Abstracts

Workshop
Engelberg, September 2–4, 2015

Renormalized quantum dimension and multivariable link invariants

Cristina Ana-Maria Anghel

In this talk we will describe the multivariable link invariants introduced by N. Geer and B. Patureau in [2] and their relations with other previously known polynomial type invariants. This construction has as an algebraic input a super Lie algebra of type one. Because of the algebraic particularities of this kind of algebras, the classical Reshetikhin-Turaev construction vanishes, the reason being the fact that the quantum dimension of an object is often zero. First of all, we will present the definition of the renormalized quantum dimension, which in a Reshetikhin-Turaev type construction leads to non-vanishing invariants. After that, we will explain the polynomial behavior of this renormalized construction and the Geer and Patureau's multivariable link invariants. Secondly, we will present how the renormalized multivariable invariants recover the multivariable Alexander polynomials, specialize to the ADO invariants [1] and contain the Kashaev's invariants [3].

[1] Y. Akutsu, T. Deguchi, and T. Ohtsuki - Invariants of colored links. J. Knot Theory Ramifications 1 (1992), no. 2, 161-184.

[2] N. Geer, B. Patureau Multivariable link invariants arising from Lie super algebras of type I, Journal of Knot Theory and Its Ramifications, Vol. 19, No. 1 (2010), 93-115.

[3] N. Geer, B. Patureau-Mirand, On the Colored HOMFLY-PT, Multivariable and Kashaev Link Invariants, Commun. Contemp. Math., 10 (2008), suppl. 1, 993-1011.

Quantum invariants, Skein relations and Volume Conjecture

Qingtao Chen

In knot theory, Jones, HOMFLY and Kauffman polynomials share a common feature that they can be defined via a purely combinatorial method called skein relation. In this way, a knot polynomial can be computed recursively by reducing its crossing numbers. But it's rather hard to search skein relations for quantum invariants such as colored HOMFLY and colored Jones invaraints. Recently, we proposed several conjectures of (congruent) skein relations at roots of unity for colored HOMFLY invariants and colored Jones polynomials etc. We have proved series of examples for these new conjectures, especially the knot case for colored Jones polynomials. The motivation behind this phenomenon involves several areas of mathematics as well as string theory, which may also shed some new light on the Volume conjecture. This is a joint work with Kefeng Liu, Pan Peng and Shengmao Zhu. Finally I will introduce a very recent discovery of new Volume Conjectures for hyperbolic 3-manifolds through Turaev-Viro type invariants and Reshetikhin-Turaev invariants.

This is a joint work with Tian Yang.

A bicategorical version of the Universal Construction

Marco De Renzi

An ETQFT (Extended Topological Quantum Field Theory) is a symmetric monoidal functor from a bicategory of cobordisms to an algebraic bicategory. In this setting cobordisms can be further decomposed into even simpler pieces, so that a relevant feature which is improved while upgrading a TQFT to an ETQFT is computability. The Universal Construction introduced by Blanchet, Habegger, Masbaum and Vogel in the seminal paper *Topological Quantum Field Theories Derived from the Kauffman Bracket* is one of the most natural ways of looking for TQFTs extending a given quantum invariant, and it was originally used to obtain a TQFT for the Witten-Reshetikhin-Turaev quantum $\mathfrak{sl}_2$ invariant. In this talk I will give the definition of a suitable bicategory of extended cobordisms (similar to ones defined by Kerler and Lyubashenko, by Morton and by Schommer-Pries) and outline how we can actually follow the Universal Construction of BHMV to obtain a proper ETQFT for the WRT quantum invariant associated with any modular category. This construction is aimed to a future adaptation to the non-semisimple case and, hopefully, to the definition of an ETQFT for the Costantino-Geer-Patureau non-semisimple quantum invariant associated with any relative G-modular category.

Re-normalization Turaev-Viro invariants

Nathan Geer

Many of the standard techniques in quantum topology require that the underlying category be semi-simple. Over the past ten years working in various collaborations with C. Blanchet, F. Costantino, B. Patureau-Mirand, N. Reshetikhin, V. Turaev and A. Virelizier we have constructed a theory of re-normalized quantum invariants from categories which are not semi-simple. The goal of my talks is to give an outline of the construction of re-normalization Turaev-Viro invariants. I will do this in three major steps. First, I will discuss the renormalized graph invariants which are obtained by replacing the vanishing quantum dimensions with non-zero modified quantum dimensions. An example of this construction (for links) will be discussed by Cristina Anghel in the case of super Lie algebras. Second, I will discuss how these graph invariants lead to re-normalized 6j-symbols. Finally, I will present the usual Turaev-Viro construction of an invariant of 3-manifolds arising from a modular category. It is based on a state sum associated to a triangulation of the manifold. This is a sum over all isomorphism class of simple objects of a modular category. I will say how one can modify this construction to the setting of certain non-semi-simple categories. The main obstacle is that these categories can have infinitly many non-isomorphic simple objects. In the whole talk, I will focus on the example of nilpotent representations of quantized $sl(2)$ at a root of unity. Clinton Reece is working on showing that in this example the re-normalization Turaev-Viro invariants should be related to the re-normalized Reshetikhin-Turaev quantum invariants. The later invariants will be discussed in the talks of Konstantinos Karvounis and Marco De Renzi.

Investigating the invariants $\mathrm{N_r}$ for $r \geq 3$

Konstantinos Karvounis

In [1] invariants of coloured links and 3-manifolds have been constructed through non semi-simple categories. For $r = 2$, it is proved that the Alexander polynomial is recovered by the invariant N_2 of coloured links (see [2]). The cases $r \geq 3$ remain under investigation. In this talk, we discuss the case $r = 3$, when all components of a link are coloured by the "Kashaev module". By computing the R-matrix (via the help of a computer), we try to recover a skein relation for the invariant of coloured links, which may involve trivalent graphs. Finally, we briefly discuss the possibility to similarly examine the case for $r = 5$.

References

[1] F. Costantino, N. Geer, B. Patureau-Mirand *Quantum invariants of 3-manifolds via link surgery presentations and non-semi-simple categories*, Journal of Topology (2014) **7 (4)**, 1005-1053.

[2] C. Blanchet, F. Costantino, N. Geer, B. Patureau-Mirand *Non semi-simple TQFTs, Reidemeister torsion and Kashaev's invariants.*, `arXiv:1404.7289`

On categorical traces and invariants of links in a solid torus

Krzysztof Karol Putyra

A trace of a category is a set of its endomorphisms considered up to conjugation. For example, the trace of the category of tangles is formed by links in a solid torus: every such link is a closure of a some tangle and closures of two tangles coincide if and only if the tangles agree up to conjugation. The trace is functorial: a functor between categories induces a map between their traces. In particular, we can obtain invariants of links in a solid torus from a functor on the category of tangles. In my talk I will discuss the tangle invariant due to Chen and Khovanov and its trace, recovering from the latter the annular Khovanov homology, defined by Asaeda, Przytycki, and Sikora. This construction equips immediately APS homology with an action of SL_2. At the end of my talk I will briefly describe how to obtain an infinite family of annular Khovanov homologies, showing that the trace of the Chen-Khovanov homology is much richer than the APS homology alone.

This is a joint work with Anna Beliakova.