Abstracts invited talks

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Some recent results about a class of Logarithmic Schrödinger equations

Claudianor Alves, Federal University of Campina Grande, Brazil

Abstract. In this talk we will show some recent results involving a class of Logarithmic Schrödinger equations in whole $\mathbb{R}^N$. By using a Variational Methods developed by Szulkin, we prove the existence, multiplicity and concentration of positive solutions.

The results that will be shown are in some papers that were written in collaboration with D. de Morais Fillho (Brazil), G. Figueiredo (Brazil), C. Ji (China) and V. Ambrosio (Italy).

Non classical memory kernels in viscoelasticity problems

Sandra Carillo, Sapienza University of Rome, Italy

Abstract. Materials with memory are modelled via integro-differential equations exhibiting an integral term which takes into account the past 'history' of the material. In particular, a body is termed viscoelastic when its mechanical response is determined not only by the present status but also by its deformation history. The kernel of the integral term, in the integro-differential equation to study, represents the relaxation function G which characterises the viscoelastic material. The relaxation function is assumed to satisfy regularity requirements, weaker than the classical ones to model wider classes of materials. Specifically, the cases, thermodynamically admissible, of an unbounded kernel at the origin and of a non-increasing and convex kernel are discussed. Results of existence and, possibly, uniqueness are provided. Some results on magneto-viscoelasticity, i.e. when the viscoelastic material is also subject to a magnetic field are also mentioned.

The presented works are part of a joint research program with V. Valente, G. Vergara Caffarelli and, recently, also M. Chipot.

Nonlinear Korn inequalities on a surface

Philippe Ciarlet, City University of Hong Kong

Abstract. It is well known that a surface can be recovered from its two fundamental forms if they satisfy the Gauss and Codazzi–Mainardi compatibility equations on a simply-connected domain, in which case the surface is uniquely determined up to isometric equivalence. It is less known that in this case the surface becomes a continuous function of its fundamental forms, again up to isometric equivalence, for various topologies, such as the Fréchet topology of continuously differentiable functions, or those corresponding to various Sobolev norms. In this talk, we will review such continuity results obtained during the past fifteen years, with special emphasis on those that can be derived by means of nonlinear Korn inequalities on a surface. We will also mention potential applications of such results, such as the intrinsic approach to nonlinear shell theory, where the unknowns are the fundamental forms of the deformed middle surface of a shell.

A maximal dissipation condition for dynamic fracture with an existence result in a constrained case

Gianni Dal Maso, SISSA, Italy

Abstract. We consider a model of elastodynamics with crack growth, based on an energy-dissipation balance and a maximal dissipation condition. We prove an existence result in the case of planar elasticity, where the maximal dissipation condition is satisfied among suitably regular competitor cracks. The case of kinks is also considered.

Fine structure of measures satisfying a PDE constraint

Guido De Philippis, SISSA, Italy

Abstract. In this talk I will present some new result concerning the structure of measure satisfying a linear PDE constraint. In 2016, in collaboration with Filip Rindler, we prove a first structural result concerning the singular part of measure subject to PDE constraint. This turned out to have several applications in GMT and in Geometric Analysis. Recently, in a joint work with Adolfo Arroyo Rabasa, Jonas Hirsch and Filip Rindler we improve upon this result proving a more precise structure on the low dimensional part of the measure. As a corollary we recover several known rectifiability results. In this talk I will try to give an overview of both these results and of their applications.

A case for 3D geometric non linear models for material defects

Adriana Garroni, Sapienza University of Rome, Italy

Abstract. We study a nonlinear three dimensional elastic model which includes incompatibilities of the deformation matrix. The incompatibilities represent topological singularities and characterise an important class of line defects in crystals, the dislocations. In particular the model provides a nonlinear semi-discrete energy for dislocations in 3d which incorporates invariance under rigid motions. The aim is to capture, at different scales, different effects (e.g., surface tension at grain boundaries or low energy dislocation structures). I will give a brief overview of the type of results that have been obtain in this directions in a two dimensional framework. The 3D analysis is substantially more complex. I will present some of the results in 3d underlining the main difficulties.

Bistable transition fronts in unbounded domains

Franҫis Hamel, Aix-Marseille University, France

Abstract. The standard notions of reaction-diffusion fronts can be viewed as examples of generalized transition fronts describing the invasion of a state by another one. These notions involve uniform limits, with respect to the geodesic distance, to a family of time-dependent hypersurfaces. The existence of transition fronts has been proved in various contexts where the standard notions of fronts make no longer sense. Even for homogeneous equations, fronts with various non-planar shapes or with varying speeds are known to exist. In this talk, I will report on some recent existence results and qualitative properties of transition fronts for bistable equations. I will also discuss their mean speed of propagation in various domains, such as the whole space, exterior domains or domains with cylindrical branches.

The talk is based on some joint works with H. Berestycki, H. Guo, H. Matano and W.-J. Sheng.

A free boundary tumor model with time dependent nutritional supply

Xiaoying Han, Auburn University, USA

Abstract. In this talk I will introduce a non-autonomous free boundary model for tumor growth. The model consists of a nonlinear reaction diffusion equation describing the distribution of vital nutrients in the tumor and a nonlinear integro-differential equation describing the evolution of the tumor size. Existence and uniqueness of a transient solution, as well as the steady state solution will be shown. The long time behavior the the solutions will also be discussed.

Monotone Hopf Harmonics

Tadeusz Iwaniec, Syracuse University, USA

Abstract. I will present my recent joint work with Jani Onninen. We introduced the concept of monotone Hopf-harmonics in 2D as an alternative to harmonic homeomorphisms. It opens a new area of study in Geometric Function Theory (GFT) and Nonlinear Elasticity (NE). The question we are concerned with is whether or not a Dirichlet energy-minimal mapping between Jordan domains with prescribed boundary homeomorphism remains injective in the domain. The classical theorem of Rado-Kneser-Choquet asserts that this is the case when the target domain is convex. An alternative way to deal with arbitrary target domains is to minimize the Dirichlet energy subject to only homeomorphisms and their limits. This leads to the so called *Hopf–Laplace Equation*. Among its solutions (some rather surreal) are continuous monotone mappings called *Monotone Hopf–Harmonics*.

Spreading speed of nonlocal diffusion KPP equations in almost periodic media

Xing Liang, University of Science and Technology of China, China

Abstract. In this talk, I will introduce our work on the spreading speed of nonlocal diffusion KPP equations in almost periodic media.

In this work, the diffusion kernel is just assumed to be the generalized density function of a probability measure. In this work, we show that if the heterogeneity of the media can be averaged by the the diffusion, then $(*)$ has spreading speeds.

Elliptic and parabolic equations under general and $p, q-$growth conditions

Paolo Marcellini, University of Florence, Italy

Abstract. We consider *variational solutions* to the Cauchy-Dirichlet problem

$$\begin{cases} \partial_t u = \mathrm{div} D_\xi f(x, u, Du) - D_u f(x, u, Du) & \text{in } \Omega_T \\ u = u_0 \quad \text{on } \partial_{\mathrm{par}} \Omega_T \end{cases}$$

where the function $f = f(x, u, \xi)$, $f : \mathbb{R}^n \times \mathbb{R}^N \times \mathbb{R}^{N \times n} \to [0, \infty)$, is convex with respect to (u, ξ) and coercive in $\xi \in \mathbb{R}^{N \times n}$, but f not necessarily satisfies a growth condition from above. A motivation to consider a class of such energy functions f can be also easily found in the stationary case, where a large literature in the *calculus of variations* is devoted to the minimization of $p, q-$*growth* problems. In the parabolic context the notion of variational solution, introduced by Bögelein-Duzaar-Marcellini, is compatible with the lack of *the same polynomial growth* from below and from above.

Existence theorems in nonlinear elasticity

Cristinel Mardare, Sorbonne University, France

Abstract. The deformation of an elastic body induced by applied forces independent of time can be computed by minimizing a functional, representing the “total energy” of the body, over an appropriate set of vector fields, representing the “admissible deformations” of the body. But the existence of minimizers is not guaranteed in nonlinear elasticity since in that case the total energy of the body is not convex. In three-dimensional elasticity, where the admissible deformations of the body are vector fields defined over a three-dimensional domain, the existence of minimizers has been established by John Ball. It relies on the observation that the functional to be minimized is “polyconvex” in a specific sense for a large class of nonlinearly elastic materials. In this talk, I will briefly review the existence theory of John Ball, then will show how this theory can be extended to nonlinearly elastic “shells”, where the admissible deformations of the body are vector fields defined over a surface.

Helmholtz equation with rapidly oscillating coefficients

Stefan Sauter, University of Zurich, Switzerland

Abstract. We present new results on the stability of the Helmholtz equation with non- smooth and rapidly oscillating coefficients on bounded domains for the heterogeneous Helmholtz equation. Injectivity of the problem is proved for a large class of coefficients by the unique continuation principle, however, this does not give directly a coefficient-explicit energy estimate.

In this talk, we will present a new theoretical approach for the one-dimensional and radial-symmetric problems and find that for a class of oscillatory and discontinuous coefficients, the stability constant (i.e., the norm of the solution operator applied to a r.h.s. in L^2) is bounded by a term independently of the number of discontinuities.

We present examples of coefficients so that the solution has exponentially increasing local energy with respect to the frequency at any predetermined location inside the domain, showing that our estimates are sharp.

Anisotropic Variant of the BMO–type seminorms

Roberta Schiattarella, University of Naples Federico II, Italy

Abstract. The purpose of this talk is to present the relation between certain BMO–type seminorms and the total variation of SBV functions.

In [3] the Authors introduced a new BMO–type space $B \subset L^1(Q)$ on the unit cube $Q \subset \mathbb{R}^n$, by mean of the seminorm

$$\|f\|_B = \sup_{0<\varepsilon<1} [f]_\varepsilon \tag{1}$$

where $[f]_\varepsilon$ is defined with a suitable maximization procedure. The space B contains BMO and the space BV of functions of bounded variation (see also [5]).

Later in [1], a new characterization of the perimeter of sets in terms of this seminorm considering characteristic functions of sets, was studied. Further results characterizing total variation of SBV functions and norm of Sobolev functions, independent of theory of distributions, were given in [6], [7] (see also [4], [2]).

Using a different approach, by considering in (1), instead of cubes, covering families by translations of a given open bounded set with Lipschitz boundary, we give a representation formula of the total variation of SBV function.

References

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Stability of vortex columns for the 3D Euler equation

Didier Smets, Sorbonne University, France

Abstract. Vortex columns are stationary solutions of the Euler equations with a particular physical relevance. Lord Rayleigh established two criteria regarding their stability : one which is applicable for 2D perturbations, and a second for axisymmetric perturbations. After reviewing these, the talk will focus on recent results obtained in collaboration with Thierry Gallay (Grenoble) which establish the spectral stability of vortex columns with respect to arbitrary 3D perturbations.

Spiralling and other solutions in limiting profiles of competition-diffusion systems

Susanna Terracini, University of Turin, Italy

Abstract. Reaction-diffusion systems with strong interaction terms appear in many multi-species physical problems as well as in population dynamics. The qualitative properties of the solutions and their limiting profiles in different regimes have been at the center of the community's attention in recent years. A prototypical example is the system of equations

$$\begin{cases} -\Delta u + a_1 u = b_1 |u|^{p+q-2} u + cp|u|^{p-2}|v|^q u, \\ -\Delta v + a_2 v = b_2 |v|^{p+q-2} v + cq|u|^p |v|^{q-2} v \end{cases}$$

in a domain $\Omega \subset \mathbb{R}^N$ which appears, for example, when looking for solitary wave solutions for Bose-Einstein condensates of two different hyperfine states which overlap in space. The sign of b_i reflects the interaction of the particles within each single state. If b_i is positive, the self interaction is attractive (focusing problems). The sign of c, on the other hand, reflects the interaction of particles in different states. This interaction is attractive if $c > 0$ and repulsive if $c < 0$. If the condensates repel, they eventually separate spatially giving rise to a free boundary. Similar phenomena occurs for many species systems. As a model problem, we consider the system of stationary equations:

$$\begin{cases} -\Delta u_i = f_i(u_i) - \beta u_i \sum_{j \neq i} g_{ij}(u_j) \\ u_i > 0 \, . \end{cases}$$

The cases $g_{ij}(s) = \beta_{ij} s$ (Lotka-Volterra competitive interactions) and $g_{ij}(s) = \beta_{ij} s^2$ (gradient system for Gross-Pitaevskii energies) are of particular interest in the applications to population dynamics and theoretical physics respectively.

Phase separation and has been described in the recent literature, both physical and mathematical. Relevant connections have been established with optimal partition problems involving spectral functionals. The classification of entire solutions and the geometric aspects of phase separation are of fundamental importance as well. We intend to focus on the most recent developments of the theory in connection with problems featuring anomalous diffusions, non-local and non symmetric interactions.

Nonlinear problems and sandpile models

Maria Agostina Vivaldi, Sapienza University of Rome, Italy

Abstract. In this talk we deal with theoretical and numerical aspects of some non-linear problems related to sandpile models. In particular we do comparisons and comments on recent articles by V. Barbu and by U. Mosco and on previous results by J. W. Barrett and L. Prigozhin. Numerical results are discussed.

References

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Dynamics and control for multi-agent networked systems

Enrique Zuazua, DeustoTech, Autonomous University of Madrid, Spain
& Sorbonne University, France

Abstract. In recent years, significant attention has been paid to models in collective dynamics. Often times, assuming that the number of interacting agents is large, mean field limit models have been derived.

We shall first discuss the various limit models of this kind and their subordination relations. We then consider a practical guidance-by-repulsion model in which a driver interacts with a number of evaders willing to escape. An analysis of this model will be presented, addressing it large time asymptotics and the possible strategies of control by means of circumvention motions of the driver.

This is a joint work in collaboration with Umberto Biccari, Ramón Escobedo, Aitziber Ibaez and Dongnam Ko.