A unified IMEX strategy for hyperbolic systems with multiscale relaxation

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Several systems of evolutionary partial differential equations may contain stiff terms, which require an implicit treatment. Typical examples are hyperbolic systems with stiff hyperbolic or parabolic relaxation and kinetic equations in regimes close to fluid dynamic limit. In the hyperbolic-to-hyperbolic relaxation (HSHR) a natural treatment consists in adopting implicit-explicit (IMEX) schemes, in which the relaxation is treated by an implicit scheme, while the hyperbolic part is treated explicitly [1]. In the hyperbolic-to-parabolic relaxation (HSPR) standard methods relax to an explicit scheme for the parabolic limit, thus suffering from parabolic CFL restriction. This drawback can be overcome by a penalization method, consisting in adding and subtracting the same term, so that the system appears as the limit relaxed system plus a small perturbation, [2, 3]. In this talk we present a unified IMEX approach for systems which may admit both limits. This generalizes the two approaches: HSHR and HSPR. The methodology is illustrated in the case of the simple 2x2 system

\[
\begin{aligned}
    u_t + v_x &= 0, \\
    \varepsilon^\alpha v_t + \frac{1}{\varepsilon^\alpha} u_x &= -\frac{1}{\varepsilon}(v - f(u))
\end{aligned}
\]

depending of an additional parameter \(\alpha\) which modifies the nature of the asymptotic behaviour which can be either hyperbolic (\(\alpha = 0\), gives HSHR) or parabolic (\(\alpha = 1\), gives HSPR). The main idea is to treat the variable \(v\) in the first equation implicitly and to discretize the time by globally stiffly accurate IMEX schemes. The modified equation associated to the scheme has bounded characteristic speeds. This approach is capable to capture the correct asymptotic limit of the system independently of the scaling used. Several examples will be presented.

References

