concept of O'Hara's definition and the analytical reatment of critical points which leads to C^∞ regularity of local minimizers. The latter involves the study of bilinear Fourier multipliers defined on fractional Sobolev spaces.

Liouville-type results for a class of fully nonlinear equations

Luca Rossi, University of Padova, Italy

Abstract. We will present some non-existence results for positive viscosity supersolutions of some fully nonlinear equations in unbounded domains. We will discuss in particular the interplay between the growth of the supersolutions and the "size" of the domain.

Second order Moser type inequalities: a borderline case

Bernhard Ruf, Università degli Studi di Milano, Italy

Abstract. We study optimal embeddings for the space of functions whose Laplacian belongs to L^1. This function space turns out to be strictly larger than the Sobolev space $W^{2,1}$ in which the whole set of second order derivatives is considered. In particular, in the limiting Sobolev case, when $N = 2$, we establish a sharp embedding inequality into the Zygmund space L_{exp}. On one hand, this result enables us to improve the regularity estimate of Brezis–Merle for the Dirichlet problem with L^1 data; on the other hand, it represents a borderline case of D. R. Adams generalization of Trudinger–Moser type inequalities to the case of higher order derivatives.

A singular weighted mean curvature flow in the plane: a Hamilton–Jacobi equation with an unusual free boundary

Piotr Rybka, University of Warsaw, Poland
(joint work with Y. Giga and P. Gorka)

Abstract. We want to gain insight into the details of evolution of 3-D ice crystals which are hexagonal prisms (or are approximated by circular cylinders). The evolutions in question is governed by a Stefan-type problem with the Gibbs–Thomson law and a kinetic undercooling on the growing surface.

This gives us a motivation to study evolution of graphs of functions under the weighted curvature flow. We consider an example of a crystalline anisotropys and a general kinetic term. Then, the Gibbs–Thomson law degenerates to a first order equation with a free boundary. It turns out that the resulting problem is a Hamilton–Jacobi equation with spatially nonlocal and discontinuous Hamiltonian. Our goal is to show existence and uniqueness of solutions. For this purpose we have to study stability and a comparison principle for properly defined viscosity solutions.

Variational characterization of the Abrikosov lattice

Etienne Sandier, Université Paris-Est, France

Abstract. The Abrikosov Lattice is the name of the hexagonal lattice in the context of superconductivity, where superconducting vortices arrange themselves in this way. In a joint work with S.Serfaty we derive from the Ginzburg–Landau model an energy for configurations of points in the plane for which the optimal lattice is the Abrikosov lattice. The question of minimization among all possible configurations remains open.

Asymptotically regular problems

Christoph Scheven, University Erlangen-Nuremberg, Germany

Abstract. In the talk I will report on a joint work with Thomas Schmidt. We investigate the regularity properties of minimizers $u \in W^{1,p}(\Omega, \mathbb{R}^N)$ of variational integrals

$$F[u] := \int_\Omega f(Du)\, dx,$$

where f is an integrand of growth rate $p \geq 2$ that is convex *only near infinity*. Heuristically, it is plausible that for proving qualitative bounds for Du, only the behavior of the integrand close to infinity should be relevant. We show by Calderón-Zygmund type arguments that this heuristical idea can be made precise as far as global estimates in the scale of Lebesgue spaces are concerned. More precisely, we prove that for a minimizer $u \in W^{1,p}(\Omega, \mathbb{R}^N)$ of an asymptotically regular functional as above, there holds

- $Du \in L^{\frac{np}{n-2}+\kappa}_{\mathrm{loc}}(\Omega, \mathbb{R}^{Nn})$ for some $\kappa > 0$ if $n \geq 3$, and
- Du is locally bounded on the whole domain for $n = 2$ or $N = 1$.

Concerning partial regularity, it turns out that the heuristic reasoning from above is misleading and the situation is more complicated than in the case of convex integrands. We prove that in general, Du stays locally bounded only on an open and dense subset of Ω. Moreover, we demonstrate by a counterexample, based on a recent counterexample by Sverak and Yan, that this set may not have full measure.

Moreover, similar results apply to solutions of quasilinear systems of PDE's that are regular only close to infinity.

On a phase-field model for two-phase compressible fluids

Giulio Schimperna, University of Pavia, Italy

Miranville–Grasselli Session:

Abstract. In this talk we will present a global existence result for a model coupling the Allen–Cahn equation with the compressible Navier-Stokes system. The model is aimed to describe the motion of a compressible two-phase substance in two or three dimensions of space and is characterized by a variational structure referred to a suitable energy functional. In particular, the configuration potential energy turns out to present a singular dependence both with respect to the density and to the phase variable.

During the talk we will present the main ideas of the existence proof, which is based on a multi-level approximation scheme and on rather refined compactness tools, and we will also discuss some ideas in view of a long-time analysis of the model.

The results discussed in this talk have been obtained in collaboration with Eduard Feireisl and Hana Petzeltova (Academy of Sciences of Czech Republic, Prague) and with Elisabetta Rocca (University of Milano)

On the Cahn–Hilliard model with inertial effects

Giulio Schimperna, University of Pavia, Italy

Amar–Andreucci–Bisegna–Gianni Session:

Abstract. The Cahn–Hilliard equation has a central importance in material science since it can properly describe the evolution of the order parameter in a wide class of two-component materials (in particular, metallic alloys). Recently, on the basis of real-world experiments, some modifications of the equation have been proposed in order to include "inertial effects"in the model. These effects can be represented either by a term depending on the second time derivative of the unknown or by a time convolution with respect on some memory kernel with suitable regularity and dissipativity properties.

From the mathematical point of view, the inertial Cahn–Hilliard model presents a number of common features with the damped wave equation with polynomial nonlinearity; however, it is analytically much more involved. For instance, up to our knowledge, in three space dimensions no well-posedness result is available, even in the case of a globally Lipschitz nonlinear term.

In the talk we will present a number of results on existence of solutions in various regularity classes, uniqueness, dissipativity, and long-time behavior of the associated dynamical system. These results will provide a fairly complete theory in the two dimensional case, while in three space dimensions our analysis is subordinated to a smallness assumption on the initial data.

The results discussed in this talk have been obtained in collaboration with Maurizio Grasselli (Politecnico di Milano), Sergey Zelik (University of Surrey), and (partly) with Antonio Segatti (University of Pavia). We will also briefly discuss some related results obtained by other authors.

Regularity for quasiconvex integrals with (p, q)-growth

Thomas Schmidt, University Erlangen-Nuremberg, Germany

Abstract. I will report on results from [10], [11], [12]:

For a bounded domain Ω in $\mathbb{R}^n$ $(n \geq 2)$ we consider the minimization problem for variational integrals

$$F[u] := \int_\Omega f(Du)\, dx$$

among all vectorial functions $u : \Omega \to \mathbb{R}^N$ $(N \geq 1)$ with prescribed boundary values. We assume that the integrand $f : \mathbb{R}^{Nn} \to \mathbb{R}$ is a smooth strictly quasiconvex function satisfying the (p, q)-growth conditions introduced in [8]:

$$\gamma|z|^p \leq f(z) \leq \Gamma(1 + |z|^q) \qquad \text{for all } z \in \mathbb{R}^{Nn}$$

with some constants $1 < p \leq q < \infty$ and $\gamma, \Gamma > 0$. As a model case our assumptions include the case

$$f(z) = |z|^p + h(\det\, z),$$

where h is a smooth convex function with $\frac{q}{n}$-growth.

In the above setting we identify the $W^{1,p}$-quasiconvexity condition of [2] as the right one for proving both, the existence and the partial regularity of minimizers of F. Moreover, following a natural extension procedure — already studied in [8, 6, 4] — we discuss similar results for relaxed minimizers.

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The 3d Navier–Stockes equations

Vladimir Semenov, Kuzbass Regional Institute of Education Employees, Russia

Abstract. Necessary and sufficient conditions of existence of global solutions.

Elliptic equations with Hardy terms

Raffaella Servadei, Università della Calabria, Italy

Abstract. I will study some quasilinear elliptic equations with Hardy terms via variational methods and I will give some existence results of nontrivial weak solutions.

Qualitative results for integro-differential equations

Daniela Sforza, Sapienza Università di Roma, Italy

Abstract. We show some results concerning qualitative features of solutions for second order equations with integral convolution terms. More precisely, we give decay estimates at infinity for the energy of weak solutions, depending on the analogous decay for the convolution kernel.

In addition, we provide a reachability result for integro-differential equations with exponential type kernels.

Such equations serve as a model for concrete examples. In particular, our findings can be applied to some problems arising in viscoelasticity.

Linear and nonlinear problems for parabolic equations modeling atmospheric dispersion of pollutants

Aida Shahmurova and Veli Shakhmurov, Okan University, Turkey

Abstract. The boundary value problems for linear and non linear degenerate elliptic differential-operator equations of second order are studied. The principal part of differential operators generated by these problems possess varying coefficients and are non self-adjoint. Several conditions for the separability, R-positivity and the fredholmness in abstract L_p-spaces are given. By using these results the existence, uniqueness and the maximal regularity of boundary value problems for nonlinear degenerate parabolic differential-operator equations are established. In applications mixed boundary value problems for degenerate-diffusion systems, appearing on atmospheric dispersion of pollutants are studied.

Separable abstract elliptic equations with parameters and applications

Veli Shakhmurov, Okan University, Turkey

Abstract. The boundary value problems for elliptic partial differential-operator equations with parameters are studied. The principal part of the appropriate differential operators are not self-adjoint. Several conditions for the uniform separability in weighted Banach-valued L_p-spaces are given. The sharp estimates for the resolvent of the corresponding differential operator are obtained. Particularly, the positivity and R-positivity of these operators are established. In applications the separability of degenerate DOEs, maximal regularity for degenerate abstract parabolic problem with parameters, the uniform separability of finite and infinite systems for degenerate anisotropic partial differential equations with parameters are studied.

Asymptotic analysis for micromagnetics of thin films governed by indefinite material coefficients

Ken Shirakawa, Kobe University, Japan
(joint work with Hadiji, Rejeb, University of Paris 12, France)

Abstract. Let S be a two-dimensional bounded domain with a smooth boundary, and for any $h > 0$, let Ω_h be a three-dimensional domain, given by $\Omega_h := S \times (0, h)$. Let $p : \mathbb{R}^3 \longrightarrow [0, +\infty)$ be a given Lipschitz continuous function.

In this talk, we suppose that h is very small, and consider the minimization problem for the following energy functional:

$$\mathcal{E}^{(h)}(m) := \fint_{\Omega_h} \left(p|\nabla m|^2 + \varphi(m) + \frac{1}{2}\nabla\zeta \cdot m \right) dx,$$
$$\forall m \in L^2(\Omega_h; \mathbb{R}^3) \cap W^{1,2}_{\rm loc}(\Omega_h \setminus p^{-1}(0); \mathbb{R}^3);$$

subject to:

$$\mathrm{div}(-\nabla\zeta + \bar{m}) = 0 \text{ in } \mathbb{R}, \text{ and } |m| = m_s \text{ in } \Omega_h.$$

The energy $\mathcal{E}^{(h)}$, for each $0 < h \ll 1$, is supposed to be the free energy, per unit volume, of ferromagnetic thin film. In the context, h and Ω_h are, respectively, the thickness and the domain of the film. $m : \Omega_h \longrightarrow \mathbb{R}^3$ is the magnetization on Ω_h, $\bar{m}$ denote the zero-extension of m onto $\mathbb{R}^3$, and $m_s > 0$ is the constant of the saturation magnetization. p is the material coefficient that is spatial indefinite on Ω_h.

The main focus, in this talk, is on the limiting observation for the above minimization problem as $h \searrow 0$. Then, the indefinite coefficient p will be given in the following form:

$$p(x_1, x_2, x_3) := p_0(x_1, x_2) + x_3^\alpha, \quad \forall (x_1, x_2, x_3) \in \mathbb{R}^3;$$

with an exponent $\alpha > 0$, and a function $p_0 : \mathbb{R}^2 \longrightarrow [0, +\infty)$, which may degenerate on some negligible set with respect to Lebesgue measure. Consequently, a limiting free energy, described in the integral form on S, will be derived with some continuous dependence of the minimizers as $h \searrow 0$.

Modelling of an age-structured population dynamics with child care

Vladas Skakauskas, University of Vilnius, Lithuania
(joint work with Sarunas Repsys)

Abstract. The Sharpe-Lotka-McKendrick-von Foerster one-sex age-structured population model and Fredrickson-Hoppebsteadt-Staroverov two-sex age-structured population one are well known in mathematical biology. But they do not describe dynamics of populations taking child care of their offsprings. In recent years, some models were proposed to describe dynamics of a wild population taking care of offsprings. They are based on the notion of the density of offsprings under maternal or parental care. However such models do not ensure the fact that offsprings under maternal (or parental) care move together with their mother (or both parents). In recent years, to solve this problem some models of a sex-age-structured population, based on the discrete set of newborns were proposed and some analytical results were published.

Numerical schemes for solving of a one-sex age-structured population model with and without spatial diffusion taking into account a discrete set of offsprings and child care and some numerical results will be discused.

Parabolics for the Sitnikov problem

Chouhaid Souissi, Monastir University, Tunisia

Abstract. We show, using a variational method, a topological functional convergence theorem and a comparison argument, the existence of homoclinic orbits at infinity for the following "Sitnikov" problem

$$\ddot{z}(t) + \alpha.\frac{z(t)}{(z(t)2 + r2(t))^{1+\alpha/2}} = 0;$$

where $0 < \alpha \leq 1$, r is T-periodic for some $T > 0$ and $r(t) > 0, \forall t \in \mathbb{R}$.

Regularity of differential forms via $\mathcal{A}$-harmonic approximation

Bianca Stroffolini, University "Napoli Federico II", Italy

Abstract. I will present some regularity results contained in a joint paper with Lisa Beck, University of Erlangen.

Let M be a regular open region of a closed, oriented , Riemannian manifold $(\mathcal{R}, g)$ of dimension $n \geq 2$. We consider differential ℓ-forms $\omega \in L^p(M, \Lambda^\ell)$, $1 < p < \infty$, which are weak solutions to the elliptic system

$$d^* A(x, \omega) = 0 \qquad \text{and} \qquad d\omega = 0\,, \tag{1}$$

for a vectorfield $A\colon M \times \Lambda^\ell \to \Lambda^\ell$ satisfying the following structure conditions: the mapping $\omega \mapsto A(x, \omega)$ is of class $C^0(\Lambda^\ell, \Lambda^\ell) \cap C^1(\Lambda^\ell \setminus \{0\}, \Lambda^\ell)$, and for fixed numbers $0 < \nu \leq L$ and all $x, \bar{x} \in M$ and $\omega, \bar{\omega} \in \Lambda^\ell$ there hold the following assumptions concerning growth, ellipticity and continuity:

(H1) A is Lipschitz continuous with respect to ω with

$$|A(x, \omega) - A(x, \bar{\omega})| \leq L\,(|\omega|^2 + |\bar{\omega}|^2)^{\frac{p-2}{2}}\,|\omega - \bar{\omega}|\,,$$

(H2) $D_\omega A$ is Hölder continuous with some exponent

$$\alpha \in \begin{cases} (0, p-2) & \text{if } p > 2\,, \\ (0, 2-p) & \text{if } p \in (1,2)\,, \end{cases}$$

such that there holds for $p > 2$

$$|D_\omega A(x, \omega) - D_\omega A(x, \bar{\omega})| \leq L\,(|\omega|^2 + |\bar{\omega}|^2)^{\frac{p-2-\alpha}{2}}\,|\omega - \bar{\omega}|^\alpha\,,$$

whereas in the subquadratic case $p \in (1,2)$ there holds for all $\omega, \bar{\omega} \neq 0$

$$|D_\omega A(x, \omega) - D_\omega A(x, \bar{\omega})| \leq L\,|\omega|^{p-2}|\bar{\omega}|^{p-2}(|\omega|^2 + |\bar{\omega}|^2)^{\frac{2-p-\alpha}{2}}\,|\omega - \bar{\omega}|^\alpha,$$

(H3) A is degenerately monotone:

$$\langle A(x, \omega) - A(x, \bar{\omega}), \omega - \bar{\omega}\rangle \geq \nu\,(|\omega|^2 + |\bar{\omega}|^2)^{\frac{p-2}{2}}\,|\omega - \bar{\omega}|^2\,,$$

(H4) A is Hölder continuous with respect to its first argument with exponent $\beta \in (0,1)$:

$$|A(x, \omega) - A(\bar{x}, \omega)| \leq L\,|\omega|^{p-1}\,|x - \bar{x}|^\beta\,,$$

(H5) A is of Uhlenbeck structure at 0, i. e., there exists a non-decreasing function $\widetilde{\mu} : \mathbb{R}^+ \to \mathbb{R}^+$ such that for all $\tilde{\omega} \in \Lambda^\ell$ with $|\tilde{\omega}| \leq \widetilde{\mu}(t)$ there holds

$$|A(x, \tilde{\omega}) - g_x(|\tilde{\omega}|)\,\tilde{\omega}| \leq t\,|\tilde{\omega}|^{p-1}$$

uniformly for all $x \in M$,

where g_x is a family of functions satisfying Uhlenbeck-type conditions.

Uhlenbeck in her fundamental paper in proving interior Hölder continuity for weak solutions ω to the nonlinear elliptic system of p-growth. Hamburger extended these results up to the boundary for functionals given in a special form corresponding to the Uhlenbeck system. In both proofs, estimates of Moser for sub- and supersolutions play a crucial role. Moreover, Hamburger provides a regularity result for weak solutions of general systems via an indirect blow up argument.

We obtain partial Hölder regularity of ω via the method of $\mathcal{A}$-harmonic approximation. Therefore, first we give an appropriate formulation of the $\mathcal{A}$-harmonic approximation lemma in the setting of differential forms. The next step is then to establish an excess decay estimate to finally deduce the desired regularity result from Campanato's characterization of Hölder continuity in a version for differential forms. We here refer to Duzaar, Grotowski, Kronz, Mingione papers for the case of functions.

Homogenization of nonstationary periodic equations

(Tatiana Suslina, St. Petersburg State University, Russia)
joint work with M. Sh. Birman

Abstract. We study homogenization for the Cauchy problem for nonstationary Schrödinger equation and hyperbolic equation with rapidly oscillating coefficients. We prove qualified estimates in $L_2(R^d)$ for the difference of the solution and the solution of the corresponding homogenized problem. Estimates are uniform with respect to the norms of initial data in appropriate Sobolev space H^s.

An Extension and modification for the Gauss pseudospectral method and its application for solving two point boundary value problems

Hamid Reza Tabrizidooz, Isfahan University of Technology, Iran

Abstract. It is well known that the pseudospectral methods provide highly accurate approximations for the solutions of nonlinear operator equations in the function spaces, provided that these solutions are sufficiently smooth. In this talk we present, simultaneously, an extension and also a modification for the well-known Gauss pseudospectral method in which the boundary conditions can be applied directly in the construction of corresponding operational matrix of derivative without losing the order of accuracy. We introduce the composite interpolation formula as an extension for the Lagrange interpolation formula based on the Legendre-Gauss-type points and also discuss some approximation results. According to the weak representations of the derivative operator, we then construct the corresponding operational matrix of derivative as a modification for that in the Gauss pseudospectral method. To demonstrate the applicability and efficiency of the method for discretizing differential equations, we examine two numerical problems in the two point boundary value problems and compare the obtained results with the available and exact solutions.

Explicit solutions for the Stefan Problem with temperature dependent thermal conductivity and a convective term

Domingo Tarzia, Conicet - University Austral, Argentina
(joint paper with M.F. Natale)

Abstract. We study a one-phase Stefan problem for a semi-infinite material with temperature dependent thermal conductivity and a convective term with a positive constant temperature boundary condition or a heat flux boundary condition of the type $-q_0/\sqrt{t}$ ($q_0 > 0$) at the fixed face $x = 0$. We obtain in both cases sufficient conditions for data in order to have a parametric representation of the solution of the similarity type for $t \geq t_0 > 0$ with t_0 an arbitrary positive time. We improve the results given in Rogers-Broadbridge, ZAMP, 39 (1988), 122-129 and Natale-Tarzia, Int. J. Eng. Sci., 41 (2003), 1685-1698 obtaining explicit solutions through the unique solution of a Cauchy problem or an integral equation with the time as a parameter and we also give an algorithm in order to compute the explicit solution in both cases.

Asymptotic behavior of solutions of some fractional differential equations

Nasser-eddine Tatar, King Fahd University of Petroleum and Minerals, Saudi Arabia

Abstract. In this talk we intend to investigate the asymptotic behavior of solutions of some fractional differential equations. Namely, we will prove polynomial decay of solutions under some weak conditions. To this end we need to generalize some well-known inequalities involving derivatives to the case of fractional order derivatives to derive some upper bounds. The polynomial stability of solutions of some fractional differential equations become an easy and immediate consequence.

Ginzburg–Landau vortex dynamics driven by an applied boundary current

Ian Tice, Brown University, USA

Abstract. This talk concerns recent results on the time-dependent Ginzburg–Landau equations on a smooth, bounded 2-D domain, subject to both an applied magnetic field and an applied boundary current. The boundary current is modeled with a gauge-invariant inhomogeneous Neumann boundary condition. The energy of such solutions does not dissipate, but it is possible to estimate the growth of the energy. This estimate then allows for the derivation of the dynamics of the vortices in the limit as the Ginzburg–Landau parameter vanishes. In an appropriate time scale, I will show that the vortex motion is driven by a novel Lorentz force term induced by the presence of the applied boundary current.

Completion of generalized elliptic and parabolic problems

Jukka Tuomela, University of Joensuu, Finland

Abstract. The notion of ellipticity (parabolicity) was generalized by Douglis and Nirenberg in 50s (Solonnikov in 60s). We show that by completing the systems they become elliptic/parabolic in the standard sense. This new completion procedure is based on finite resolution of modules. The actual computations can be carried out with Gröbner basis techniques. The completed systems are in general overdetermined, and hence somewhat nonstandard. In spite of that this approach has turned out to be interesting also in numerical computations and we give some examples related to flow problems.

Regularity results for minimizers of functionals with general growth

Anna Verde, "Federico II" of Naples, Italy

Abstract. We prove $C^{1,\alpha}$-regularity for local minimizers of functionals with general growth, providing also the decay estimate for the "excess". In particular we present a unified approach in the case of power type functions. As an application of the excess decay estimate, we prove Lipschitz continuity for local minimizers of asymptotically convex problems.

Comparison principles and Holder gradient estimates for viscosity solutions

Antonio Vitolo, University of Salerno, Italy

Abstract. Our aim here is to discuss the extension of the Holder estimates of L. A. Caffarelli for the gradient of C-viscosity solutions of second order uniform elliptic equations with lower order terms. In our approach the basic estimate for equations independent of the spatial variables is obtained establishing a maximal inequality for the derivatives from which comparison principles in unbounded domains between C-viscosity solutions can be also deduced without additional regularity assumptions. As a consequence of the above Holder gradient estimates, we can furtherly generalize the so-called Gleaser inequality recently considered by Y. Li and L. Nirenberg. The subject is framed in the thematic of interest for a national research project in progress at University of Rome 1 and addressed in collaboration with Italo Capuzzo Dolcetta.

Elliptic Differential-Operator Problems with Application to Elliptic PDEs

Yakov Yakubov, Tel-Aviv University, Israel

Abstract. We consider boundary value problems for elliptic differential-operator equations with the spectral parameter and operator-boundary conditions containing the parameter of the same order as the equation. An isomorphism and the corresponding estimate of a solution (with respect to the space variable and the parameter) are obtained. Then, discreteness of the spectrum and completeness of a system of root functions of the corresponding homogeneous problem are established. Finally, an application is given of the abstract results obtained on the isomorphism and the completeness to boundary value problems for elliptic partial differential equations with a parameter in non-smooth domains.

The Asymptotic Behavior of Elliptic Variational Inequalities with Constraint on the Gradient in Cylinders

Karen Yeressian, University of Zurich, Switzerland

Abstract. In a cylinder we consider an elliptic variational inequality with constraint on the gradient and two solutions with different boundary conditions on the ends of the cylinder. It is proved that these two solutions converge to each other by polynomial rate of convergence as the length of the cylinder grows. This is a result similar to Saint-Venant's principle which holds for linear elliptic equations.

An Approach to Solvability of Generalized Navier–Stokes Equations

Vasily Zhikov

Abstract. The Navier–Stokes equation with a non-linear viscous term will be considered, $p > 1$ is the exponent of non-linearity. An existence theorem is proved for the case when the convection term is not subordinate to the viscous term, in particular for the previously open case $p < 2$. The method of proof is based on ideas of the geometric measure theory and compensated compactness. A space-time measure may have a singular component. A connection between this singularity and the known results the partial regularity theory is discussed, in particular for the classical case $p = 2$.