Abstracts: short communications

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Existence of positive solutions for a class of semipositone quasilinear problems through Orlicz–Sobolev space

Jefferson Abrantes dos Santos, Federal University of Campina Grande, Brazil

Abstract. In this paper we show the existence of weak solution for a class of semipositone problem of the type

$$\begin{cases} -\Delta_\Phi u &= f(u) - a & \text{in} & \Omega, \\ u(x) &> 0 & \text{in} & \Omega, \\ u &= 0 & \text{on} & \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 2$, is a smooth bounded domain, $f : [0, +\infty) \to \mathbb{R}$ is a continuous function with subcritical growth, $a > 0$, $\Delta_\Phi u$ stands for the Φ-Laplacian operator. By using variational methods, we prove the existence of solution for a small enough.

This is a joint work with Claudianor O. Alves and Angelo R. F. de Holanda.

Stability properties of stochastic maximal L^p-regularity

Antonio Agresti, Sapienza University of Rome, Italy

Abstract. Stochastic maximal L^p-regularity plays a central role in the modern treatment of semi- and quasi-linear stochastic parabolic evolution equations. Nowadays it is known that a large class of elliptic operators satisfies this property but a little is known about the property itself. the aim of this talk is to discuss some aspects of stochastic maximal L^p-regularity such as exponential stability and analyticity of the semigroup, suitable variants, weighted inequalities and optimal space-time regularity.

This is a joint work with Mark Veraar (Delft university of technology).

Continuous and discrete sandpile models: comparison and numerical tests

Carlo Alberini, Sapienza University of Rome, Italy

Abstract. In this note we shall do a comparison between the article Self-organized criticality of cellular automata model; absorbtion in finite-time of supercritical region into the critical one (2013) by V. Barbu and the article Finite-time self-organized-criticality on synchronized infinite grids (2018) by U. Mosco. In the first one we shall analyze the main principle theoretical continuous aspects presented for SOC models while, in the latter, the following discrete analytic model made by U. Mosco on this theory that also provides a generalization for the same models in $2-D$ and $3-D$ cases. We shall retrace the main development lines for each article using, as possible, also the authors mathematical notation, and we shall analyze their main results. Finally, numerical results about both articles shall be presented.

Replicator-mutator equations in evolutionary biology

Matthieu Alfaro, University of Montpellier, France

Abstract. Since the works of Kimura (60'), replicator mutator equations are key models in evolutionary biology. I will present some recent mathematical results on such equations (unbounded fitness vs. confining fitness). In particular I will discuss the conditions for evolutionary branching to occur, and the number of selected phenotypes.

Homogenization results for heat conduction in composite media with singular sources

Micol Amar, Sapienza University of Rome, Italy

Abstract. We prove existence and homogenization results for a family of elliptic problems depending both on a small parameter $\varepsilon > 0$ (which represents the characteristic length of the microstructure underlying the model) and on a parameter $\alpha \geq -1$ (taking into account different scalings and, therefore, different physical properties appearing in the model). We focus our attention on the differences of the limit equations (characterizing the properties of the material from the macroscopic point of view), with respect to the parameter α (see, [3,4]).

Such problems describe the equilibrium for the heat conduction in composite materials (for details on the related physical models, see, for instance, [5,6] and the reference therein).

In our model, we assume that the microscopic array of the region occupied by the material is made by two phases separated by an active interface having a possibly nonlinear response.

The mathematical description is given by two non-homogeneous elliptic equations in each diffusive phase, complemented with the assumption that the heat flux is continuous across the interface and proportional to a possibly nonlinear function of the jump of the temperature. Moreover, we assume that in both phases the rate of the heat generation is given by a singular source, that is a function which blows up when the solution becomes small.

The same kind of energies can also be useful to study the electrical conduction in biological tissues (see for instance [1,2], where the related parabolic problems without singular source are studied).

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Multiple concentrating solutions for a fractional Kirchhoff equation with magnetic fields

Vincenzo Ambrosio, Marche Polytechnic University, Italy

Abstract. In this talk we deal with the multiplicity and concentration behavior of nontrivial solutions for the following fractional Kirchhoff equation in presence of a magnetic field:

$$\left(a\varepsilon^{2s} + b\varepsilon^{4s-3}[u]^2_{A/\varepsilon}\right)(-\Delta)^s_{A/\varepsilon}u + V(x)u = f(|u|^2)u \quad \text{in } \mathbb{R}^3,$$

where $\varepsilon > 0$ is a small parameter, $a, b > 0$ are constants, $s \in (\frac{3}{4}, 1)$, $(-\Delta)^s_A$ is the fractional magnetic Laplacian, $A : \mathbb{R}^3 \to \mathbb{R}^3$ is a smooth magnetic potential, $V : \mathbb{R}^3 \to \mathbb{R}$ is a positive continuous potential having a local minimum and $f : \mathbb{R} \to \mathbb{R}$ is a C^1 subcritical nonlinearity. Applying penalization techniques and Ljusternik–Schnirelmann theory, we relate the number of nontrivial solutions with the topology of the set where the potential V attains its minimum.

Moser's estimates for degenerate Kolmogorov equations with non-negative divergence lower order coefficients

Francesca Anceschi, University of Modena and Reggio Emilia, Italy

Abstract. Degenerate Kolmogorov equations arise in the theory of stochastic processes (e.g. the simplest non-trivial Kolmogorov operator is the infinitesimal generator of the Langevin's stochastic equation), kinetic theory (e.g. linear Fokker-Planck equations, non-linear Boltzmann-Landau equations) and mathematical finance (e.g. problem of pricing Asian options). The study of the regularity theory for weak solutions to this kind of equations is carried out paralleling the weak theory for parabolic equations, such as Sobolev and Caccioppoli inequalities, Moser's iteration, Hölder regularity and Harnack inequality. In particular, we consider the following second order partial differential equation of Kolmogorov type

$$\begin{aligned}&\sum_{i,j=1}^{m_0}\partial_{x_i}\left(a_{ij}(x,t)\partial_{x_j}u(x,t)\right)\\&\quad+\sum_{i,j=1}^{N}b_{ij}x_j\partial_{x_i}u(x,t)-\partial_t u(x,t)+\\&\quad+\sum_{i=1}^{m_0}b_i(x,t)\partial_i u(x,t)-\sum_{i=1}^{m_0}\partial_{x_i}\left(a_i(x,t)u(x,t)\right)+c(x,t)u(x,t)=0\end{aligned}$$

where $(x,t)=(x_1,\dots,x_N,t)=z$ is a point of $\mathbb{R}^{N+1}$, and $1\leq m_0\leq N$. (a_{ij}) is an uniformly positive symmetric matrix with bounded measurable coefficients, (b_{ij}) is a constant matrix. We apply the Moser's iteration method to prove the local boundedness of the solution u under minimal integrability assumption on the coefficients.

This is a joint work with Sergio Polidoro and Maria Alessandra Ragusa.

Asymptotic behavior of diffusion equations on inhomogeneous manifolds

Daniele Andreucci, Sapienza University of Rome, Italy

Abstract. We consider the behavior for long times of solutions to degenerate parabolic equations on Riemannian manifolds. Several problems are addressed: optimal sup estimates, finite speed of propagation, blow up of the support of (initially compactly supported) solutions. They are connected with properties of the metric, and of the equation itself.

This is a joint work with Prof. A. Tedeev (Vladikavkaz, Russia).

Norm-attaining intervals in BLO

Francesca Angrisani, University of Naples Federico II, Italy

Abstract. A real valued locally integrable function $f(x) \in L^1_{loc}(\mathbb{R})$ is said to have *Bounded Lower Oscillation* ($f(x) \in BLO(\mathbb{R})$) if

$$\sup_I \fint_I \Big[f(x) - \inf_I f\Big]\, dx = \sup_I \Big[f_I - \inf_I f\Big] = \|f\|_{BLO} \leq \infty. \tag{1}$$

where with inf we denote the essential infimum and I is a compact interval. In particular, a BLO function is said to have *Vanishing Lower Oscillation* ($f \in VLO([0,1])$) if it also satisfies:

$$\limsup_{|I|\to 0} \Big[f_I - \inf_I f\Big] = 0. \tag{2}$$

Recently we have been able to prove, following a result from Leibov in the larger class of functions with bounded mean oscillation, that the supremum in the definition of BLO is attained (hence is a maximum) by a specific sub-interval I of $\mathbb{R}$ if we work in the hypothesis that the function f is also VLO.

Polyconvexity and existence theorem for nonlinearly elastic shells

Sylvia Anicic, University de Haute Alsace, France

Abstract. We present an existence theorem for a large class of nonlinearly elastic shells with low regularity in the framework of a two-dimensional theory involving the mean and Gaussian curvatures. We restrict our discussion to hyperelastic materials, that is to elastic materials possessing a stored energy function. Under some specific conditions of polyconvexity, coerciveness and growth of the stored energy function, we prove the existence of global minimizers. In addition, we define a general class of polyconvex stored energy functions which satisfies a coerciveness inequality.

Picone identity for variable exponent operators and its applications

Rakesh Arora, University of Pau and Pays de l'Adour, France

Abstract. In this work, we establish a new Picone identity for anisotropic quasilinear operators, such as the $p(x)$-Laplacian. Our extension provides a new version of the Diaz–Saa inequality and new uniqueness results to some quasilinear elliptic equations with variable exponents. This new Picone identity can be also used to prove some accretivity property to a class of fast diffusion equations involving variable exponents. Using this, we prove for this class of parabolic equations a new weak comparison principle. We also study qualitative properties of the quasi linear elliptic equations like stabilization in the presence of both autonomous and non-autonomous term.

On subspaces of some Morse spaces

Giacomo Ascione, University of Naples Federico II, Italy

Abstract. In [1] the authors proved that for $p > 2$ and E an infinite dimensional subspace of L^p, either E is isomorphic to ℓ^2 and complemented in L^p or E contains a subspace that is isomorphic to ℓ^p and is complemented in L^p. This result has been extended to various Banach spaces, such for instance non-reflexive Orlicz spaces (see [2, 3]) and the space VMO (see [4]). We will present a similar result for infinite dimensional subspaces of some Morse spaces. The Morse space $M^\Psi(Q)$ on the unit cube $Q = [0,1]^n \subseteq \mathbb{R}^n$ is defined as the closure of $L^\infty(Q)$ in the Orlicz space $L^\Psi(Q)$ for some finite Young function Ψ. In particular, following the lines of [4], we will show that if Ψ satisfies the Δ^0 growth condition and $t \in [0,1] \mapsto \Psi^{-1}(1/t)$ belongs to $L^\Psi([0,1])$, then any infinite dimensional closed subspace E of $M^\Psi(Q)$ is either isomorphic to ℓ^2 and complemented in $L^\Psi(Q)$ or for any $\varepsilon > 0$ it contains a subspace that is ε-isometric to c_0 and complemented in E. In particular we will make use of an equivalent norm on $L^\Psi(Q)$ described in [5].

This is a joint work with Francesca Angrisani and Gianluigi Manzo.

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The Chern–Simon equation in the zero mass case

Antonio Azzollini, University of Basilicata, Italy

Abstract. We will present some results concerning the following Chern-Simon equation in the zero mass case

$$\begin{cases} -\Delta u + \left(\dfrac{h_u^2(|x|)}{|x|^2} + \displaystyle\int_{|x|}^{+\infty} \dfrac{h_u(s)}{s} u^2(s)\, ds \right) u = |u|^{p-1}u \quad \text{in } \mathbb{R}^2, \\ u(x) \to 0, \qquad \text{as } |x| \to +\infty, \end{cases} \tag{$\mathcal{P}$}$$

where $u : \mathbb{R}^2 \to \mathbb{R}$ is radially symmetric, $p > 3$ and

$$h_u(s) = \int_0^s r u^2(r)\, dr, \quad s \geq 0.$$

New examples on Lavrentiev gap using fractals

Anna Kh. Balci, Bielefeld University, Germany

Abstract. We construct new examples on Lavrentiev phenomenon using fractal contact sets. Comparing to the well-known examples of Zhikov it is not important that at the saddle point the variable exponent crosses the threshold dimension. As a consequence we give the negative answer to the well-known conjecture that the dimension plays a critical role for the Lavrentiev gap to appear. As an application we present new counterexamples to the density of smooth functions in variable exponent Sobolev spaces and to the regularity of the functional with double-phase potential.

The talk is based on joint work with Lars Diening and Mikhail Surnachev.

The Campanato nearness condition revisited

Annamaria Barbagallo, University of Naples Federico II, Italy

Abstract. In this presentation we propose to survey the theory of nearness between operators acting on normed spaces X and Y and developed by S. Campanato at the end of the eighties (see [1]). The aim of S. Campanato was to study existence and regularity results for some differential elliptic equations. This talk adresses the natural question of the symmetry of the nearness condition. At the best of our knowledge, this question has never been asked. We observe that when Y is an inner product space and the map a is near the map b, then b is near a. We show that when the dimension of Y is greater or equal to three, then the three following properties are equivalent: Y is an inner product space, the Birkhoff–James orthogonality is symmetric, and the Campanato nearness is symmetric. In a second part of the talk, we introduce a concept of nearness between set-valued mappings, extending Campanatos definition. After that we investigate which properties of set-valued mappings are preserved by nearness. Moreover, we discuss some examples and deduce that the surjectivity and the bijectivity properties are preserved with some additional conditions on the set-valued mappings. Such remarks leave some open questions which will give new research ideas.

References

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The sharp exponent in the study of the nonlocal Hénon equation in $\mathbb{R}^N$

Begoña Barrios, University La Laguna, Spain

Abstract. We will consider the nonlocal Hénon equation

$$(-\Delta)^s u = |x|^\alpha u^p, \quad \mathbb{R}^N,$$

where $(-\Delta)^s$ is the fractional Laplacian operator with $0 \le s \le 1$, $-2s \le \alpha$, $p \ge 1$ and $N \ge 2s$. We prove a Liouville result for positive solutions in the optimal range of the nonlinearity, that is, when

$$1 \le p \le p^*_{\alpha,s} := \frac{N + 2\alpha + 2s}{N - 2s}.$$

Moreover, we prove that a bubble solution, that is a fast decay positive radially symmetric solutions, exists when $p = p^*_{\alpha,s}$.

Relaxation for Optimal Design Problems with Non-Standard Growth

Ana Cristina Barroso, University of Lisbon, Portugal

Abstract. In this talk I will present some joint work with E. Zappale where we investigated the possibility of obtaining a measure representation for two functionals arising in the context of optimal design problems, under non-standard growth conditions and a perimeter penalization.

We show that one of the functionals under consideration only admits a weak measure representation, whereas for the other a strong measure representation holds. Under some convexity assumptions, we provide a partial characterization of the corresponding measures, a full representation is obtained in the one-dimensional setting.

On the global bifurcation diagram of the Gelfand problem

Daniele Bartolucci, University of Rome Tor Vergata, Italy

Abstract. For a large class of domains (which need not be neither simply connected nor symmetric) we describe the qualitative behavior of the global bifurcation diagram of the unbounded branch of solutions of the Gelfand problem crossing the origin. At least to our knowledge this is the first result about the exact monotonicity of the branch of non-minimal solutions which is not just concerned with radial solutions and/or with symmetric domains.

This is part of a joint research project in collaboration with A. Jevnikar.

Non-uniqueness of blowing-up solutions to the Gelfand problem

Luca Battaglia, Roma Tre University, Italy

Abstract. I will consider blowing-up solution for the Gelfand problem on planar domains. It is well known that blow up at a single point must occur at a critical point x of a "reduced functional" F, whereas uniqueness of blowing up families has been recently shown provided x is a non-degenerate critical point of F. We showed that, if x is a degenerate critical point of F and satisfies some additional generic condition, then one may have two solutions blowing up at the same point. Solutions are constructed using a Lyapunov–Schmidt reduction.

This is a joint work with Massimo Grossi and Angela Pistoia.

Approximation of the Mumford-Shah functional with phase fields of bounded variation

Sandro Belz, Technical University of Munich, Germany

Abstract. We introduce a new phase field approximation of the Mumford–Shah functional following the idea of the well-known Ambrosio–Tortorelli functional. However, in our setting the phase field variable is allowed to be a function of bounded variation, instead of a Sobolev function. Hence, the phase field variable can have jumps, which we exploit for the construction of the recovery sequence. In some numerical examples, in the context of segmential image segmentation, we see that this also leads to to sharper edge detections.

The presentation is based on a work together with Kristian Bredies.

Boundary null-controllability of one-dimensional coupled parabolic systems with Robin conditions

Kuntal Bhandari, University of Toulouse, France

Abstract. This work is concerned with the boundary null-controllability property of some 2×2 linear coupled parabolic system with Robin boundary conditions. More precisely, we consider a cascade system with a linear coupling of first component inserted into the second equation and here we exert a single control input only on the first component at one boundary point. Moreover, we discuss that as the Robin parameter goes to 0 or ∞ (at least up to a subsequence), the corresponding solution converges in some sense to a solution of *Neumann* or *Dirichlet* control problem. The later one is important in the sense of a *penalization approach* to solve a non-homogeneous Dirichlet boundary problem.

This is a joint work with Franck Boyer.

Fractional Schrödinger equations with singular potentials

Bartosz Bieganowski, Nicolaus Copernicus University, Poland

Abstract. We look for solutions to a fractional Schrödinger equation with sum of bounded and Hardy-type potentials and sign-changing nonlinearities. Under suitable conditions we find a ground state solution, i.e. a critical point of the energy being minimizer on the Nehari manifold. We are also interested in an asymptotic behaviour of solutions.

The Analysis of Bending Thin Periodically Perforated Plates

Joe Bishop, University of Cardiff, Wales

Abstract. We rigorously derive the correct Γ-limit that describes the energy of a thin, periodically perforated elastic plate as the period of the perforations and thickness of the plate tend to zero simultaneously. The limit energy functional is obtained via simultaneous homogenisation and dimension reduction from the fully non-linear elastic energy functional for a 3D perforated elastic plate. The limit of the bending energy regime studied results in Kirchhoff's nonlinear model for plates. The form of the limit homogenised functional depends on the scaling between the period of the perforations and the thickness of the plate.

Long-term dynamics of the coupled suspension bridge system with localized Kelvin–Voigt dissipation

Ivana Bochicchio, University of Salerno, Italy

Abstract. We consider the one dimensional beam composed of N parts, each part is composed of one of three different components: a Kelvin–Voigt viscoelastic material, a purely elastic material (no dissipation), and an elastic material inserted with a frictional damping mechanism. The main result is that the position of beam components (optimal design) play an important role in the stabilization. Precisely, we show that the model is exponential stable if and only if all the elastic components are connected with one component with frictional damping. Otherwise the solution of the model is not exponentially stable and the corresponding solution decays to zero polynomially as $1/t^2$.

Existence and uniform bounds of pullback attractors for semilinear evolution equations with almost sectorial operators

Maykel Boldrin Belluzi, Federal University of So Carlos, Brazil & University of Seville, Spain

Abstract. This talk is devoted to the study of the asymptotical behavior of semilinear evolutions equations in which the linear operator is a time-dependent family of uniformly almost sectorial operators. We shall give conditions under which this problem has a global solution and then apply this to a Reaction-diffusion equation in a Dumbbell Domain, proving that the solution of this problem generates a process with pullback attractor.

Uniform attractors for model of viscoelastic media with memory motion

Aleksandr Boldyrev, Voronezh State University, Russia

Abstract. We study attractors existence of weak solutions to viscoelastic media with memory motion model in non-autonomous case. The theory of trajectory attractors for non-invariant trajectory spaces is applied and the existence of uniform trajectory attractor and uniform global attractor for this system is proved. The proof of existence theorems is based on the approximation-topological method.

This research was supported by the Ministry of Science and Higher Education of the Russian Federation (grant 14.Z50.31.0037)

On a class of Robin problems with variable exponents

Maria-Magdalena Boureanu, University of Craiova, Romania

Abstract. We are concerned with elliptic anisotropic problems with variable exponents, but we also make connections to the isotropic case and to other related types of problems. We work over a general class of bounded domains which includes non-Lipschitz domains. By relying on the critical point theory, we discuss the existence and uniqueness of the weak solution; then we are able to establish a global boundedness result. This talk is based on a joint work with Alejandro Vélez-Santiago (University of Puerto Rico at Mayagüez).

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Spectral condensation and controllability of parabolic PDEs

Franck Boyer, University of Toulouse, France

Abstract. In this talk I will describe some recent results concerning the influence of spectral condensation phenomena on the controllability properties of parabolic PDEs. I will be in particular interested in the existence or not of a positive minimal null-control time.

This talk is based on a joint work with A. Benabdallah and M. Morancey.

An isoperimetric inequality for the harmonic mean of the lowest Neumann eigenvalues of the Laplace–Beltrami operator for domains in a hemisphere

Barbara Brandolini, University of Naples Federico II, Italy

Abstract. We prove an isoperimetric inequality for the harmonic mean of the first $N-1$ non-trivial Neumann eigenvalues of the Laplace–Beltrami operator for domains contained in a hemisphere of $(S)^N$.

This is a joint work with Francesco Chiacchio and Rafael D. Benguria.

Large solutions to quasilinear problems involving the p–laplacian as p diverges

Stefano Buccheri, Sapienza University of Rome, Italy

Abstract. Let Ω be a smooth bounded domain of $\mathbb{R}^N$ with $N \geq 1$, assume $p \in (1, \infty)$ and consider the following quasilinear large solution problem

$$\begin{cases} u - \Delta_p u + \beta |\nabla u|^q = 0 & \text{in } \Omega\,, \\ u = \infty & \text{on } \partial\Omega. \end{cases} \tag{1}$$

It is knonw that, if $p - 1 < q \leq p$ and $\beta > 0$, problem (1) admits a unique solution (see the seminal work [LL] for $p = 2$ and the more recent [LP] for the general case). Here we wonder what happens to the solution $u = u_p$ as p diverges to infinity.

There exists a limit function u_∞ such that $u_p \to u_\infty$ in some suitable sense?

There exists a limit problem solved by u_∞?

The surprising fact is that the answer to the previous questions depends on the *size* of Ω.

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Localization and multiplicity in the homogenization of nonlinear problems

Renata Bunoiu, University of Lorraine, France

Abstract. We propose a method for the localization of solutions for a class of nonlinear problems arising in the homogenization theory. The method combines concepts and results from the linear theory of PDEs, linear periodic homogenization theory, and nonlinear functional analysis. A significant gain in the homogenization theory of nonlinear problems is that our method makes possible the emergence of finitely or infinitely many solutions.

This is a joint work with Radu Precup, Babeş–Bolyai University, Cluj, Romania.

Asymptotics for time-changed diffusions

Raffaela Capitanelli, Sapienza University of Rome, Italy

Abstract. We consider time-changed diffusions and their connections with PDEs. We relate the asymptotic analysis of time-changed processes to the convergence of corresponding Dirichlet forms. Finally, we provide some examples and applications.

This talk is based on joint works with Mirko D'Ovidio.

Regularity results for a class of elliptic equations with discontinuous coefficients

Claudia Capone, Istituto per le Applicazioni del Calcolo M. Picone, Italy

Abstract. We present some higher differentiability and integrability results for the gradient of the solutions of elliptic equations of the form

$$divA(x, Du) + b(x)u = f \quad \text{in}\,\Omega \subset \mathbb{R}^n$$

where we will assume that the map $\xi \to A(x, \xi)$ satisfies the usual ellipticity and growth conditions, the map $x \to A(x, \xi)$ belongs to a suitable Sobolev space, and, finally, that $b(x)$ and $f(x)$ are related by $f(x) \leq Qb(x)$, for a.e. $x \in \Omega$ and for some positive constant Q.
Moreover, assuming $b = f = 0$ and the map $x \to A(x, \xi)$ belonging to a suitable Orlicz-Sobolev space, slightly smaller than $W^{1,n}$, we are able to prove an higher integrability result for the second derivatives of the solutions.

Exact controllability in projections for the bilinear Schrödinger equation

Marco Caponigro, CNAM, France

Abstract. We consider the bilinear Schrödinger equation with a discrete-spectrum free Hamiltonian. We present a general methods for the approximate controllability based on Lie-algebraic control techniques applied to suitable finite dimensional approximation of the Galerkin-type. Under some regularity assumptions on the Hamiltonians and generic conditions on the controllability of the finite dimensional Galerkin approximations we show exact controllability in projection on a arbitrary given number of eigenstates.

Long time behavior of fractional impulsive stochastic differential equations with unbounded delay

Tomás Caraballo, University of Seville, Spain

Abstract. This talk is first devoted to the local and global existence of mild solutions for a class of fractional impulsive stochastic differential equations with infinite delay driven by both $\mathbb{K}$-valued Q-cylindrical Brownian motion and fractional Brownian motion with Hurst parameter $H \in (1/2, 1)$. A general framework which provides an effective way to prove the continuous dependence of mild solutions on initial value is established under some appropriate assumptions. Furthermore, it is also proved the exponential decay to zero of solutions to fractional stochastic impulsive differential equations with infinite delay. Finally, some comments and remarks will be mentioned concerning the existence of attractings sets.

This is a joint work with Jiaohui Xu.

On the Gradient Structure of a Non-autonomous Chafee–Infante Like Problem

Alexandre Carvalho, University of São Paulo, Brazil

Abstract. In this work we prove that some non-autonomous scalar one dimensional semi-linear parabolic problems have an associated skew-product semigroup with gradient structure similar to that observed for the autonomous Chafee–Infante problem. The aim is to exhibit a non-autonomous problem for which the asymptotic dynamics can be fairly well described. The tools involved are symmetry, invariance, comparison and the lap-number.

Lipschitz continuity results for a class of obstacle problems

Michele Caselli, University of Modena and Reggio Emilia, Italy

Abstract. We prove Lipschitz continuity results for solutions to a class of obstacle problems under standard growth conditions of p-type, $p \geq 2$. The main novelty is the use of a linearization technique going back to Fuchs (1985) in order to interpret our constrained minimizer as a solution to a nonlinear elliptic equation, with a bounded right hand side. This lead us to start a Moser iteration scheme which provides the L^∞ bound for the gradient. The application of a recent higher differentiability result by Eleuteri and Passarelli di Napoli (2018) allows us to simplify the procedure of the identi

cation of the Radon measure in the linearization technique employed in Fuchs and Mingione (2000). To our knowdledge, this is the

rst result for non-automonous functionals with standard growth conditions in the direction of the Lipschitz regularity.

This is a joint project with Prof. Carlo Benassi (Università degli Studi di Modena e Reggio Emilia).

Ancient solutions of superlinear heat equations on Riemannian manifolds

Daniele Castorina, John Cabot University, Italy

Abstract. We study some qualitative properties of ancient solutions of superlinear heat equations on a Riemannian manifold, with particular interest in positivity and constancy in space.

This talk is based on a joint work with Carlo Mantegazza (Napoli).

Some canonical Riemannian metrics: rigidity and existence

Giovanni Catino, Polytechnic University of Milan, Italy

Abstract. In this talk, which is the second part of a joint seminar with P. Mastrolia (Università degli Studi di Milano), I will present some results concerning rigidity and existence of canonical metrics on closed (compact without boundary) four manifolds. In particular I will consider Einstein metrics, Harmonic Weyl metrics and some generalizations. These are joint works with P. Mastrolia (UniMI), D.D. Monticelli and F. Punzo (PoliMi).

On correctors for elliptic problems in cylinders

Adrien Ceccaldi, University of Rouen Normandie, France

Abstract. The talk is divided in two parts: In the first part of the talk, we discuss some results on the convergence to the solution of a linear elliptic problem in an infinite cylinder of the solutions of the same problem taken on larger and larger truncations of the cylinder. In the second part, we construct correctors that help us to improve the results presented in the first part. Thanks to the estimates obtained when these correctors are involved, we can also prove the optimality of the convergence results described in the first part in the sense that, without the use of correctors, the length of the cylinders where the convergence takes place can't be increased.

About the O. A. Ladyzhenskaya modification of liquids

Gregory Chechkin, M.V. Lomonosov Moscow State University, Russia

Abstract. Using the approach suggested by O. A. Ladyzhenskaya for modification of the classical Navier–Stokes equations, we modify the Prandtl system of equations for thin boundary layer of fluids. We consider the inhomogeneous magnetic boundary layer and study the shift of the separation point because of the magnetic field. Also we prove the existence and uniqueness theorems for the modified Prandtl system.

Homogenization of multivalued monotone operators with variable growth exponent

Valveria Chiadò Piat, Polytechnic University of Turin, Italy

Abstract. We consider the Dirichlet boundary value problem for an elliptic multivalued maximal monotone operator satisfying growth estimates of power type with a variable exponent. This exponent and also the symbol of the operator oscillate with a small period with respect to the space variable. We prove a homogenization formula for this problem, thus, generalizing previous results for multivalued operators with standard growth or for single-valued operators with variable growth.

This is a joint work with S. Pastukhova, Russian Technological University, Moscow.

Conservation law models for supply chains on a network with finite buffers

Maria Teresa Chiri, University of Padova, Italy

Abstract. We introduce a new model for supply chains on a network based on conservation laws with discontinuous flux evolving on each arc (sub-chain) and on buffers of limited capacity in every junction (separating sub-chains). The dynamics of every arc is governed by a continuity equation describing the evolution of the density of objects processed by the supply chain. The flux is discontinuous at the maximal density since it admits different values according with the free or congested status of the supply chain. We provide a definition of viscosity solution on each arc for the corresponding (discontinuous) Hamilton-Jacobi equation and we show that the space derivative of such solutions are entropy weak solutions of the conservation law that keep trace of the (free-congested) status transition in the point of discontinuity of the flux. We then establish existence and uniqueness of solutions of the Hamilton-Jacobi equations on each arc and show that the map associating to a given buffer-queue the corresponding solution at the junction and then the resulting buffer-queue is a contractive transformation. This yields the well-posedness of the Cauchy problem at the junction with bounded and integrable initial data.

This is a joint work with Fabio Ancona (Universitá di Padova)

Renormalized solutions for parabolic problems with strongly nonstandard growth

Iwona Chlebicka, University of Warsaw, Poland

Abstract. We provide existence and uniqueness of renormalized solutions to a general nonlinear parabolic equation

$$\partial_t u - \operatorname{div} A(t,x,\nabla u) = f \in L^1(\Omega_T),$$

where the growth of A is governed by a fully anisotropic N-function inhomogeneous in time and in space. In turn, our studies cover (not studied so far) cases in variable exponent spaces

$$\partial_t u - \operatorname{div}(b(x,t)|\nabla u|^{p(x,t)-2}\nabla u) = f \in L^1(\Omega_T),$$

for log-Hölder continuous exponent $p : \Omega_T \to (1,\infty)$, as well as problems posed in fully anisotropic Orlicz spaces (under no growth conditions), weighted Sobolev spaces (with bouded weights), and double-phase spaces within the range of parameters sharp for density of smooth functions. Based on joint works

- I. Chlebicka, P. Gwiazda, A. Zatorska-Goldstein, *Parabolic equation in time and space dependent anisotropic Musielak-Orlicz spaces in absence of Lavrentiev's phenomenon*, **Annales de l'Institut Henri Poincaré. Analyse Non Lineairé (C)**, 2019. DOI:10.1016/j.anihpc.2019.01.003
- I. Chlebicka, P. Gwiazda, A. Zatorska-Goldstein, *Renormalized solutions to parabolic equation in time and space dependent anisotropic Musielak-Orlicz spaces in absence of Lavrentiev's phenomenon*, **J. Differ. Equations**, 2019. DOI:10.1016/j.jde.2019.02.005.

See also

- P. Gwiazda, I. Skrzypczak, A. Zatorska-Goldstein, *Existence of renormalized solutions to elliptic equation in Musielak-Orlicz space*, **J. Differ. Equations** 264 (1) (2018), 341–377.
- Y. Ahmida, I. Chlebicka, P. Gwiazda, A. Youssfi, *Gossez's approximation theorems in Musielak-Orlicz-Sobolev spaces*, **J. Functional Analysis** 275 (9) (2018), 2538–2571.
- I. Chlebicka, P. Gwiazda, and A. Zatorska-Goldstein. *Well-posedness of parabolic equations in the non-reflexive and anisotropic Musielak-Orlicz spaces in the class of renormalized solutions*, **J. Differ. Equations** 265 (11) (2018), 5716–5766.

A rigorous derivation of the stationary compressible Reynolds equation via a compressible Navier–Stokes system

Ionel Sorin Ciuperca, University of Lyon 1, France

Abstract. We provide a rigorous derivation of the compressible Reynolds equation as a singular limit of a compressible Navier–Stokes system on a thin domain. We consider a pressure low of hard sphere type, where the admissible range of density is confined to a bounded interval. Finally, the uniqueness for the limit problem is established in the 1D case.

General decay properties of abstract linear viscoelasticity

Monica Conti, Polytechnic University of Milan, Italy

Abstract. We establish new stability results concerning the decay of the energy associated with a linear Volterra integro-differential equation of hyperbolic type in a Hilbert space, which is an abstract version of the equation

$$u_{tt}(t) - \Delta u(t) + \int_0^t \mu(s)\Delta u(t-s)ds = 0$$

describing the motion of linearly viscoelastic solids. We provide sufficient conditions for the decay to hold, without invoking differential inequalities involving the convolution kernel μ.

This is a joint work with V. Pata.

Nonlocal propagation in domain with obstacle

Jerome Coville, Inra PACA, France

Abstract. I will present some recent results on the propagation phenomena in domain with obstacles where the dispersal process is governed by a compound Poisson process. I will briefly present the construction of generalised transition wave as well as the description of the resulting stationnary solutions.

On some quasi-linear BVPs in fractal-type domains

Simone Creo, Sapienza University of Rome, Italy

Abstract. In this talk, we present some results about boundary value problems of quasi-linear type on irregular domains, in particular of fractal type. We will consider BVPs for quasi-linear operators in divergence form of order $p \geq 2$ in domains with a fractal boundary of Koch type, dealing with existence, uniqueness and regularity of the solutions of such problems. In view of concrete real world applications, we will also consider suitable approximating problems in pre-fractal domains, and we will investigate the asymptotic behavior of the weak solutions of the pre-fractal problems.

Optimality criteria method for the optimal design problems in linearized elasticity

Ivana Crnjac, University of Osijek, Croatia

Abstract. In optimal design problems the goal is to find the arrangement of given materials within the body which optimizes its properties with respect to some optimality criteria. The performance of the mixture is usually measured by an integral functional, while optimality of the mixture is achieved through minimization or maximization of this functional, under constraints on amount of materials and PDE constraints that underlay involved physics. We consider optimal design problems in the setting of linearized elasticity, where the mechanical behaviour of the domain is modelled with linearized elasticity system of PDEs, and restrict ourselves to domains filled with two isotropic elastic materials. Since the classical solution usually does not exist, we use relaxation by the homogenization method in order to get a proper relaxation of the original problem. Unfortunately, the set of all possible homogenized elastic materials is not known (the famous G-closure problem), not even for mixtures of two isotropic phases. However, in the case of minimizing a functional that is equal to the total elastic energy of the system, the minimization can be performed on a smaller subset made of sequential laminates, which is explicitly known. Moreover, for compliance functional, the necessary conditions of optimality are easily derived, which enables a development of optimality criteria method for finding an approximate solution. The possible issue is a necessity for explicit calculation of Hashin–Shtrikman bounds, that naturally arise in the optimality conditions. We explicitly calculate lower Hashin–Shtrikman bound on complementary energy in the two- and three-dimensional case, and using necessary conditions of optimality, we develop a new variant of optimality criteria method for the single state compliance minimization problems.

The equation $\operatorname{div} u + \langle a, u\rangle = f$

Gyula Csató, UPC, Spain

Abstract. Consider the following classical and broadly treated problem: Given a function f on $\Omega \subset \mathbb{R}^n$, find a vector field u such that

$$\begin{cases} \operatorname{div} u = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

It is obvious that a necessary condition is $\int_\Omega f = 0$. This is also a sufficient condition. Let us generalize the differential operator and introduce the boundary value problem

$$\begin{cases} \operatorname{div} u + \langle a, u\rangle = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where a is a given vector field and $\langle\ ,\ \rangle$ is the scalar product. What is now the necessary and sufficient condition for solvability? What is the expected regularity result? The answer is easy, if a is of the special form $a = \operatorname{grad} A$. We present some results about the general case. This is joint work with B. Dacorogna and P. Bousquet appearing in the following references:

References

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[2] Bousquet P. and Csató G., The equation $\operatorname{div} u + \langle a, u\rangle = f$, to appear in *Comm. in Contemporary Math.*.

Everywhere regularity of vectorial minimizers of some non-convex functionals

Giovanni Cupini, University of Bologna, Italy

Abstract. The convexity of the integrand of a functional of the calculus of variations is equivalent to the lower semicontinuity of the functional in the scalar case, but it is only a sufficient condition in the vectorial case. Moreover, the convexity of the integrand turns out to be a too strong and unrealistic assumption in applications, as for instance in mathematical models in nonlinear elasticity. In the vectorial framework more appropriate and weaker conditions than the convexity are the polyconvexity and the quasiconvexity. Under these assumptions, many results were proved concerning the partial regularity of minimizers, but the results concerning the everywhere regularity are very few and mainly in low dimensions ($n = N = 2$). We will discuss recent everywhere regularity results of vectorial minimizers for some classes of polyconvex and quasiconvex functionals ($n, N > 2$) obtained in collaboration with F. Leonetti and E. Mascolo (local boundedness) and with them and M. Focardi (Holder continuity). The proofs rely on the power and elegant (typically scalar) method by De Giorgi (1957).

Modeling and simulation of the cardiovascular system

Maria Pia D'Arienzo, University of Salerno, Italy

Abstract. Modeling and simulation of the cardiovascular system The study of mathematical models for vascular networks is very important because of its applications in medicine and pharmacology. The best way to study the arterial system would be to use a 3-D model, but it requires a very high computational power, so we use a 1-D reduced model based on Navier–Stokes equations with appropriate boundary conditions, which still allows to capture the major features under study, such as arterial blood pressure and flow [3, 5]. We choose the elastic model for the pressure. As inflow conditions we use a model for the aortic valve [1,4]. The valve opens when the pressure is lower than the left ventricular one, in which case the pressure at the inflow gets prescribed, and it closes when the velocity becomes negative, in which case the velocity at the inflow is prescribed to be zero. As terminal condition, we have used a model with terminal reflection coefficient R_t, (see [2]), which is based on the assumption that the backward characteristic is proportional to the forward one. At the junctions we use continuity conditions for the Bernoulli pressure and the continuity of the forward and backward characteristics for the hyperbolic system. This model is solved numerically with a discontinuous Galerkin scheme, using Gauss-Legendre method to approximate integrals and Newton method to find the solution of the Riemann problem at junctions. Some simulation results are presented, including optimization problems and the the numerical model of the entire 55-edge network to study the effects of the variable environment on the cardiovascular system performance, showing that the obtained results are consistent with the expected ones.

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Nonlinear Schrödinger–Bopp–Podolsky system

Pietro d'Avenia, Polytechnic University of Bari, Italy

Abstract. We consider the following nonlinear Schrödinger–Bopp–Podolsky system

$$\begin{cases} -\Delta u + \omega u + q^2\phi u = |u|^{p-2}u \\ -\Delta \phi + a^2\Delta^2\phi = 4\pi u^2 \end{cases} \quad \text{in } \mathbb{R}^3$$

with $a, \omega \geq 0$. We discuss existence and nonexistence results depending on the parameters q, p.

Joint work with Gaetano Siciliano (Universidade de São Paulo, Brazil)

An Asymptotic Analysis of Spectral Stiff Problems

Lorenza D'Elia, Polytechnic University of Turin, Italy

Abstract. This talk is concerned with an asymptotic analysis of the eigenvalues and the eigenfunctions of the Neumann stiff spectral problem for the Laplace operator in a bounded domain $\Omega \in \mathbb{R}^d$. Let Ω_0, Ω_1 be two connected bounded domains of $\mathbb{R}^d$ with smooth boundaries Γ_0, Γ_1 such that $\Omega = \Omega_0 \cup \Omega_1 \cup \Gamma_0$ and $\partial\Omega = \Gamma_1$. The density and the stiffness constants are of order $O(\varepsilon^{-2m})$ and $O(\varepsilon^{-1})$ in the core Ω_0 while they are of order $O(1)$ in the annulus Ω_1. Here $m \in \mathbb{R}$ is a fixed parameter reflecting the dead-weight of the material and $\varepsilon \in (0, 1)$.
Depending on the orders of the relative density in Ω_0, we show a different asymptotic behavior of the eigenpairs $(\lambda^\varepsilon, u^\varepsilon)$ as ε tends to 0.

This is a joint work with Prof. S. A. Nazarov (Saint-Petersburg State University-Institute of Problems Mechanical Engineering) and Prof. V. Chiadò Piat (Politecnico di Torino).

Delayed and Rushed motions

Mirko D'Ovidio, Sapienza University of Rome, Italy

Abstract. We consider time-changed processes, that is processes obtained through random time changes given by subordinators and their inverses. Such processes can be considered in order to solve fractional partial differential equations. In this context, very often, we refer to the characterization in terms of subdiffusion, superdiffusion, normal diffusion which is usually given in terms of mean square displacement (instead of velocity correlation!). However, this definition includes a number of very different dynamics. A further characterization of such dynamics can be given by considering the random time as a new clock for the base process. Then, along the trajectory of the base process, we focus on exit times after time changes. In particular, we introduce a definition of delayed and rushed processes and provide some examples which are, in some cases, counter-intuitive. Our analysis shows that, quite surprisingly, inverse processes are not necessarily leading to delayed processes.

Multiplicity of multi-bump type nodal solutions for a class of elliptic problems with exponential critical growth in $\mathbb{R}^2$

Denilson da Silva Pereira, Federal University of Campina Grande, Brazil

Abstract. In this paper, we establish the existence and multiplicity of multi-bump nodal solutions for the following class of problems

$$\begin{cases} -\Delta u + (\lambda V(x) + 1)u = f(u), \text{ in } \mathbb{R}^2, \\ \\ u \in H^1(\mathbb{R}^2) \end{cases}$$

where λ is a positive parameter, (s) is a continuous function with exponential critical growth in the sense of the Moser–Trudinger. The nonnegative potential $\mathbb{R}^2 \to \mathbb{R}$ is a continuous function and has the property that $\operatorname{int} V^{-1}(0)$ consists of bounded smooth domain Ω_i with positive distance between any two components. The mains result says that for λ suficiently large there exist a multi-bump nodal solutions λ which, for $\lambda \to \infty$, converge to least energy nodal solutions of $\Delta u + u = f(u), \in H_0^1(\cup_{j\in\Lambda}\Omega_j)$ for some $\Lambda \subset \{1, \cdots, k\}$.

This is a joint work with C. O. Alves.

On a uniformly-valid asymptotic plate model

Hui-Hui Dai, City University of Hong Kong, China

Abstract. Three mathematical approaches for deriving plate models from 3D formulations are well-known. One way is to apply asymptotic expansions to the 3D weak formulation, which was developed by Ciarlet and his associates. Another way, initiated by Goldenveizer is to apply asymptotic expansions to the 3D differential formulation. The third way is by Gamma convergence, which is concerned with the limiting two-dimensional variational problem for small thickness. It is also well-known that these three approaches need to impose , as a priori, the assumption on scaling of external loads with respect to the thickness of the plate. For different kinds of scalings, a hierarchy of plate models are obtained. One natural question is: Can a single plate model be derived in a consistent manner without these scaling assumptions, which contains the above-mentioned hierarchy of models as special cases? In this work, we derive a uniformly-valid plate model, independent of the magnitudes of applied loads, based on the series expansions about the bottom surface of the plate. The key is the establishment of the recursive relations for the expansions coefficients. It is further shown by asymptotic expansions that the obtained plate model is uniformly-valid, in the sense that the hierarchy of plate models, such as membrane, von Kármán, Föppl membrane and Kirchhoff–Love plate models, arise as special cases when different specific scalings of applied loads are imposed. We believe that such a uniformly-valid plate model (derived without ad hoc assumptions) is very much needed in applications as it combines stretching, bending and rotation (and their coupling) effects in a single framework.

Sectional symmetry of solutions of elliptic systems in cylindrical domains

Lucio Damascelli, University of Rome Tor Vergata, Italy

Abstract. We prove a kind of axial/rotational symmetry for solutions of semilinear elliptic systems in some bounded cylindrical domains. The symmetry theorems obtained hold for low-Morse index solutions whenever the nonlinearities satisfy some convexity assumptions. These results extend and improve those obtained in previous papers on foliated Schwarz symmetry of solutions of elliptic problems.

This is a joint work with F. Pacella.

An abstract linking theorem applied to indefinite operators via spectral properties

Liliane de Almeida Maia. University of Brasília, Brazil

Abstract. An abstract linking result proved for Cerami sequences without the Cerami condition is presented. It is applied directly in order to obtain the existence of critical points for a class of indefinite problems in infinite dimensional Hilbert Spaces. The main applications are given to Hamiltonian systems and Schrödinger equations. Here spectral properties of the operators are exploited and hypotheses of monotonicity on the nonlinearities are discarded.

These are works in collaboration with Mayra Soares (UnB, Brazil).

Partial regularity for manifold constrained $p(x)$-harmonic maps

Cristiana De Filippis, University of Oxford, United Kingdom

Abstract. We study the regularity features of local minimizers of the functional

$$W^{1,p(\cdot)}(\Omega,\mathcal{M})\ni w\mapsto \mathcal{E}(w,\Omega):=\int_\Omega k(x)|Dw|^{p(x)}\,dx\,,$$

where $p(\cdot)$ and $k(\cdot)$ are Hölder continuous functions and $\mathcal{M}\subset\mathbb{R}^N$ is a compact submanifold endowed with a suitable topology. Our first result is that there exists a relatively open set $\Omega_0\subset\Omega$ of full n-dimensional Lebesgue measure such that if $u\in W^{1,p(\cdot)}(\Omega,\mathcal{M})$ is any constrained local minimizer of $\mathcal{E}(\cdot)$, then $u\in C^{1,\beta_0}_{loc}(\Omega_0,\mathcal{M})$ and $\Sigma_0(u):=\Omega\setminus\Omega_0$ has Hausdorff dimension at the most equal to $n-\inf_{x\in\Omega}p(x)$. Moreover, after imposing some restrictions on the variable exponet $p(\cdot)$, we are able to provide a further reduction of the Hausdorff dimension of the singular set of $\mathcal{M}$-constrained minimizers of the model energy

$$w\mapsto\int_\Omega |Dw|^{p(x)}\,dx,$$

thus recovering analogous results to those already available for the p-Laplacean case.

Sign-changing prescribed Gaussian curvature

Francesca De Marchis, Sapienza University of Rome, Italy

Abstract. I will consider the problem of prescribing the Gaussian curvature (under pointwise conformal change of the metric) on surfaces with conical singularities. This question has been first raised by Troyanov and it is a generalization of the Kazdan–Warner problem for regular surfaces, known as the Nirenberg problem on the sphere. From the analytical point of view, this amounts to solve a singular Liouville-type equation on the surface. I will focus on the case of a prescribed sign-changing Gaussian curvature with singularities of arbitrary signs.

The talk is based on joint works with S. Kallel, R. Lòpez-Soriano and D. Ruiz.

Symmetrization with respect to mixed volumes

Francesco Della Pietra, University of Naples Federico II, Italy

Abstract. The aim of the talk is to introduce new symmetrization with respect to mixed volumes or anisotropic curvature integral, which generalizes the one with respect to quermassintegral due to Talenti (Ann. Scuola Norm. Sup. Pisa, 1981) and Tso (J. Anal. Math. 1989). We show that such symmetrization diminishes the anisotropic Hessian integral. Moreover, we use it to show several other applications about anisotropic Hessian operators.

The talk is based on joint works with N. Gavitone (Napoli) and C. Xia (Xiamen).

Stationary optimal harvesting strategies

Giuseppe Devillanova, Polytechnic University of Bari, Italy

Abstract. This talk deals with the stationary solution to the time dependent optimal harvesting problem addressed by Bressan, Coclite, etc. (see [1], [2]) in a multidimensional bounded, open, connected and smooth domain $\Omega \subset \mathbb{R}^N$, with $N \geq 2$, in a time interval $[0, T]$

$$\begin{cases} \partial_t \varphi &= \Delta\varphi - \varphi\mu + (h(x) - \varphi)\varphi & \text{in }]0,T] \times \Omega \\ \partial_{\mathbf{n}} \varphi &= 0 & \text{on } [0,T] \times \partial\Omega \\ \varphi(0,x) &= \varphi_0(x) & \text{in } \Omega, \end{cases} \qquad (\mathrm{P}(\mu))$$

where the function $h(x) \in L^\infty(\Omega)$ is the spatially heterogeneous environment capacity, the measure μ is the strategy used while harvesting the amount of fish $\varphi(x,t)$ in the point x at time t, $\mathbf{n} = \mathbf{n}(x)$ stands, for every point $x \in \partial\Omega$, for the unit outer normal vector to $\partial\Omega$, and $\varphi_0(x)$ is the initial density of fish.

In particular, in [3] the authors deal with the leisure harvesting problem, i.e. with the problem of finding an harvesting strategy μ^* which maximizes the functional

$$J_0(\mu) := \sup \int_\Omega u d\mu$$

(where the supremum is taken over the stationary solutions u of Problem $(\mathrm{P}(\mu))$) on the set of smooth Radon measures μ. They find that the problem still makes sense and provide a different formulation of the problem (which actually is independent on the strategy used). Furthermore, this new formulation admits a relaxed version and while, from on side, the relaxed problem always has a unique solution which, under suitable assumptions (expressed in terms of the environment capacity), is also a solution of the initial problem, on the other side, some examples of environment capacities are provided for which the corresponding original problem has no solution. Finally on the space of smooth measures a notion of convergence in sense of strategies is introduced and a comparison with the usual narrow convergence of measures is ruled out.

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Fully anisotropic elliptic equations

Giuseppina di Blasio, University of Campania L. Vanvitelli, Italy

Abstract. We deal with a class of nonlinear fully anisotropic Dirichlet problems, whose anisotropy is governed by a general N-dimensional Young function of the gradient. In the classical theory of regularity, the anisotropic condition depends on differential operators whose growth with respect to the partial derivatives of the unknown is governed by different powers. Problems governed by fully anisotropic growth conditions have been recently studied. We outline some results for a class of of nonlinear fully anisotropic Dirichlet problems, whose anisotropy is governed by a general N-dimensional Young function which need neither be radial, nor have a polynomial growth, and are not even assumed to satisfy the so-called Δ_2-condition.

Higher integrability of minimizers of degenerate functionals in Carnot–Carathéodory spaces

Patrizia Di Gironimo, University of Salerno, Italy

Abstract. We prove a higher integrability result for the horizontal gradient of a minimizer of a functional of the type

$$I(\Omega, u) = \int_\Omega \sum_{i,j} a_{ij} X_i u X_j u \, dx$$

whose matrix of the coefficients $A(x) = {}^tA(x)$ satisfies the anisotropic bounds

$$\frac{|\xi|^2}{K(x)} \leq \langle A(x)\xi, \xi\rangle \leq K(x)|\xi|^2 \qquad \forall \xi \in \mathbb{R}^n, \text{ for a.e. } x \in \Omega,$$

where the ellipticity function $K(x) \in A_2 \cap RH_\tau$, τ opportunely related to the homogeneous dimension, and is such that the pair $\left(K, \frac{1}{K}\right) \in A_1$.

This is a joint work with Flavia Giannetti.

Asymptotic analysis of a multiscale parabolic problem with a rough fast oscillating interface

Patrizia Donato, University of Rouen Normandie, France

Abstract. We present some results in collaboration with E. Jose and D. Onofrei concerning the problem of well posedness and homogenization for a multiscale parabolic problem in a cylinder Q. A rapidly oscillating non-smooth interface inside Q separates the cylinder in two heterogeneous connected components. The interface has a periodic microstructure and it is situated in a small neighborhood of a hyperplane which separates the two components. The problem models a time-dependent heat transfer in two heterogeneous conducting materials with an imperfect contact between them. At the interface, we suppose that the flux is continuous and that the jump of the solution is proportional to the flux. On the exterior boundary, homogeneous Dirichlet boundary conditions are prescribed. The peculiarity of this time dependent problem is apparent in the lack of regularity for the time-derivative of the solution which further complicates the homogenization procedure in general, and in particular the identification of the initial data. This is overcome by proving a suitable compactness result. We also derive a corrector result showing the accuracy of our approximation in the energy norm.

Null controllability of the Grushin equation

Michel Duprez, Sorbonne University, France

Abstract. It is well known that it is possible to steer the solution of the heat equation to zero on a time interval as small as we want and without geometrical condition on the control region. It is not the case for all parabolic systems such as for instance the Grushin equation which admits a degeneracy in one space direction. Concerning this equation, up to now, only rectangular control regions have been considered. In this talk, we will show how adapt and combine the known results to treat the case of non-rectangular control region. We will give the minimal time of null controllability in a large number of situations. We will establish the positive result thanks to the fictitious control method and the negative one by interpreting the associated observability inequality as an L^2 estimate on complex polynomials. We will finish with some open problems.

Existence and nonexistence results for singular elliptic equations

Riccardo Durastanti, Sapienza University of Rome, Italy

Abstract. In this talk I present recent results for the asymptotic behavior, as γ tends to infinity, of solutions for the homogeneous Dirichlet problem associated to

$$-\Delta u = \frac{f(x)}{u^\gamma} \text{ in } \Omega,$$

where Ω is an open, bounded subset of $\mathbb{R}^N$ and f is a bounded function. We prove existence and nonexistence of a limit equation under two different assumptions on f: either strictly positive on every compactly contained subset of Ω or only nonnegative. Through this study we deduce optimal existence and nonexistence results of positive solutions for the homogeneous Dirichlet problem associated to

$$-\Delta v + \frac{|\nabla v|^2}{v} = f \text{ in } \Omega.$$

Traveling water waves with exponentially localized vorticity

Mats Ehrnstrom, Norwegian University of Science and Technology, Norway

Abstract. We study traveling and stationary waves in a two-dimensional body of water that rests above a flat ocean bed and below vacuum. An external gravitational force acts in the bulk of the fluid, and the upper boundary is a free surface along which the pressure is constant and the effects of surface tension are felt. This system is described by the Euler equations with a moving boundary. Our main result states that there exists large families of such waves that exhibit an exponentially localized distribution of vorticity. This is accomplished using ideas drawn from the theory of spike-layer solutions to singularly perturbed elliptic equations, together with a delicate contraction mapping argument.

This is joint work with Samuel Walsh (University of Missouri) and Chongchun Zeng (Georgia Institute of Technology).

Regularity results for a class of non-differentiable obstacle problems

Michela Eleuteri, University of Modena and Reggio Emilia, Italy

Abstract. In this paper we prove the higher differentiability in the scale of Besov spaces of the solutions to a class of obstacle problems of the type

$$\min\left\{\int_\Omega F(x,z,Dz) : z \in \mathcal{K}_\psi(\Omega)\right\}.$$

Here Ω is an open bounded set of $\mathbb{R}^n$, $n \geq 2$, ψ is a fixed function called *obstacle* and $\mathcal{K}_\psi(\Omega)$ is set of admissible functions $z \in W^{1,p}(\Omega)$ such that $z \geq \psi$ a.e. in Ω. We assume that the gradient of the obstacle belongs to a suitable Besov space. The main novelty here is that we are not assuming any differentiability on the partial maps $x \mapsto F(x,z,Dz)$ and $z \mapsto F(x,z,Dz)$, but only their Hölder continuity.

This is a joint project with Antonia Passarelli di Napoli.

On the Moving Plane Method for Singular Solutions to some Semilinear and Quasilinear Elliptic Problems

Francesco Esposito, University of Calabria, Italy

Abstract. In this talk we will consider positive singular solutions to semilinear or quasilinear elliptic problems. We will deduce symmetry and monotonicity properties of the solutions via the moving plane procedure, that goes back to the two celebrated papers of Alexandrov and Serrin. In particular, we will consider the problem

$$\begin{cases} -\Delta_p u = f(u) & \text{in} \quad \Omega \setminus \Gamma \\ u > 0 & \text{in} \quad \Omega \setminus \Gamma \\ u = 0 & \text{on} \quad \partial\Omega \end{cases}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, $1 < p < +\infty$, is the p-Laplace operator, Ω is a bounded smooth domain of $\mathbb{R}^n$ with $n \geq 2$, which is convex and it is symmetric with respect to the hyperplane $\{x_1 = 0\}$; $\Gamma \subset \{x_1 = 0\}$ is a closed set of vanishing p-capacity. Furthermore, the nonlinearity f will be assumed to be locally Lipschitz continuous from above far from the singular set in the case $p = 2$ and with some additional assumption when $p \neq 2$, that will be discussed.

Log-determinants in conformal geometry

Pierpaolo Esposito, Roma Tre University, Italy

Abstract. I will report on a recent result, in collaboration with A. Malchiodi, concerning a four-dimensional PDE of Liouville type arising in the theory of log-determinants in conformal geometry. The differential operator combines a linear fourth-order part with a quasi-linear second-order one. Since both have the same scaling behavior, compactness issues are very delicate and even the "linear theory" is problematic. For the log-determinant of the conformal laplacian and of the spin laplacian we succeed to show existence and logarithmic behavior of fundamental solutions, quantization property for non-compact solutions and existence results via critical point theory.

Abstract. Singular problems arise in the study of non-Newtonian fluids, boundary layer phenomena for viscous fluids, chemical heterogeneous catalysts, as well as in the theory of heat conduction in electrically conducting materials. An increasing attention to singular stationary or evolution equations has been paid in the last decades.

In the present talk we deal with a singular elliptic problem involving a nonlinearity which is indefinite in sign:

$$\begin{cases} -\Delta u = (\lambda u^{s-1} - u^{r-1})\chi_{\{u>0\}} & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P_λ}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $0 < r \leq 1 < s < 2$, or $0 < r < s < 1$, λ is a positive parameter and $\chi_{\{u>0\}}$ the characteristic function corresponding to the set $\{u > 0\}$.

We prove the following:

Theorem 1. *There exists $\Lambda > 0$ such that problem (P_λ) has no positive solution for $\lambda < \Lambda$ and two distinct nontrivial nonnegative weak solutions for $\lambda > \Lambda$. One of them belongs to $\mathrm{int}(C_0^1(\overline{\Omega})_+)$ and corresponds to a local minimum point of the energy functional associated to problem (P_λ).*

We point out that the energy functional associated to (P_λ) is not Gâteaux differentiable in the Sobolev space $W_0^{1,2}(\Omega)$ and the classical critical point theory does not apply. In our proof truncation techniques and variational arguments are employed.

The talk is based on the papers [1] and [2].

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Evolution problems with very singular convection term

Fernando Farroni, University of Naples Federico II, Italy

Abstract. We prove existence and uniqueness of a weak solution to some evolution problem with very singular convection term. We also discuss the asymptotic behavior of solutions. A model case of our problem corresponds to the initial-boundary value problem for the so-called Fokker–Planck equation. Such equation plays a key role in the theory of the mean field games.

Asymptotic stability of the critical pulled front in a Lotka–Volterra competition model

Grégory Faye, Paul Sabatier University, France

Abstract. In this presentation, I will report on joint work with Matt Holzer (George Mason University, USA) regarding the nonlinear asymptotic stability of the critical pulled front of Lotka–Volterra competition systems. More precisely, we show that perturbations of the critical front decay algebraically with rate $t^{-3/2}$ in a weighted L^∞ space. Our proof relies on pointwise semigroup methods and utilizes in a crucial way that the faster decay rate $t^{-3/2}2$ is a consequence of the lack of an embedded zero of the Evans function at the origin for the linearized problem around the critical front.

Unique continuation principles for higher order fractional equations

Veronica Felli, University of Milano-Bicocca, Italy

Abstract. In this talk we discuss strong unique continuation principle and unique continuation from sets of positive measure for solutions of higher order fractional equations in open domains. By the Caffarelli–Silvestre extension method the problem is formulated as a system of two second order equations with singular or degenerate weights in a half-space, for which asymptotic estimates are derived by a blow-up analysis and energy estimates obtained by studying an Almgren type frequency function.

The results discussed in the talk are contained in joint papers with A. Ferrero.

On hysteresis-reaction-diffusion systems

Klemens Fellner, University of Graz, Austria

Abstract. We report on recent progresses on models coupling a hysteresis operator to reaction-diffusion equations. In a first collaboration with C. Münch, we present the derivation of hysteresis-reaction-diffusion systems as singular fast-reaction limit of a suitable class of coupled ODE-PDE systems. Besides existence theory and numerical examples, we demonstrate in particular a hysteresis driven instability mechanism, where the shape of a generalised play operator may decide between large-time equilibration or large-time blow-up. In a second collaboration with M. Brokate, we prove weak-differentiability of the control-to-state mapping in a parabolic problem with hysteresis.

The Stokes paradox in inhomoegenous elastostatics

Adele Ferone, University of Campania Luigi Vanvitelli, Italy

Abstract. We prove that the displacement problem of inhomogeneos elastostatics in a two-dimensional exterior Lipschitz domain has a unique solution with finite Dirichlet integral u, vanishing uniformly at infinity if and only if the boundary datum satisfies a suitable compatibility condition.

Optimal control problems and Sub-Riemannian geometry: controllability of a (nonholonomic) macroeconomic systems

Massimiliano Ferrara, University Mediterranea of Reggio Calabria, Italy

Abstract. This paper studies optimal control problems and sub-Riemannian geometry on a nonholonomic macroe- conomic system. The main results show that a nonholonomic macroeconomic system is controllable either by trajectories of a single-time driftless control system (single-time bang-bang controls), or by nonholonomic geodesics or by sheets of a two-time driftless control system (two-time bang-bang controls). They are strongly connected to the possibility of describing a nonholonomic macroeconomic system via a Gibbs-Pfaff equation or by four associated vector fields, based on a contact structure of the state space and our isomorphism between thermodynamics and macroeconomics that praises three laws of a nonholonomic macroeconomic system.

A geometric condition for the continuity of the eigenvalues of higher order elliptic operators

Francesco Ferraresso, University of Bern, Switzerland

Abstract. Are the eigenvalues and eigenfunctions of higher order selfadjoint elliptic differential operators with compact resolvent stable under (possibly singular) domain perturbations? It was proved that under certain conditions on the convergence of the functional spaces associated with the quadratic forms, the spectrum is continuous with respect to the perturbation. In this talk we discuss a geometric reformulation of such functional condition. The main focus is on intermediate boundary conditions, for which very few results are available. The geometric condition is proved to be optimal by analyzing in detail a boundary homogenization problem for polyharmonic operators subject to homogeneous boundary conditions of intermediate type.

Based on joint works with J. M. Arrieta and P. D. Lamberti.

Some remarks on the comparison principle in Kirchhoff equations

Giovany Figueiredo, University of Brasília, Brazil

Abstract. In this paper we study the validity of the comparison principle and the sub-supersolution method for Kirchhoff type equations. We show that these principles do not work when the Kirchhoff function is increasing, contradicting some previous results. We give an alternative sub-supersolution method and apply it to some models.

Approximate controllability for semilinear reaction-diffusion equations governed by bilinear controls

Giuseppe Floridia, University of Naples Federico II, Italy

Abstract. In this talk we present some global approximate controllability results for semilinear reaction-diffusion equations governed via the coefficient of the reaction term (bilinear control). Before, we consider a one-dimensional uniformly parabolic problem and we show in [1] that the system can be steered from an initial continuous state that admits a finite number of points of sign change to a target state with the same number of changes of sign in the same order. Our method uses a technique introduced in [1] employing the shifting of the points of sign change by making use of a finite sequence of initial-value pure diffusion problems. The above result can be also extended to degenerate reaction-diffusion equations (see [2] and [3]) with application to some energy balance models in climatology (see, e.g., the Budyko–Sellers model) and some models in population genetics (see, e.g., the Fleming–Viot model).

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Interpolation results related to G-Gamma spaces and some applications to PDEs

Maria Rosaria Formica, Parthenope University of Naples, Italy

Abstract. We present some new results, based on a joint work with A. Fiorenza, A. Gogatishvili and J. M. Rakotoson, concerning the computation of the K-functional related to couples of spaces as small or classical Lebesgue spaces or Lorentz–Marcinkiewicz spaces. This computation allows to determine the interpolation space, in the sense of Peetre, for such couples. As an application, we obtain a regularity estimate for the very weak solution of a linear equation in a domain with data in the space of the integrable functions with respect to the distance function to the boundary of the domain.

Asymptotics and estimates for p-Laplacian obstacle problems

Salvatore Fragapane, Sapienza University of Rome, Italy

Abstract. In this talk we deal with obstacle problems involving p-Laplace type operators in domains with fractal boundary Ω and in approximating (non-convex) domains Ω_n. We present asymptotic results for both $p \to \infty$ and $n \to \infty$. Moreover, we show estimates for the corresponding FEM-solutions.

Sadowsky models for ribbons and Möbius bands by Γ-convergence

Lorenzo Freddi, The University of Udine, Italy

Abstract. A long and thin parallelepiped whose length is much larger than the diameter of its cross-section can be considered as the reference configuration of several different structures. It may be, for instance, an elastic string, an inextensible ribbon, a thin-walled beam, a rod or, possibly, even another kind of body.

Among such structures, each one exhibits a peculiar mechanical behavior due to the ratio δ between the length of the two sides of the rectangular cross-section and the magnitude ε of the elastic energy which, in turn, depends on the applied loads.

Starting from the energy functional of 3D nonlinear elasticity and letting the sides of the cross-section go to zero, an asymptotic analysis by Γ-convergence, done in collaboration with Peter Hörnung, Mara Giovanna Mora and Roberto Paroni, shows that the 1D Γ-limit energy depends on the parameters δ and ε. Thus, it turns out that the set of limit 1D models is not a hierarchy, meant in the sense of an ordered set; in fact, they occupy different regions of a diagram $\delta - \varepsilon$ looking like a geographic map. The resulting diagram turns out to be divided in three main macroregions, the diagonal and the two regions besides it.

My talk is devoted to illustrate the most interesting part of the diagram, that is the diagonal. It is a region in which the ratio between the magnitude of the energy and the thickness of the cross section is critical, and corresponds to nonlinear director theories of thin-walled beams and ribbons. Among them it falls the model proposed by Sadowsky in 1930 for Möbius bands and recently corrected and extended to a larger class of ribbons (including non-Euclidean ribbons).

Outside the diagonal we have a region in which the energy is small compared to the thickness of the cross section that is occupied by linearized theories. On the other side, where the cross section is thinner, we have a simple model of inextensible strings.

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Sufficient conditions for the observability of N wave equations on compact manifolds with interfaces

Ludovick Gagnon, INRIA Grand-Est, France

Abstract. In this talk we consider N wave equations defined on compact manifolds $(M_i, g_i), i = 1, ..., N$ without boundary. We moreover assume that the manifolds intersect each other. At the intersection the continuity of the solutions of the wave equations are imposed. Due to the possible discontinuity of the different metric g_i at the interface, rays intersecting the interface are reflected and transmitted according to the Snell law. The observability problematic in such a configuration is difficult as the propagation of the rays get more complex. We provide sufficient geometrical conditions for the observability to hold.

Unique Solvability of Some Nonlinear Inequalities with Fractional Laplacian

Evgeny Galakhov, Peoples' Friendship University of Russia & Olga Salvieva, Moscow State Technological University "Stankin", Russia

Abstract. Let $s = [s] + \{s\} \in \mathbb{R}_+$, where $[s]$ is the integer part of s, and $\{s\}$ the fractional one. We define the operator $(-\Delta)^s$ in the standard way by the formula

$$(-\Delta)^s u(x) \stackrel{\text{def}}{=} c_{n,s} \cdot (-\Delta)^{[s]} \left(\text{p.v.} \int\limits_{\mathbb{R}^n} \frac{u(y) - u(x)}{|x-y|^{n+2\{s\}}}\, dy \right), \tag{1}$$

where $c_{n,s}$ is an appropriate positive constant, for all functions such that the right-hand side of (1) makes sense at least in the distributional setting.

We consider the nonlinear elliptic inequality

$$(-\Delta)^s u \geq c|u|^q (1+|x|)^\alpha \quad (x \in \mathbb{R}^n), \tag{2}$$

where $q > 1$ and $\alpha \in \mathbb{R}$.

Using a modification of the test function method [1], we obtain the following theorem.

Theorem 1. *The trivial solution of* (2) *is unique in the class of nonnegative weak solutions for* α *and* q *such that*

$$\alpha > -2s \text{ and } 1 < q < \frac{n+\alpha}{n-2s}. \tag{3}$$

Similar results were obtained for some other types of nonlinear problems with fractional Laplacian, including systems of elliptic inequalities and respective parabolic ones. Some of these results were published in [2].

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Band Structures for Periodic Fractional Schrödinger Operators and Applications to Nonlinear Ground States

Elias Gasmi, Karlsruhe Institute of Technology, Germany

Abstract. In the first part of the talk a spectral theory for periodic fractional Schrödinger operators is presented. It is shown how the Floquet–Bloch theory known from the spectral theory of periodic differential operators can be extended to the fractional case. In the second part, an application to nonlinear ground states is briefly outlined.

A two-dimensional electrostatic model of interdigitated comb drive actuator in longitudinal mode

Antonio Gaudiello, University of Naples Federico II, Italy

Abstract. A periodic homogenization model of the electrostatic equation is constructed for a comb drive with a large number of fingers and whose mode of operation is in-plane and longitudinal. The model is obtained in the case where the distance between the rotor and the stator is of an order ε^α, $\alpha \geq 2$, where ε denotes the period of distribution of the fingers. The model derivation uses the two-scale convergence technique. Strong convergences are also established. This allows us to find, after a proper scaling, the limit of the electrostatic force applied to the rotor in the longitudinal direction. It is a joint work with Michel Lenczner (FEMTO- ST, Besançon, France).

Higher differentiability for a class of obstacle problems with non-standard growth conditions

Chiara Gavioli, University of Modena and Reggio Emilia, Italy

Abstract. We establish the higher differentiability of integer order of the solutions to a class of obstacle problems assuming that the gradient of the obstacle possesses an extra integer differentiability property. We deal with the case in which the solutions to the obstacle problems satisfy a variational inequality of the form

$$\int_\Omega \langle \mathcal{A}(x, Du), D(\varphi - u) \rangle \, dx \geq 0 \qquad \forall\, \varphi \in \mathcal{K}_\psi(\Omega)$$

where $\mathcal{A}$ is an operator satisfying suitable p, q-growth conditions with p and q linked by the relation

$$\frac{q}{p} < 1 + \frac{1}{n} - \frac{1}{r}, \tag{1}$$

being $r > n$. Here $\psi \in W^{1,p}(\Omega)$ is a fixed function called obstacle for which we assume $\nabla\psi \in W^{1,2q-p}_{\mathrm{loc}}(\Omega)$, and $\mathcal{K}_\psi = \{w \in W^{1,p}(\Omega) : w \geq \psi \text{ a.e. in } \Omega\}$ is the class of admissible functions. We require for the partial map $x \mapsto \mathcal{A}(x, \xi)$ a higher differentiability of Sobolev order in the space $W^{1,r}$, with $r > n$ from condition (1). This result can be used to obtain Lipschitz continuity for a class of obstacle problems under non-standard growth conditions.

A quantitative Weinstock inequality

Nunzia Gavitone, University of Naples Federico II, Italy

Abstract. Inthis talk I will discuss about a quantitative Weinstock inequality in higherdimension for the first non trivial Steklov eigenvalue of the Laplace operator for convex sets. The key role is played by a quantitative isoperimetric inequality which involves the boundarymomentum, the volume and the perimeter of a convex open set of $\mathbb{R}^n$,$n \geq 2$.

This is a joint work with Domenico Angelo La Manna, Gloria Paoli and Leonardo Trani.

Dynamics of multiphase models with density constraints

Maria Stella Gelli, University of Pisa, Italy

Abstract. We approximate the dynamics of a multiphase model (for instance two populations) given as a gradient flow of non local energies subject to a global density constraint by particle systems whose motion is ruled by suitable constrained energies.

Compactness and non compactness for Yamabe Problem on manifold with boundary

Marco Gipo Ghimenti, University of Pisa, Italy

Abstract. The Yamabe problem for scalar-flat metrics on manifolds with boundary was firstly raised by Escobar. Once the existence of solution is settled it is natural to ask if the set of solution is compact and if the problem is stable with respect to liner perturbation of the mean curvature term. We will summarize a list of previous result and we will present a series of paper in collaboration with Anna Maria Micheletti and Angela Pistoia in which we focus on the case of umbilic boundary.

Recent results about singular fractional elliptic and parabolic equations

Jacques Giacomoni, University of Pau and Pays de l'Adour, France

Abstract. will present recent contributions on elliptic and parabolic problems involving nonlocal operators as the fractional Laplace operator and singular nonlinearities (existence, uniqueness/multiplicity, Hlder regularity, stability, etc.).

Elliptic problems with growth in nonreflexive Orlicz spaces and with L^1 data

Flavia Giannetti, University of Naples Federico II, Italy

Abstract. We consider nonlinear elliptic Dirichlet problems of the type

$$\begin{cases} -\mathrm{div}A)(x,u,\nabla u) = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded Lipschitz domain in $\mathbb{R}^N$ and $A(x,z,\xi) : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^N$ is a Caratéodory function. Assuming that the monotone vector field A has a growth with respect to the (z,ξ) variables expressed through general N-functions and that the right hand side datum f is merely integrable, we prove the existence and the uniqueness of the so called approximable solution to (1).

Propagating terraces in multidimensional and spatially periodic domains

Thomas Giletti, University of Lorraine, France

Abstract. This talk will be devoted to the existence of pulsating travelling front solutions for spatially periodic heterogeneous reaction-diffusion equations in arbitrary dimension, in both bistable and more general multistable frameworks. In the multistable case, the notion of a single front is not sufficient to understand the dynamics of solutions, and we instead observe the appearance of a so-called propagating terrace. This roughly refers to a finite family of stacked fronts connecting intermediate stable steady states whose speeds are ordered. Surprisingly, for a given equation, the shape of this terrace (i.e., the involved intermediate steady states or even their number) may depend on the direction of the propagation.

The presented results come from a series of work with A. Ducrot, H. Matano and L. Rossi.

Thermodynamically consistent vectorial models in deformable ferroelectrics

Claudio Giorgi, University of Brescia, Italy

Abstract. Within the framework of continuum thermodynamics, we propose a vector-valued model describing the temperature-induced phase transition in a deformable ferroelectric material. The constitutive functions are required to be consistent with the second law of thermodynamics. Among other results, the second law implies a general form of the kinetic equation for the polarization vector which looks like a Ginzburg-Landau equation involving the Gibbs free energy density rescaled by temperature. Suitable choices of this thermodynamic potential unable us to describe both first and second order paraelectric-ferroelectric transitions. The specific Ginzburg–Landau-Devonshire model is recovered when the material lattice has either octahedral or transversely isotropic symmetries.

Besov regularity for solutions of elliptic equations

Raffaella Giova, Parthenope University of Naples, Italy

Abstract. We consider a nonlinear elliptic equation in divergence form of the type

$$\mathrm{div} A(x, Du) = \mathrm{div} F$$

under a p-growth or $p(x)$-growth assumption on the operator A. We establish an extra fractional differentiability for the solutions of the equation in the scale of Besov spaces assuming that the partial map $x \mapsto A(x, \xi)$ and the datum F possess a Besov or a Orlicz–Besov type regularity.

This is a joint work with Albert Clop and Antonia Passarelli di Napoli.

Lagrange multipliers and non-constant gradient constrained problem

Sofia Giuffré, Mediterranea University of Reggio Calabria, Italy

Abstract. Aim of the talk is to study a gradient constrained problem associated to a linear operator. In particular, we are able to prove two kinds of results: first, we prove, under a suitable condition on the gradient constraint, the equivalence of a non-constant gradient constraint problem to a suitable obstacle problem, where the obstacle solves a Hamilton–Jacobi equation in the viscosity sense and, second, we obtain the existence of Lagrange multipliers associated to the problem.

Partial regularity for quasiconvex functionals with linear growth from below

Franz Gmeineder, University of Bonn, Germany

Abstract. In this talk I present partial regularity results for a class of variational problems which have linear growth from below. In this situation, we have to work directly on the relaxed problem, and discuss a novel Caccioppoli inequality for the (almost) optimal ratio between the upper and lower growth exponents of the integrands. The method equally works for problems depending on other differential operators than the full gradient, e.g., the symmetric gradient. The talk comprises joint work with Jan Kristensen (Oxford).

The effect of topology on the number of positive solutions of elliptic equation involving Hardy–Littlewood–Sobolev critical exponent

Divya Goel, Indian Institute of Technology Delhi, India

Abstract. In this talk, we discuss the effect of topology on the existence and multiplicity of the following elliptic equation

$$-\Delta u = \left(\int_\Omega \frac{|u(y)|^{2^*_\mu}}{|x-y|^\mu} dy\right) |u|^{2^*_\mu - 2}u + f \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega,$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$ $(N \geq 3)$, $2^*_\mu = \frac{2N-\mu}{N-2}$ is the Hardy-Littlewood-Sobolev critical exponent. Here f is a function of subcritical growth. We first discuss the cases $f = 0$ or $f \in L^\infty(\Omega)$ with Ω is an annulus. Furthermore, we discuss the case $f(u) = |u|^{q-2}u$, Ω is a smooth bounded domain and $q \in [2, 2^*)$.

Local effects for some spectral stiff problems

Delfina Gómez, University of Cantabria, Spain

Abstract. We analyze the asymptotic behavior for the eigen-elements of a second order differential operator, with piecewise constants coefficients which depend on a small parameter ε, in a planar domain. Certain components of this planar structure may also depend on this small parameter. Considering the range of the low, middle and high frequencies, we provide asymptotics for the eigenvalues and the corresponding eigenfunctions when ε tends to zero. In particular, depending on the boundary conditions, we highlight the frequencies for which certain localization phenomena arise.

This is a joint work with S. Nazarov and M.-E. Pérez-Martínez.

An embedding theorem for BV-functions

Luigi Greco, University of Naples Federico II, Italy

Abstract. We prove an embedding theorem for functions of bounded variation. We give an application to the study of Poisson integral on the disk.

Elliptic operators with unbounded coefficients

Federica Gregorio, University of Salerno, Italy

Abstract. Recently the interest in operators with unbounded coefficients has grown considerably due to their numerous applications in many fields of science, such as quantum mechanics, fluid dynamics, e.g., in the study of Navier-Stokes equations with a rotating obstacle. This class of operator is a generalisation of the operators with bounded coefficients and historically, in the mathematical literature, the subject is studied using several approaches, with ideas and methods from partial differential equations, Dirichlet forms, stochastic processes, stochastic differential equations.

We focus on the elliptic operator

$$A = (1 + |x|^\alpha)\Delta + b|x|^{\alpha-1}\frac{x}{|x|} \cdot \nabla - c|x|^\beta.$$

We prove that the realization A_p in $L^p(\mathbb{R}^N)$, $1 < p < \infty$, of A with domain $D(A_p) = \{u \in W^{2,p}(\mathbb{R}^N) \,|\, Au \in L^p(\mathbb{R}^N)\}$ generates a strongly continuous analytic semigroup $T(\cdot)$ provided that $\alpha > 2$, $\beta > \alpha - 2$ and any constants $b \in \mathbb{R}$ and $c > 0$. Moreover we show that $T(\cdot)$ is consistent, immediately compact and ultracontractive.

Recent result for the Moser–Trudinger problems

Massimo Grossi, Sapienza University of Rome, Italy

Abstract. We discuss some recent results for positive and sign-changing solutions or Moser–Trudinger type problems.

Optimal reinforcing networks for elastic membranes

Serena Guarino Lo Bianco, University of Naples Federico II, Italy

Abstract. We study the optimal reinforcement of an elastic membrane, fixed at its boundary, by means of a network (connected one- dimensional structure), that has to be found in a suitable admissible class. We show the existence of an optimal solution that may present multiplicities, that is regions where the optimal structure overlaps. Some numerical simulations are shown to confirm this issue and to illustrate the complexity of the optimal network when their total length becomes large.

Asymptotic behaviour of parabolic equations in non-cylindrical domains

Senoussi Guesmia, University of the Bahamas, The Bahamas

Abstract. We here investigate the asymptotic behaviour of some non-homogeneous parabolic equations defined on non-cylindrical domains becoming unbounded in many directions when the time t tends to infinity. In fact, the considered problem is not standard since one cannot expect a convergence on the whole section above t. Then we have to envisage the steady state problem for the limit behaviour far away from the boundary layer and we need to define another different problem to describe the limit behaviour on the neighborhood of the boundary layer.

Semicoercive Hemivariational Inequalities and Nonmonotone Contact Problems

Joachim Gwinner, Bundeswehr University Munich, Germany

Abstract. Semicoercive variational inequalities model unilateral contact problems in solid mechanics where the elastic body is not fixed by prescribed displacements (Dirichlet boundary conditions), but is only subject to loads and friction along the contact boundary and where the body nevertheless cannot escape because of geometric conditions on the load.

In this communication we are concerned with existence theory for this delicate situation where coercivity of the elastic energy is lost. Here we base our analysis on a Garding inequality for the elliptic operator. As a concrete example we consider the deformation of an elastic block sliding in a rusty rail.

String solutions of the self-dual Einstein–Maxwell–Higgs equation on compact surfaces

Jongmin Han, Kyung Hee University, South Korea

Abstract. In this talk, I present the self-dual Einstein–Maxwell–Higgs equation on compact surfaces and give a reduction process to a single elliptic equation. Then, I give a recent result on the existence of of solutions of topological and non-topological types.

Existence Results and A Priori Bounds for Positive Solutions of Discrete Nonlinear Elliptic Equations

Carlos Hauser, Karlsruhe Institute of Technology (KIT), Germany

Abstract. For dimensions $n \in \mathbb{N}$, we define the finite difference Laplacian Δ_h with respect to the grid size $h > 0$ by

$$\Delta_h u(x) = \frac{1}{h^2} \sum_{j=1}^{n} [u(x + he_j) - 2u(x) + u(x - he_j)]$$

for $x \in \mathbb{R}_h^n := h\mathbb{Z}^n$. In this talk we consider the discrete Emden inequality

$$-\Delta_h u \geq u^p \quad \text{on } \Omega \cap \mathbb{R}_h^n$$

for a class of infinite cones Ω. We show the existence of a critical value p_* (depending on the geometry of Ω) such that there is only the zero solution for all p smaller than p_*, whereas there are positive solutions for p bigger than p_*. From the non-existence result we infer a priori bounds for non-negative solutions of the boundary value problem

$$\begin{cases} -\Delta_h u = f(u) & \text{in } D \cap \mathbb{R}_h^n, \\ u = 0 & \text{on } \partial D \cap \mathbb{R}_h^n, \end{cases}$$

where the nonlinearity $f(u)$ is of the form $u^p\big(1 + \mathrm{o}(1)\big)$ with $\mathrm{o}(1) \to 0$ as $u \to \infty$ and the bounded domain D locally coincides with an infinite cone Ω at the boundary.

Harnack's inequality for a porous medium type equation

Andreas Herán, University of ErlangenNuremberg, Germany

Abstract. Based on the technique of "Expansion of Positivity" we present the proof for a Harnack inequality to the porous medium equation $\partial_t u - \operatorname{div} \mathbf{A}(x,t,u,Du^m) = \operatorname{div} F$. Thereby we focus on the degenerate case $m > 1$ and assume that u^m is weakly differentiable with respect to the space variable for weak solutions u. This extends the result of DiBenedetto, Gianazza and Vespri in "Harnack's inequality for Degenerate and Singular Parabolic equations", where the authors assume weak differentiability of $u^{\frac{m+1}{2}}$ in the degenerate case and of u^m in the singular case.

Some controllability results for shadow models

Víctor Hernández-Santamaría, University of Toulouse, France

Abstract. In this talk, we present some controllability results for shadow models, which arise as a limit process for reaction-diffusion systems. This leads to a coupled system where one component solves a parabolic partial differential equation (PDE) and the other one is an ordinary differential equation (ODE). We analyze these shadow systems from a controllability perspective. First, by employing Carleman inequalities and ODE arguments, we prove that the null controllability of the shadow model holds. We also obtain a uniform Carleman estimate for the reaction-diffusion equations which allows us to obtain the null control for the shadow system as a limit when the diffusivity tends to infinity in one of the equations.

On pointwise estimates

Ritva Hurri-Syrjanen, University of Helsinki, Finnland

Abstract. I will discuss the validity of some pointwise estimates for functions defined in irregular domains in the Euclidean n-space. As an application we will show that there exist embeddings into suitable Orlicz spaces from the L^1_p-space such that the corresponding Orlicz norm depends on the geometry of the given domain. The results are sharp for L^1_1-functions.

My talk is based on joint work with Petteri Harjulehto.

Continuity of a weak solution of degenerate diffusion equations with drifts

Sukjung Hwang, Yonsei University, South Korea

Abstract. We discuss the continuity of a weak solution of porous medium type equations with drift terms. In other words, a sharp L^p type class of drift vector fields to have the uniform or Hölder continuity of a weak solution is the main ingredient of this talk.

On the uniqueness of positive solutions of the Lane–Emden problem in planar domains

Isabella Ianni, University of Campania Luigi Vanvitelli, Italy

Abstract. The question of the uniqueness for the positive solutions of the Lane–Emden problem arose since the famous symmetry result by Gidas, Ni, Nirenberg (1979), which implies uniqueness when the domain is a ball. A conjecture on the uniqueness in any convex domain was then formulated during the eighties, but only partial results have been proven so far. In a recent work we have solved this conjecture in any smooth bounded domain in the planar case and for p large enough. This is the first complete answer to this longstanding open problem. In this talk we will describe the main ideas behind our proof, which relies on the computation of the Morse index of the solutions for large values of p and ultimately on the characterization of their asymptotic behavior.

This is from joint works with Franesca De Marchis, Massimo Grossi and Filomena Pacella (University Sapienza of Roma, Italy).

Null-controllability of the linear Kuramoto–Sivashinsky equation on star-shaped trees

Liviu Ignat, Institute of Mathematics Simion Stoilow, Romania

Abstract. In this talk we discuss null-controllability properties for the linear Kuramoto–Sivashinsky equation on a star-shaped tree with two types of coupling conditions, with boundary controls acting on the external vertices of the tree. We show that with few exceptions (when the so-called anti-diffusion parameter belongs to a countable critical set) at any positive time the system is null-controllable when acting with controls on a part of the external vertices. We point out that the critical set for which the null-controllability fails differs from the first model to the second one.

This is a joint work with Cristian Cazacu and Ademir Pazoto.
Partially supported by CNCS-UEFISCDI Grant No. PN-III-P4-ID-PCE-2016-0035.

The geometry of the free boundary near the fixed boundary generated by a fully nonlinear uniformly elliptic operator

Emanuel Indrei, Purdue University, USA

Abstract. This talk focuses on the regularity problem of the free boundary near the fixed boundary for the fully nonlinear obstacle problem in higher dimensions.

Donati compatibility conditions on a surface – Application to shell theory

Oana Iosifescu, University of Montpellier, France

Abstract. An intrinsic approach to a mathematical model of a linearly or nonlinearly elastic body consists in considering the strain measures found in the energy of this model as the sole unknowns, instead of the displacement field in the classical approach. Such an approach thus provides a direct computation of the stresses by means of the constitutive equation. The main problem therefore consists in identifying specific compatibility conditions that these new unknowns, which are now matrix fields with components in L^2, should satisfy in order that they correspond to an actual displacement field. Such compatibility conditions are either of Saint–Venant type, in which case they take the form of partial differential equations, or of Donati type, in which case they take the form of orthogonality relations against matrix fields that are divergence-free.

We show in this talk how an intrinsic approach can be successfully applied to the well-known Koiters model of a nonlinearly elastic "shallow" shell, thus providing the first instance of a mathematical justification of this approach applied to a nonlinear shell model. More specifically, we first identify and justify compatibility conditions of Donati type guaranteeing that the nonlinear strain measures found in Koiters model correspond to an actual displacement field. Second, we show that the associated intrinsic energy attains its minimum over a set of matrix fields that satisfy these Donati compatibility conditions, thus providing an existence theorem for the intrinsic approach; the proof relies in particular on an interesting per se nonlinear Korn inequality on a surface. Incidentally, this existence result (once converted into an equivalent existence theorem for the classical displacement approach) constitutes a significant improvement over previously known existence theorems for Koiters model of a nonlinearly elastic shallow shell.

This talk is based on joint work with Philippe Ciarlet.

Multiplicity and concentration results for some nonlinear Schrödinger equations with the fractional p-Laplacian

Teresa Isernia, Marche Polytechnic University, Italy

Abstract. In this talk we consider a class of parametric Schrödinger equations driven by the fractional p-Laplacian operator and involving continuous positive potentials and nonlinearities with subcritical or critical growth. By using variational methods and Ljusternik–Schnirelmann theory, we study the existence, multiplicity and concentration of positive solutions for small values of the parameter.

Concentration and effective behaviour of brittle damage

Flaviana Iurlano, Sorbonne University, France

Abstract. This talk is concerned with an asymptotic analysis of a variational model of brittle damage, when the damaged zone concentrates into a set of zero Lebesgue measure and, at the same time, the stiffness of the damaged material becomes arbitrarily small. Concentration leads to a limit energy with linear growth whose singular part can be easily described; conversely, the identification of the bulk part of the limit energy requires a subtler analysis of the concentration properties of the displacements.

This is a joint work with J.-F. Babadjian and F. Rindler.

Composite elastic plate via general homogenization theory

Jelena Jankov, J. J. Strossmayer University of Osijek, Croatia

Abstract. Homogenization theory is one of the most successful approaches for dealing with optimal design problems (in conductivity or linearized elasticity), that consists in arranging given materials such that obtained body satisfies some optimality criteria, which is mathematically usually expressed as minimization of some (integral) functional under some (PDE) constrains. General, non-periodic homogenization theory is well developed for second order elliptic partial differential equations, where the key role plays the notion of H-convergence. It was introduced by Spagnolo through the concept of G-convergence (1968) for the symmetric case, and further generalized by Tartar (1975) and Murat and Tartar (1978) for non-symmetric coefficients under the name H-convergence. Some aspects for higher order elliptic problems were also considered by Zhikov, Kozlov, Oleinik and Ngoan (1979).

More recently, prompted by possible applications in optimal design problems, Antonić and Balenović (1999, 2000) considered a direct approach to homogenization in the context of the stationary plate equation. These results were pushed forward by Burazin, Jankov and Vrdoljak (2018), and a number of properties of H-convergence were proved, such as locality, irrelevance of the boundary conditions, corrector result, etc.

Using this newly developed theory, we prove that the set of composites obtained by periodic homogenization is dense in the set of all possible composites. Also, we derive expressions for elastic coefficients of composite plate obtained by mixing two materials in thin layers, also known as laminated materials, and for mixing two materials in low-contrast regime. Moreover, we also derive optimal bounds on the effective energy of a composite material, known as Hashin–Shtrikman bounds. In the case of two-phase isotropic materials, explicit optimal Hashin–Shtrikman bounds are calculated. We show that an analogous results can be derived for the complementary energy.

This is a joint work with Kresimir Burazin and Marko Vrdoljak.

Strongly nonlinear multiplicative inequalities and applications to PDEs

Agnieszka Kalamajska, Polish Academy of Science & University of Warsaw, Poland

Abstract. We will discuss the variants of multiplicative inequalities like:

$$\int_\Omega |\nabla f(x)|^p h(f(x))\,dx \le C \int_\Omega \left(\sqrt{|\nabla^{(2)} f(x) \mathcal{T}_h(f(x))|}\right)^p h(f(x))\,dx,$$

where $\Omega \subseteq \mathbb{R}^n$, $\mathcal{T}_h(s)$ is certain transform of function $h(\cdot)$ such that it retrieves the power functions up to the constant, under certain assumptions. They imply the classical Gagliargo–Nirenbergs interpolation inequality when $p \ge 2$. We will show applications of that inequality to the regularity theory for degenerated PDEs of elliptic type and focus on some models where such inequlities can be applied. The example is the model of electrostatic micromechanical systems (MEMS).

The talk will be based on the series of my joint works with Jan Peszek, Tomasz Choczewski, Katarzyna Mazowiecka, Katarzyna Pietruska-Paluba, Alberto Fiorenza and Claudia Capone.

Maximal Operator in Musielak–Orlicz spaces

Anna Kaminska, University of Memphis, USA

Abstract. Hardy–Littlewood maximal operator in Musielak–Orlicz spaces will be discussed. The Musielak–Orlicz spaces are generalizations of variable Lebesgue spaces $L^{p(x)}$, and it is the important class of non-symmetric function spaces. In the recent years the maximal operator and its applications in $L^{p(x)}$ have been studied intensively in various contexts. In view of several examples of Musielak–Orlicz spaces that are neither symmetric nor variable Lebesgue spaces, the studies of the maximal operator with its applications in Musielak–Orlicz spaces seem to be very natural and useful.

Inverse problem for elastic body with thin inclusion

Alexander Khludnev, Lavrentyev Institute of Hydrodynamics SB RAS & Novosibirsk State University, Russia

Abstract. An inverse problem for an elastic body with a thin elastic inclusion is investigated. It is assumed that the inclusion crosses the external boundary of the elastic body. A connection between the inclusion and the elastic body is characterized by the damage parameter. We study a dependence of the solutions on the damage parameter. In particular, passages to infinity and to zero of the damage parameter are investigated. Limit models are analyzed. Assuming that the damage and rigidity parameters of the model are unknown, inverse problems are formulated. Sufficient conditions for the inverse problems to have solutions are found. Estimates concerning solutions of the inverse problems are established.

Stability results for the log Sobolev inequality

Daesung Kim, Purdue University, USA

Abstract. The logarithmic Sobolev inequality states that the Fisher information is bounded below by the relative entropy. Equality holds if and only if a measure is Gaussian. We are interested in measuring how far a probability measure is away from Gaussian measures when it is close to achieving equality. To this end, we find a lower bound of the deficit, which is the difference between the Fisher information and the relative entropy, in terms of distances. In this talk, we discuss deficit bounds for the log Sobolev inequality in terms of the Wasserstein distances and L^1 distance. We also show that these results are best possible by giving an explicit example.

This is based on joint work with Emanuel Indrei.

Semilinear elliptic equations with Dirichlet operator and singular nonlinearities

Tomasz Klimsiak, Nicolaus Copernicus University, Poland

Abstract. We consider elliptic equations of the form $-Au = u^{-\gamma} \cdot \mu$, where A is the operator associated with a regular symmetric Dirichlet form, μ is a positive nontrivial measure and $\gamma > 0$. We present the existence and uniqueness results for solutions of such equations as well as some regularity results. We also study stability of solutions with respect to the convergence of measures on the right-hand side of the equation. For this purpose, we introduce some type of functional convergence of smooth measures, which in fact is equivalent to the quasi-uniform convergence of associated potentials.

New Optimal Control Problems in Density Functional Theory motivated by Photovoltaics

Michael Kniely, IST Austria, Austria

Abstract. We present and study novel optimal control problems motivated by the search for photovoltaic materials with high power-conversion efficiency. The material must perform the first step: convert light (photons) into electronic excitations. We formulate various desirable properties of the excitations as mathematical control goals at the Kohn-Sham-DFT level of theory, with the control being given by the nuclear charge distribution. We prove that nuclear distributions exist which give rise to optimal HOMO-LUMO excitations, and present illustrative numerical simulations for 1D finite nanocrystals. We observe pronounced goal-dependent features such as large electron-hole separation, and a hierarchy of length scales: internal HOMO and LUMO wavelengths $<$ atomic spacings $<$ (irregular) fluctuations of the doping profiles $<$ system size.

This is a joint work with Gero Friesecke.

Dynamics and control for the "Guidance by repulsion" model

Dongnam Ko, DeustoTech Bilbao, Spain

Abstract. We control the "Guidance by repulsion" model based on the two-color agents framework: dogs try to drive sheep into a certain area. On the one hand, this model is originated from the hyperbolic PDE (sheep) controlled by the ODE system (dogs) through repulsive interactions. On the other hand, the system can be understood as a bilinear type partial control problem in terms of the characteristic curves. We study the well-posedness and asymptotic dynamics of the simplified system to observe the controllability on the evader's position. For the asymptotic stability, the Lyapunov functions are constructed using hypo-coercivity arguments.

On roughness effect in optimal boundary control problem for ill-posed elliptic equation in domain with oscillating boundary

Peter Kogut, Oles Honchar Dnipro National University, Ukraine

Abstract. Let ε be a small positive parameter. Let Ω_ε and Ω be given bounded open subsets of R^N $(N>2)$ with $C^{1,1}$-boundaries confined in a fixed bounded domain D. We assume that Ω lies locally on one side of $\partial\Omega$ and the boundary $\partial\Omega$ consists of two disjoint parts $\partial\Omega=\Gamma_D\cup\Gamma_N$ such that the sets Γ_D and Γ_N have positive $(N-1)$-dimensional measures, $\Gamma_D\subset\partial D$. We make the similar assumption with respect to the boundary of Ω_ε, though each of domains Ω_ε may have rather rough part of the boundary $\partial\Omega_\varepsilon=\Gamma_D\cup\Gamma_{N,\varepsilon}$, where $\Gamma_D\cap\Gamma_{N,\varepsilon}=\emptyset$ and Γ_D-part does not depend on $\varepsilon>0$. Since we consider Ω_ε as some perturbation of Ω that excludes the case of perforated domains, we assume that the family $\{\Omega_\varepsilon\}_{0<\varepsilon\le\varepsilon_0}$ approaches an open bounded set Ω as follows $\lim_{\varepsilon\to0}\left(\sup_{x\in\Omega\Delta\Omega_\varepsilon}d(x,\partial\Omega)\right)=\lim_{\varepsilon\to0}\left(\sup_{x\in\Omega\Delta\Omega_\varepsilon}\inf_{y\in\partial\Omega}|x-y|_{\mathbb{R}^N}\right)=0$.

We consider the following optimal control problem for a nonlinear elliptic equation provided one of the control zones is located along a rugose $\Gamma_{N,\varepsilon}$-part of the boundary $\partial\Omega_\varepsilon$:

$$\text{Minimize } J_\varepsilon(u,g,y)=\frac12\int_{\Omega_\varepsilon}|y-y_d|^2\,dx+\frac12\int_{\Gamma_{N,\varepsilon}}|u|^2\,d\mathcal{H}^{N-1}+\frac1q\int_{\Omega_\varepsilon}|g-g_d|^q\,dx, \tag{1}$$

subject to constraints

$$-\Delta y=f(y)+g\quad\text{in }\ \Omega_\varepsilon, \tag{2}$$

$$y=0\quad\text{on }\ \Gamma_D,\quad \partial_\nu y=u\quad\text{on }\ \Gamma_{N,\varepsilon}, \tag{3}$$

$$g\in\mathfrak{G}_{ad}=L^q(D),\quad y\in W_0^{1,2}(\Omega_\varepsilon;\Gamma_D), \tag{4}$$

$$u\in\mathfrak{A}_{ad}(\Gamma_{N,\varepsilon})=\left\{v|_{\Gamma_{N,\varepsilon}}\ :\ v\in W^{1,r}(D),\ \|v\|_{W^{1,r}(D)}\le\beta\right\}. \tag{5}$$

It is assumed that $N>2$, $g_d\in L^q(D)$ and $y_d\in L^2(D)$ are given distributions, $q\ge\frac{2N}{N+2}$, $r>N$, $f(y)=F'(y)$, $f:\mathbb{R}\to(0,\infty)$ is a strictly convex function, and $F\in C^2(K)$ for any compact set $K\subset\mathbb{R}$, where F is a non-decreasing positive function such that

$$F'(z)\ge C_FF(z),\quad\forall\,z\in\mathbb{R}\ \text{ and }\ \left|\int_{-\infty}^0zF'(z)\,dz\right|<+\infty. \tag{6}$$

for some constant $C_F>0$.

The main characteristic feature of the indicated BVP is the fact that because of the specificity of non-linearity $f(y)$, we can not expect to have a solution of the state equation for a given control. After having defined a suitable functional class in which we look for solutions, we prove the consistency of the original optimal control problem and show that it admits a unique optimal solution. Then we derive a first order optimality system assuming the optimal solution is slightly more regular. The main novelty of our case is the fact that we have two different types of controls — distributed and boundary, and the control zone for one of them is supported along of a rough part of the boundary of Ω_ε. Using the methods of variational convergence of constrained minimization problems, we study the asymptotic behaviour of solutions to the considered optimal control problem with mixed boundary conditions (3) in the presence of boundary oscillations along the control zone, and derive the sufficient conditions allowing to recognize the limit problem in an explicit form.

Breather Solutions for a Quasilinear 1+1dim Wave Equation

Simon Kohler, Karlsruhe Institute for Technology, Germany

Abstract. For the $1+1$-dimensional quasilinear wave equation

$$g(x)\partial_t^2u-\partial_x^2u+\Gamma(x)\partial_t((\partial_tu)^3)=0,$$

with $(x,t)\in\mathbb{R}^2$, we prove the existence of a breather solution, i.e., a solution which is localized in space and periodic in time.
Here $g\in L^\infty(\mathbb{R})$ is a periodic potential such that 0 lies in a spectral gap of the operators $L_k=-\frac{\mathrm{d}^2}{\mathrm{d}x^2}-k^2\omega^2g(x)$ on $L^2(\mathbb{R})$. A typical example for g is a periodic step potential. The main feature of the problem is the choice of $\Gamma(x)$ as a multiple of a delta-distribution located at 0. Using a Fourier ansatz in time we obtain the breather solution as a minimizer of a functional on a suitable sequence space of Fourier coefficients. The analytical results are complemented by numerical simulations.

Quantum graphs: optimization problem

Sylwia Kondej, University of Zielona Góra, Poland

Abstract. The talk is devoted to the analysis of quantum mechanical systems with the potentials supported on the star shaped graphs in $\mathbb{R}^3$. The corresponding Hamiltonian is mathematically defined by the Schrödinger operator with certain boundary conditions. We show that the maximum of the ground state energy is reached for configurations known from Thomson problem. Moreover, we discuss conditions under which the discrete spectrum disappears.

Analysis of Hooke-like isotropic hypoelasticity models based on corotational stress rates

Sergey Korobeynikov, Lavrentyev Institute of Hydrodynamics SB RAS, Russia

Abstract. This study presents an analysis of the constitutive relations of Hooke-like isotropic hypoelastic material models in Lagrangian and Eulerian forms generated using corotational stress rates with associated spin tensors from the family of material spin tensors. Explicit expressions were obtained for the Lagrangian and Eulerian tangent stiffness tensors for the hypoelastic materials considered. The main result of this study is a proof that these fourth-order tensors have full symmetry only for material models generated using two corotational stress rates: the Zaremba–Jaumann and the logarithmic ones. In the latter case, the Hooke-like isotropic hypoelastic material is simultaneously the Hencky isotropic hyperelastic material. For the material models considered, basis-free expressions for the material and spatial tangent stiffness tensors are obtained that can be implemented in FE codes. In particular, new basis-free expressions are derived for the tangent stiffness (elasticity) tens ors for the Hencky isotropic hyperelastic material model.

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Variational Problems for Non-Penetrating Cracks in Elastic Bodies whose Material Moduli depend on the Mean Normal Stress

Victor Kovtunenko, University of Graz, Austria & Lavrentyev Institute of Hydrodynamics SB RAS, Russia

Abstract. The well-posedness of the problem of non-penetrating crack in an elastic body whose material moduli depends on the mean normal stress is studied. The type of models considered is based on a new implicit theory for the response of elastic bodies. The underlying nonlinear problem is studied within the context of a variational inequality.

Acknowledgements: The work is supported by the Austrian Science Foundation (FWF) project P26147-N26: "Object identification problems: numerical analysis" (PION). V.A.K. thanks the Faculty of Mathematics, Physics and Geodesy at the Graz University of Technology, the RFBR and JSPS research project 19-51-50004 for partial support.

Metrizable compactification of an arbitrary subset of an Euclidean space with applications to DiPerna–Majda measures theory

Piotr Kozarzewski, University of Warsaw, Poland

Abstract. It is known, that for every sufficiently regular space X and a set F of bounded, continuous, real-valued functions on X there exists a compactification EX, such that every function from F may be extended to a continuous function on EX. The known proofs are, however, non-constructive. Moreover, they don't provide any clues on metrizability of EX.

We present an easy and direct proof of this fact in particular setting, when X is a subset of an Euclidean space and F is a separable subset of $C(X)$. As a construction of EX and its metrizability are also established, we apply the result in the theory of DiPerna–Majda measures to generalize and deepen the existing results, as well as verify positively some classical, so far not confirmed, assumptions.

Relaxation of nonlocal supremal functionals

Carolin Kreisbeck, Utrecht University, The Netherlands

Abstract. Nonlocal functionals in the form of double integrals appear naturally in models of peridynamics. In the homogeneous case, separate convexity of the integrands has recently been identified as a necessary and sufficient condition for weak lower semicontinuity. When it comes to relaxation, though, a characterization of the weak lower semicontinuous envelopes is still largely open. It is in particular unclear whether they can be represented as double integrals. Motivated by these interesting developments, this talk addresses a related question by discussing homogeneous supremal functionals in the nonlocal setting. We show that weak* lower semicontinuity holds if and only if the level sets of a symmetrized and suitably diagonalized version of the supremand are separately convex. Unlike for double integrals, the supremal structure is guaranteed to be preserved in the process of relaxation. The proof of this statement relies on the connection between supremal and indicator functionals, which reduces the problem to studying weak* closures of a class of nonlocal inclusions. We give examples of explicit relaxation formulas for different multi-well functions.

This is joint work with Elvira Zappale (University of Salerno).

Quasistatic viscoelasticity with self-contact at large strains

Stefan Krömer, UTIA, Czech Academy of Sciences, Czech Republic

Abstract. We study a model for the evolution of non-simple viscoelastic materials that prevents self-interpenetration locally and globally. Inertia is neglected. The main result is the existence of a weak solution of the associated initial-boundary value problem.

This is a joint work with Tomas Roubicek.

Derivation of von Kármán plate theory in the framework of three-dimensional viscoelasticity

Martin Kruzik, Czech Academy of Sciences, Czech Republic

Abstract. We apply a quasi-static nonlinear model for non-simple viscoelastic materials at a finite-strain setting in the Kelvin's-Voigt's rheology to derive a viscoelastic plate model of von Kármán type. We start from solutions to a model of three-dimensional viscoelasticity where the viscosity stress tensor complies with the principle of time-continuous frame-indifference. Combining the derivation of nonlinear plate theory by Friesecke, James and Müller, and the abstract theory of gradient flows in metric spaces by Sandier and Serfaty we perform a dimension-reduction from 3D to 2D and identify weak solutions of viscoelastic form of von Kármán plates.

This is a joint work with M. Friedrich (Muenster).

Existence of solution to the fractional mean curvature flow

Domenico La Manna, University of Cassino and Southern Lazio, Italy

Abstract. The aim of this talk is to discuss about the issue of existence of a classical solution to the fractional mean curvature flow.

This is a joint work with Vesa Julin.

Spectral stability and boundary homogenization for the biharmonic operator subject to Steklov boundary conditions

Pier Domenico Lamberti, University of Padova, Italy

Abstract. We consider two Steklov-type problems for the Biharmonic operator and study their spectral stability upon domain perturbation. One of the two problems is the classical DBS – Dirichlet Biharmonic Steklov – problem, the other one is a variant. Under a comparatively weak condition on the convergence of the domains, we prove the stability of the resolvent operators for both problems, which implies the stability of the eigenvalues and the eigenfunctions. Our condition turns out to be sharp at least for the variant of the DBS problem in which case the sharpness is proved by studying a natural boundary homogenization problem. In the case of the DBS problem, we prove stability of a suitable Dirirchlet-to-Neumann type map under very weak conditions on the convergence of the domains and we formulate an open problem.

This talk is based on a joint work with Alberto Ferrero.

Nonlocal diffusion processes in irregular domains

Maria Rosaria Lancia, Sapienza University of Rome, Italy

Abstract. We consider nonlocal diffusion processes in non smooth domains of fractal type as well as in the corresponding smoother approximating domains . Existence, uniqueness and regularity issues will be discussed. The asymptotic behaviour of the smoother solutions, if any, will be discussed.

Total variation flow of curves in Riemannian manifolds

Michal Lasica, University of Warsaw, Poland

Abstract. We consider the functional of total variation defined on maps from an interval I into a complete, connected Riemannian manifold N. We investigate well-posedness of the steepest descent flow of this functional. Unless N is of non-positive sectional curvature (NPC), it fails critically to be semiconvex, hence Ambrosio–Gigli–Savare theory of gradient flows in metric spaces is not readily applicable. We introduce a notion of solution to flow equations for general N that coincides with the one of Ambrosio–Gigli–Savare for NPC manifolds. We show that these solutions exist under a mild condition on the size of jumps of initial datum. The proof uses some novel tools such as a "completely local" a priori estimate and a variant of Sobolev inequality with covariant derivative.

This is based on a joint project with Lorenzo Giacomelli and Salvador Moll.

Remarks on Lyapunov operator in infinite dimensional setting

Martin Lazar, University of Dubrovnik, Croatia

Abstract. We analyse boundedness and coercivity properties of the Lyapunov operator defined as

$$L(P) = AP + PA, \quad P \in \mathcal{L}(X),$$

Here A is a given self-adjoint, unbounded operator on Hilbert space X with compact resolvent. As the image of the Lyapunov operator is not contained within $\mathcal{L}(X)$, careful analysis and the appropriate functional setting are required in order to obtain the above mentioned properties of L. The result will be compared to existing eigenvalue decay estimates of solutions to operator Lyapunov equations. Relation to the control theory will be given on an example of the Gramian operator for the heat equation.

Optimal location of a thin rigid inclusion for a problem describing equilibrium of a composite plate with a crack

Niurgun Lazarev, North-Eastern Federal University, Russia

Abstract. A nonlinear model describing equilibrium of a cracked inhomogeneous plate with a thin rigid inclusion is studied. Deformation of the plate is described by the classical theory of plates based on the Kirchhoff–Love hypothesis. We assume that a Signorini-type boundary condition, ensuring non-penetration of the crack faces, is satisfied. For a family of corresponding variational problems, we analyze the dependence of their solutions on the location of the rigid inclusion. We formulate an optimal control problem with a cost functional defined by an arbitrary continuous functional on the solution's space and a control parameter describing location of the inclusion. The existence of a solution of the optimal control problem is proven.

Lipschitz regularity for orthotropic p-harmonic functions

Chiara Leone, University of Naples Federico II, Italy

Abstract. We present some regularity results for the gradient of solutions to very degenerate equations, which exhibit a great lack of ellipticity. In particular we show that local weak solutions of the orthotropic p-harmonic equation are locally Lipschitz, for every $p \geq 2$ and in every dimension.

The results presented in this talk have been obtained in collaboration with Pierre Bousquet (Toulouse), Lorenzo Brasco (Ferrara) and Anna Verde (Napoli).

On the positive semigroups generated by Fleming–Viot type differential operators

Vita Leonessa, University of Basilicata, Italy

Abstract. This talk is devoted to a class of degenerate second-order elliptic differential operators arising in the application of Fleming-Viot processes to some models of population dynamics. Such operators are defined by setting, for every $u \in C^2([0,1]^d)$ $(d \geq 1)$ and $x \in [0,1]^d$,

$$A(u)(x) = \sum_{i=1}^{d} x_i(1-x_i)\frac{\partial^2 u}{\partial x_i^2}(x) + [a_i + 1 - (a_i + b_i + 2)x_i]\frac{\partial u}{\partial x_i}(x)\,, \qquad (1)$$

where $a_i, b_i \in \mathbb{R}$ satisfy $a_i > -1, b_i > -1$ for all $i = 1, \ldots, d$.

By using techniques of approximation theory, we show that their closures generate positive semigroups both in the space of all continuous functions and in Jacobi weighted L^p-spaces. Moreover, these semigroups are obtained as limits of iterates of certain positive linear operators which generalize the Bernstein-Durrmeyer operators with Jacobi weights on $[0,1]$. The approximation formula obtained allows to get regularity properties of the generated semigroups along with their asymptotic behaviours.

Many authors dealt with differential operators of type (1) in the framework of d-dimensional simplices as well as of hypercubes. In particular, the special case where $b_i = a_{i+1}$, $a_i \in \mathbb{R}$ $(i = \ldots, d)$ has been considereed in [3].

We want to point out that all results presented are taken from [2], where the approximating operators have been introduced and studied for the first time. Recently, a new generalization of the Bernstein-Durrmeyer operators by means of an arbitrary measure has been investigated (see [2]). An open question should be to understand under which conditions these new operators might be useful for generating and approximating semigroups related to differential operators more general than A.

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Test functions for some polyconvex integrals

Francesco Leonetti, University of L'Aquila, Italy

Abstract. When proving regularity for minimizers of functionals, we need suitable test functions: we will show a few of them that have been used in regularity theorems concerning polyconvex integrals.

Quasiconvergence in parabolic equations in one space dimension

Fang Li, Shanghai Normal University, China

Abstract. In this talk we first prove a quasiconvergence result for bounded solutions of general quasilinear parabolic equations in unbounded or variable domains in one space dimension. Then, we use this result to study a one dimensional heterogeneous reaction diffusion equation with combustion type of nonlinearity. We prove a trichotomy result on the asymptotic behavior of solutions of the Cauchy problem: any nonnegative solution converges as $t \to \infty$ to either a positive steady state, or the ignition point, or the trivial solution 0.

Local controllability to the trajectories of the Fokker–Planck equation with a localized control

Pierre Lissy, Paris Dauphine University, France

Abstract. We will present a new result on the control of the Fokker–Planck equation, posed on a bounded domain of $\mathbb{R}^d (d \geq 1)$. More precisely, the control is the drift force, localized on a small open subset. We prove that this system is locally null controllable to regular nonzero trajectories. The results are obtained thanks to a linearization method based on a standard inverse mapping procedure and the fictitious control method. The main novelties of the present article are twofold. Firstly, we propose an alternative strategy to the standard fictitious control method: the algebraic solvability is performed and used directly on the adjoint problem. Secondly, we prove a new Carleman inequality for the heat equation with one order space-varying coefficients: the right-hand side is the gradient of the solution localized on a subset (rather than the solution itself), and the left-hand side can contain arbitrary high derivatives of the solution.

This is a joint work with Michel Duprez.

Minimal controllability time for the heat equation under nonnegative state constraint

Jerôme Lohéac, University of Lorraine / CNRS, France

Abstract. The heat equation with homogeneous Dirichlet boundary conditions is well known to preserve non-negativity. Besides, due to infinite velocity of propagation, the heat equation is null-controllable within arbitrary small time, with controls supported in any arbitrarily open subset of the domain (or its boundary) where heat diffuses. The following question then arises naturally: can the heat dynamics be controlled from a positive initial steady-state to a positive final one, requiring that the state remains nonnegative along the controlled time-dependent trajectory? I will show that this state-constrained controllability property can be achieved if the control time is large enough, but that it fails to be true in general if the control time is too short, thus showing the existence of a positive minimal controllability time. In other words, in spite of infinite velocity of propagation, realizing controllability under the unilateral non-negativity state constraint requires a positive minimal time.

I will also present the discretized version of this result. In particular, for the discretized heat equation, we will be able to give a description of the structure of the time optimal Dirichlet boundary controls.

Singular equations with subquadratic growth in the gradient: multiplicity and uniqueness

Salvador López-Martínez, University of Granada, Spain

Abstract. In this talk we will summarize some new results, proved in [3], concerning the existence, uniqueness and multiplicity of solutions to the following elliptic problem:

$$\begin{cases} -\Delta u = \lambda u + \mu(x)\dfrac{|\nabla u|^q}{u^\alpha} + f(x) & \text{in } \Omega \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{P_λ}$$

Here, $\Omega \subset \mathbb{R}^N$ ($N \geq 3$) is a bounded domain with boundary smooth enough, $0 \lneqq \mu \in L^\infty(\Omega)$, $0 \lneqq f \in L^p(\Omega)$ with $p > \frac{N}{2}$, $1 < q < 2$, $\alpha \in [0,1]$ and $\lambda \in \mathbb{R}$. We will show that, for $\lambda > 0$, the nature of problem (P_λ) depends drastically on the "strength" of the singularity, i.e., on the size of α.

First of all, for the range $0 \leq \alpha < q-1$, an existence and *multiplicity* result can be proved for $\lambda > 0$ small. Note that the non-singular case $\alpha = 0$ is included here, so that the mentioned result extends in some sense the multiplicity result in [1] for $q = 2$. In contrast, the case $q - 1 < \alpha \leq 1$ is completely different as existence and *uniqueness* hold for $\lambda > 0$ small enough. In sum, we complement the results in [2] concerning the special case $\alpha = q - 1$.

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Integrodifferential equations and glass relaxation models

Paola Loreti, Sapienza University of Rome, Italy

Abstract. The stretched exponential function is usually employed to model glass relaxation. Thanks to the approximation of the stretched exponential function we show some results for the equation of viscoelasticity.

The results are obtained in the joint work with D. Sforza: Viscoelastic aspects of glass relaxation models, accepted for publication in Physica A.

Energy release rate and stress intensity factors in planar elasticity in presence of smooth cracks

Ilaria Lucardesi, University of Lorraine, France

Abstract. In this talk, we first analyze the singular behavior of the displacement of a linearly elastic body in dimension 2 close to the tip of a smooth crack, extending the well-known results for straight fractures. As conjectured by Griffith, the displacement behaves as the sum of an H^2-function and a linear combination of two singular functions, whose profile is similar to the square root of the distance from the tip. The coefficients of the linear combination are the so called stress intensity factors. Afterwards, we prove the differentiability of the elastic energy with respect to an infinitesimal fracture elongation and we compute the energy release rate, enlightening its dependence on the stress intensity factors. In the last part of the talk we will see some generalizations to $C^{1,1}$ fractures.

This is a joint work with S. Almi (Vienna) and G. Lazzaroni (Florence).

The regularizing effect of superlinear terms in Elliptic and Parabolic Problems

Martina Magliocca, Sapienza University of Rome, Italy

Abstract. We want to discuss the regularizing effect induced by superlinear terms in some class of Elliptic and Parabolic Equations. To give an idea, our model equations are

$$u_t - \Delta u = g(u)|\nabla u|^q + f \qquad \text{in } (0,T)\times\Omega, \tag{1}$$

and

$$-\Delta u = g(u)|\nabla u|^q + f \qquad \text{in } \Omega, \tag{2}$$

being $\Omega \subset \mathbb{R}^N$ for $N \geq 2$, $g(u) \geq 0$ and $q \leq 2$. We assume that the term

$$g(u)|\nabla u|^q$$

behaves in a superlinear way. Roughly speaking, if $g(u) \equiv$ const., then we are asking for $1 \leq q \leq 2$. In the more general case $g(u) \not\equiv$ const., the q threshold is influenced by this perturbation term and the superlinear q range depends on its growth. An important remark on this kind of problems concerns the data assumptions, which have to satisfy well precises compatibility conditions in order to have existence of solutions.
We will show that, under certain growth assumptions on $g(u)$, we can relax the regularity needed on the data w.r.t. the case $g(u) =$ const..
We will compare the cases

(P1) $g(u) \equiv$ const. in (1) [M18, 4];
(P2) $g : \mathbb{R} \to (0, +\infty)$ continuous and such that $g(u) \leq c/(1 + |u|^\alpha)$ with c, $\alpha \geq 0$ in (1) (ongoing project in collaboration with F. Oliva);

and

(E1) $g(u) \equiv$ const. in (2) [GMP14];
(E2) $g : \mathbb{R} \to (0, +\infty)$ continuous and such that $g(u) \leq c/(1 + |u|^\alpha)$ with c, $\alpha \geq 0$ in (2) (ongoing project in collaboration with S. Segura de León & M. Latorre Balado).

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H-convergence for equations depending on monotone operators in Carnot Groups

Alberto Maione, Universiy of Trento, Italy

Abstract. Let Ω be an open and bounded subset of a Carnot Group $\mathbb{G}$ and $2 \leq p < \infty$. In this talk we present some results related to the convergence of solutions of Dirichlet problems for sequences of monotone operators. The aim of this talk is to give a generalization of well-known results of Tartar [1], De Arcangelis-Serra Cassano [2] and Franchi et al. [3] in more general frameworks.

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Lack of null controllability of viscoelastic flows

Debayan Maity, Autonomous University of Madrid, Spain

Abstract. We consider controllability of linear viscoelastic flow with a localized control in the momentum equation. We show that, for Jeffreys fluids or for Maxwell fluids with more than one relaxation mode, exact null controllability does not hold. This contrasts with known results on approximate controllability.

Integral representations for solutions of the Dirichlet problem for Helmholtz equation

Angelica Malaspina, University of Basilicata, Italy

Abstract. In this talk we consider the Dirichlet problem for the n-dimensional Helmholtz equation. In particular we deal with the problem of representability of the solutions by means of simple layer potentials. The main result concerns the solvability of a boundary integral equation of the first kind. Such a result is obtained by using the theories of differential forms and reducible operators.

This talk is based on joint work with A. Cialdea and V. Leonessa.

Some Inequalities in the Spirit of Trudinger's inequality

Arka Mallick, EPFL, Switzerland

Abstract. In this talk, I would like to present some recent results regarding the behaviour of the functions which are uniformly bounded under the action of a certain class of non-convex non-local functionals. In the literature, this class of functionals happens to be a very good substitute of the first order Sobolev spaces. As a consequence various improvements of the classical Poincars inequality, Sobolevs inequality and Rellich–Kondrachovs compactness criterion were established. This talk will be focused on addressing the gap between a certain exponential integrability and the boundedness for functions which are finite under the action of these class of non-convex functionals.

Energy quantification for perturbed Moser–Trudinger functionals in dimension two

Gabriele Mancini, Sapienza University of Rome, Italy

Abstract. I will discuss some recent results obtained in collaboration with L. Martinazzi and P-D. Thizy concerning semi-linear elliptic problems involving Moser–Trudinger type nonlinearities. After summarizing some known facts regarding the standard Moser–Trudinger critical equation, I will describe how the classical results are affected by the presence of perturbation terms. In particular, I will present the answer to a question formulated by O. Druet, proving the existence of families of nonlinearities with uniformly critical growth for which the corresponding semi-linear problem admits a bubbling sequence of solutions having a nontrivial weak limit.

Variational methods and bifurcation results for nonlinear Helmholtz systems

Rainer Mandel, Karlsruhe Institute of Technology, Germany

Abstract. In this talk I will present existence results for localized vector solutions of the cubic nonlinear Helmholtz system

$$-\Delta u - \mu u = u^3 + buv^2 \qquad \text{in } \mathbb{R}^N,$$
$$-\Delta v - \nu u = v^3 + bvu^2 \qquad \text{in } \mathbb{R}^N$$

for given $\mu, \nu > 0$ and a coupling parameter $b \in \mathbb{R}$. Our results are obtained using a dual variational approach and bifurcation theory.

The talk is based on joint work with D.Scheider [1,2]. It is supported by the German Research Foundation (DFG) through CRC 1173 "Wave phenomena: analysis and numerics".

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Li & Yau estimates for some semilinear heat equations and applications

Carlo Mantegazza, University of Naples Federico II, Italy

Abstract. We will show some Li & Yau–type gradient estimates for positive solutions of the semilinear heat equation $u_t = \Delta u + u^p$ with $p > 1$, on a complete n–dimensional Riemannian manifold (M, g) with nonnegative Ricci tensor. We then discuss some applications ?to ancient and eternal solutions.

This is a joint work with Daniele Castorina.

o-O type structure for rearrangement-invariant Banach spaces

Gianluigi Manzo, University of Naples Federico II, Italy

Abstract. A way to construct an o-O type structure for many rearrangement-invariant Banach spaces will be presented, along with some consequences, with particular attention on the case of Orlicz spaces.

Nonlinear Korn inequalities and recovery of immersions from metric tensors in $W^{1,p}$

Sorin Mardare, University of Rouen, France

Abstract. A nonlinear Korn inequality in $\mathbb{R}^n$ states that the distance between two smooth enough immersions defined over an open subset Ω of $\mathbb{R}^n$ can be controlled by the corresponding distance between their metric tensors.

Under the assumption that Ω satisfies the uniform interior cone property, we establish such a nonlinear Korn inequality for immersions in the space $W^{2,p}(\Omega\,;\mathbb{R}^n)$, $p > n$. The main ingredient used in the proof of this Korn inequality is a result on the continuity with respect to the data, of the solution of a Cauchy problem associated to a Pfaff system with L^p coefficients.

If we consider the mapping resulting from the recovery of a class of immersions in $W^{2,p}(\Omega\,;\mathbb{R}^n)/\mathcal{R}$ ($\mathcal{R}$ being the relation of isometric equivalence in $\mathbb{R}^n$) from its metric tensor of class $W^{1,p}$, then a consequence of the nonlinear Korn inequality is that this mapping is locally Lipschitz-continuous.

This talk is based on a joint work with Philippe G. Ciarlet.

Uncertainty inequalities in metric measure spaces

Joaquim Martín, Autonomous University of Barcelona, Spain

Abstract. We extend the recent L^1 uncertainty inequalities obtained by G. M. Dall'ara and D. Trevisan to the metric setting. For this purpose we introduce a new class of weights, named *isoperimetric weights*, for which the growth of the measure of their level sets $\mu(\{w \leq r\})$ can be controlled by $rI(r)$, where I is the isoperimetric profile of the ambient metric space. We use isoperimetric weights, new *localized Poincaré inequalities*, and interpolation, to prove $L^p, 1 \leq p < \infty$, uncertainty inequalities on metric measure spaces. We give an alternate characterization of the class of isoperimetric weights in terms of Marcinkiewicz spaces, which combined with sharp Sobolev inequalities, and interpolation of weighted norm inequalities, give new uncertainty inequalities in the context of rearrangement invariant spaces.

Stabilization of one-dimensional wave equation with nonmonotone damping

Swann Marx, LAAS-CNRS, France

Abstract. This talk will be about the stabilization of one-dimensional wave equation with nonmonotone damping. Such a situation appears when one is faced with modelization errors on the nonlinearity in the feedback. Due to this lack of monotony, applying the so-called LaSalle's Invariance Principle is hard and we cannot moreover have good regularity results in order to conclude on the result. Our strategy to tackle this problem consists in considering the wave equation in L^p-type spaces and in obtaining some nice regularity results on the solution in order to bound the solution essentially. Thanks to this, one is able first to prove the well-posedness of the equation and, moreover, to give some decay rate estimates of the trajectory. These decay rates are obtained thanks to a Lyapunov functional, that is provided thanks to a link between time-variant system and system with nonlinear damping.

This work is a joint work with Yacine Chitour and Christophe Prieur.

Nonuniformly elliptic energy integrals with p, q-growth

Elvira Mascolo, University of Florence, Italy

Abstract. We present some recent results, obtained in collaboration with Giovanni Cupini and Paolo Marcellini, on the regularity of weak solutions to quasilinear systems and/or local minimizers to vector-valued integral functionals with $p-q$ growth conditions. In particular non-uniformly energy integral are considered where the integrand function $f = f(x, z)$ satisfies

$$\lambda(x)|z|^p \le f(x, z) \le \mu(x)(1 + |z|^q)$$

for some exponents $q \ge p \ge 1$ and with nonnegative functions λ, μ, μ satisfying some summability conditions.

Generalizations of some canonical Riemannian metrics

Paolo Mastrolia, University of Milan, Italy

Abstract. In this talk, which is the first part of a joint seminar with G. Catino (Politecnico di Milano), I will introduce some generalization of certain canonical Riemannian metrics, presenting two possible approaches (curvature conditions with potential and critical metrics of Riemannian functionals). The main result is related to the existence of a new canonical metric, which generalizes the condition of harmonic Weyl curvature, on every 4-dimensional closed manifold.

Explicit integral representations of the relaxation of non-local energies for structured deformations

José Matias, Instituto Superior Técnico, Portugal

Abstract. The theory of structured deformation in the SBV setting developed by [1] only takes into account the linear dependance on jumps along the approximating sequences. In [2] a model was proposed toward capturing the non-linear dependence on the jumps. The idea was to modify the initial energy as follows: for each $r \in (0,1)$ let

$$F^r(u) := \int_0^1 W(\nabla u(x))\,\mathrm{d}x + \sum_{z\in S_u} \psi([u](z)) + \int_0^1 \Psi\Big(\sum_{z\in S_u\cap(x-r,x+r)} \frac{[u](z)}{2r} \Big)\mathrm{d}x, \quad (1)$$

and then undergo a relaxation process in the context of structured deformations followed by taking the limit as $r \to 0^+$. In this work we extend this model for any dimensions and different types of non-local energies Ψ. Moreover, we consider more general types of averaging of the jumps.

This is a joint work with Marco Morandotti, David R. Owen, Elvira Zappale.

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Anisotropic degenerate parabolic problems in $\mathbb{R}^N$ with variable exponent and locally integrable data

Rabah Mecheter, University of M'sila, Algeria

Abstract. In this paper, we prove the existence and regularity of weak solutions for a class of nonlinear anisotropic parabolic equations in the whole $(0,T)\times\mathbb{R}^N$ with $p_i(x)$ growth conditions and locally integrable data. The functional setting involves Lebesgue–Sobolev spaces with variable exponents. Our results are generalizations of the corresponding results in the constant exponent case and some results given in Bendahmane et al.

Asymptotic analysis of the spectral Robin problem in a thick junction with the branched fractal structure

Taras Mel'nyk, National University of Kyiv, Ukraine

Abstract. The asymptotic behavior of the eigenvalues and eigenfunctions of a mixed boundary-value problem for the Laplace equation in a thick junction with the branched fractal structure and the perturbed Robin boundary conditions on the boundaries of the branches is studied. The Hausdorff convergence of the spectrum is proved, the leading terms of asymptotics are constructed and the corresponding asymptotic estimates are justified both for the eigenvalues and eigenfunctions.

Controllability of the wave equation with memory term

Sorin Micu, University of Craiova, Romania

Abstract. We study the internal null-controllability of a wave equation with memory in the principal part, defined on the one-dimensional torus. We assume that the control is acting on an open subset which is moving with a constant velocity. The main result of the paper shows that the equation is null controllable in a sufficiently large time and for initial data belonging to suitable Sobolev spaces. Its proof follows from a careful analysis of the spectrum associated with our problem and from the application of the classical moment method.

Constrained optimal rearrangement problem leading to a free boundary problem with non-local obstacle

Hayk Mikayelyan, University of Nottingham Ningbo China, China

Abstract. We consider a new type of obstacle problem in the cylindrical domain, where the obstacle is imposed on the integral of the function with respect to the axis direction. We prove existence and regularity results and show that the comparison principle does not hold for the minimizers. This problem is derived from a classical optimal rearrangement problem in a cylindrical domain, under the constraint that the force function does not depend on the variable of the cylindrical axis. This leads to a new type of obstacle problem with non-local obstacle. We will also discuss some recent results on fractional rearrangement problems.

Boundary-domain integral equations for Stokes and Brinkman PDE systems with variable coefficients in L_p-based spaces

Sergey Mikhailov, Brunel University London, United Kingdom

Abstract. We consider Boundary-Domain Integral Equations (BDIEs) associated with some boundary value problems for the stationary Stokes system in L_p-based Sobolev spaces in a bounded Lipschitz domain in $\mathbb{R}^3$ with the variable viscosity coefficient from a space of Sobolev multipliers. First, we introduce a parametrix and construct the corresponding parametrix-based variable-coefficient Stokes Newtonian and layer integral potential operators with densities and the viscosity coefficient in L_p-based Bessel potential or Besov spaces. Then we generalize various properties of these potentials, known for the Stokes system with constant coefficients, to the case of the Stokes system with variable coefficients. Next, we show that the considered boundary value problems for the Stokes system with variable coefficients are equivalent to a BDIE system. Then we analyse the Fredholm properties of the BDIE systems in L_p-based Sobolev and Besov spaces and finally prove their invertibility in corresponding spaces. Finally, we apply these result to analyse the existence of a solution of the nonlinear Stokes and Brinkman systems.

This is a joint work with Mirela Kohr and Massimo Lanza de Cristoforis.

Multiple sequences of solutions for critical polyharmonic equations

Giovanni Molica Bisci, University of Urbino Carlo Bo, Italy

Abstract. In this talk we study the following problem

$$\begin{cases} (-\Delta)^m u = |u|^{\frac{4m}{d-2m}} u \ \text{ in } \ \mathbb{R}^d \\ u \in \mathcal{D}^{m,2}(\mathbb{R}^d), \end{cases}$$

involving the polyharmonic operator $(-\Delta)^m$, with $m \in \mathbb{N}$ and $d \geq m$. By using the Palais principle of symmetric criticality and by exploiting some algebraic–theoretical arguments developed in [1, 2], we prove the existence of a finite number ζ_d sequences of infinitely many finite energy nodal solutions which are unbounded in the classical higher order Sobolev space $\mathcal{D}^{m,2}(\mathbb{R}^d)$. Taking into account the recent results contained in [3], a concrete expression of ζ_d is given in terms of the number of unrestricted partition of the Euclidean dimension expressed by the celebrated Rademacher formula. Furthermore, the asymptotic behavior of the number ζ_d obtained here is a direct consequence of the classical Hardy–Ramanujan analyis based on the circle method. The main multiplicity result represents a more precise form of [1, Theorem 1.1] for polyharmonic problems settled in higher dimensional Euclidean spaces. In conclusion, an explicit numerical comparison with [3, Theorem 4.8] is given. The above results are obtained in a recent paper in collaboration with P. Pucci [4].

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Analysis of the Kobayashi–Warren–Carter model of grain boundary motion

Salvador Moll, University of Valencia, Spain

Abstract. In this talk, a phase field system of grain boundary, known as "Kobayashi-Warren-Carter model", is considered. The model is described as two parabolic PDEs, which are derived as gradient flows of a free-energy of two unknown variables: the orientation order and the orientation angle of grain. The free-energy includes a term of unknown-dependent total variation motivated by the reproduction of the facet structure in a polycrystal.

The main objective of the talk is to report some recent results concerning the Dirichlet problem. In particular, I will present existence of energy dissipating solutions and large time behavior of the solutions, including a complete characterization of the ω-limit set in the one-dimensional case and some examples in dimension 2.

This is a joint work with Prof. K. Shirakawa (U. Chiba, Japan) and H. Watanabe (U. Oita, Japan)

Nonexistence results for elliptic problems in contractible domains

Riccardo Molle, Tor Vergata University of Rome, Italy

Abstract. In this talk I will consider nonlinear elliptic equations involving the Laplace or the p-Laplace operator and nonlinearities with supercritical growth, from the viewpoint of the Sobolev embedding. I will present some new nonexistence results in contractible and non starshaped domains. The domains that are considered can be arbitrarily close to non contractible domains and their geometry can be very complex.

The results presented are developed in collaboration with Donato Passaseo.

A limiting free boundary problem for a degenerate operator in Orlicz–Sobolev spaces

Sergio H. Monari Soares, University of São Paulo, Brazil

Abstract. A free boundary optimization problem involving the Φ-Laplacian in Orlicz–Sobolev spaces is considered for the case where Φ does not satisfy the natural conditions introduced by Lieberman. A minimizer u_Φ having non-degeneracy at the free boundary is proved to exist and some important consequences are established, namely, the Lipschitz regularity of u_Φ along the free boundary, the locally uniform positive density of positivity set of u_Φ and that the free boundary is porous with porosity $\delta \geq 0$ and has finite $(N-\delta)$-Hausdorff measure. The method is based on a truncated minimization problem in terms of the Taylor polynomial of Φ of order $2k$. The proof demands to revisit the Lieberman's proof of a Harnack inequality and verify that the associated constant with this inequality is independent of k provided that k is sufficiently large.

This is joint work with Jefferson Abrantes Santos.

Dirichlet problem for non-coercive elliptic equations in unbounded domains

Sara Monsurrò, University of Salerno, Italy

Abstract. We consider the Dirichlet problem for noncoercive linear and nonlinear elliptic equations with discontinuous coefficients on unbounded domains. Exploiting a nonlinear approach, we achieve some existence, uniqueness and regularity results.

This is a joint work with Emilia Anna Alfano and Maria Transirico.

Reaction-diffusion problems on time-dependent Riemannian manifolds: stability of periodic solutions

Dario Monticelli, Polytechnic University of Milan, Italy

Abstract. In this talk we will briefly discuss some recent results concerning the stability of time-periodic solutions of semilinear parabolic problems posed on a bounded domain of a Riemannian manifold, with homogeneous Neumann boundary conditions when the boundary of the domain is not empty. On the domain we consider metrics that vary periodically in time. The discussion is based on the principal eigenvalue of periodic parabolic operators. The study is related to biological models on the effect of growth and curvature on pattern formation. Metric properties, for instance, the Ricci curvature, play a crucial role.

These results are joint work with C. Bandle (University of Basel) and F. Punzo (Politecnico di Milano).

Symmetry properties of singular solutions to some quasilinear problems involving a first order-term

Luigi Montoro, University of Calabria, Italy

Abstract. We consider singular solutions to quasilinear elliptic equations involving a first-order term under zero Dirichlet boundary condition. We deduce symmetry and monotonicity properties of positive solutions via an improved moving plane procedure.

Global weak solutions and asymptotic behavior of a chemotaxis system with singular chemotactic sensitivity and a non-diffusible chemical

Cristian Morales-Rodrigo, University of Seville, Spain

Abstract. We study nonnegative solutions for the chemotaxis system

$$\begin{cases} u_t = \Delta u - \nabla \cdot \left(\frac{u}{v}\nabla v\right) & \text{in } \Omega \times (0,T), \\ v_t = -uv & \text{in } \partial\Omega \times (0,T), \end{cases} \tag{2}$$

in a smooth domain $\Omega \subset \mathbb{R}^n$ for $n \leq 3$ under Neumann boundary conditions. We show that for nonnegative initial data (u_0, v_0) there are global weak solutions to (2) independently of the size of the initial data. Moreover we prove that if (u, v) is a weak solution to (2) then converges in weak-$L^1(\Omega) \times L^p(\Omega)$ $(1 \leq p < +\infty)$ to the homogeneous state $(\overline{u}, 0)$, where $\overline{u}$ is the mean value of u_0.

Key words: chemotaxis, global existence, stability
AMS Classification: 35B40, 35B45, 35K55, 35K57, 92C17

Analysis of a perturbed Cahn–Hilliard model for Langmuir–Blodgett films

Marco Morandotti, Polytechnic University of Turin, Italy

Abstract. An advective Cahn–Hilliard model motivated by thin film formation is studied in this paper. The one-dimensional evolution equation under consideration includes a transport term, whose presence prevents from identifying a gradient flow structure. Existence and uniqueness of solutions, together with continuous dependence on the initial data and an energy equality are proved by combining a minimizing movement scheme with a fixed point argument. Finally, it is shown that, when the contribution of the transport term is small, the equation possesses a global attractor and converges, as the transport term tends to zero, to a purely diffusive Cahn–Hilliard equation.

This is a joint work with M. Bonacini and E. Davoli.

Blocking of propagation in a bistable reaction-diffusion equation in a star graph

Yoshihisa Morita, Ryukoku University, Japan

Abstract. We are concerned with a bistable reaction-diffusion in a domain of multiple half lines with a junction, called a star graph. We give a condition on the blocking of front propagations around the junction by proving the existence and stability of stationary solutions.

This talk is based on the recent joint work with S. Jimbo (Hokkaido University).

Boundary stabilization of parabolic type equations via fixed point theorem

Ionut Munteanu, Al. I. Cuza University, Romania

Abstract. We shall design a boundary feedback stabilizer to unbounded trajectories for semi-linear heat equation. The feedback is linear, given in a simple explicit form and involves only the eigenfunctions of the Laplace operator. The control is supported in a given open subset of the boundary of the domain. The simple form of the feedback allows to write the solution, of the equation, in a mild formulation via a kernel, then appealing to a fixed point argument the existence & stabilization result is proved.

Moser type inequalities in Gauss space

Vít Musil, University of Firenze, Italy

Abstract. We deal with a family of limiting exponential type Sobolev inequalities in Gauss space, namely the space $\mathbb{R}^n$ endowed with the Gauss probability measure γ_n given by

$$\mathrm{d}\gamma_n(x) = (2\pi)^{-\frac{n}{2}} e^{-\frac{|x|^2}{2}}\,\mathrm{d}x \quad \text{for } x \in \mathbb{R}^n.$$

The inequalities to be considered admit diverse variants. All of them concern the validity, for $\beta > 0$, of the uniform bound

$$\int_{\mathbb{R}^n} e^{(\kappa|u|)^{\frac{2\beta}{2+\beta}}}\,\mathrm{d}\gamma_n \le C \tag{1}$$

for some positive constants κ and C, and for every weakly differentiable function u in $\mathbb{R}^n$ subject to a constraint on a form of exponential integrability for $|\nabla u|^\beta$, and to the normalization that the mean value or the median of u equals to zero. The most straightforward version of the relevant constraint reads

$$\int_{\mathbb{R}^n} e^{|\nabla u|^\beta}\,\mathrm{d}\gamma_n \le M \tag{2}$$

for some constant $M > 1$. It is known that for every $\beta > 0$, the exponent $\frac{2\beta}{2+\beta}$ is the largest possible that makes the inequality (1) true for suitable κ and C.

Our main focus will be on a sharp form of inequality (1). Specifically, we investigate the optimal (i.e. largest possible) constant κ for which inequality (1) holds under the normalization condition (2) or some alternate closely related assumption. We will also discuss the main and surprising dissimilarities between this type of inequalities in Gauss and Euclidean space.

This talk is based on joint work with A. Cianchi and L. Pick.

A weighted estimate for generalized harmonic extensions

Roberta Musina, The University of Udine, Italy

Abstract. Motivated by a celebrated paper by Caffarelli and Silvestre [CPDE, 2007], we will discuss some recent results about Hardy and trace-Hardy inequalities for generalized harmonic functions on half-spaces, with applications to symmetry breaking phenomena in some nonlocal problems.

This is a research in collaboration with Alexander I. Nazarov, St.Petersburg Department of Steklov Institute and St.Petersburg State University.

A real analyticity result for the eigenvalues of the Dirichlet Laplacian in a perturbed periodic domain

Paolo Musolino, University of Padova, Italy

Abstract. In this talk we investigate a spectral Laplace-Dirichlet problem in a domain Π_ϕ, which is the plane $\mathbb{R}^2$ doubly periodically perforated by a lattice of sets which are translations by points with integer coordinates of the image of a reference domain through a diffeomorphism ϕ. By the Floquet-Bloch theory for periodic problems, one can transform the original problem into a family of eigenvalue problems depending on a parameter $\eta \in [0, 2\pi]^2$ for quasi-periodic functions in the unit cell. We prove a result of real analytic dependence for symmetric functions of the eigenvalues of the cell problem upon variation of the diffeomorphism ϕ. Then we indicate some applications of such a result to the spectrum of the original problem.

This talk is based on joint work with M. Lanza de Cristoforis and J. Taskinen.

Weak solutions for (p, q)-equations on Riemannian manifolds

Antonella Nastasi, University of Palermo, Italy

Abstract. In this talk we consider a class of quasilinear elliptic problems, driven by (p, q)-Laplace operator, on a non compact Riemannian manifold M. In particular, if M is an open subset of the Euclidian space $\mathbb{R}^n$, we obtain a homogeneous Neumann boundary value problem as a special case. We exhibit conditions on the nonlinearity in order to prove the existence of a nontrivial weak solution via variational methods.

Finite volume methods for first order elliptic systems

Michael Ndjinga, University of Paris-Saclay, France

Abstract. We review the L^2 theory for the existence and uniqueness to boundary value problem for first order elliptic systems. We then study finite volume approximations and give proof of convergence. We also give an estimate of the condition number of the associated linear system.

Normalized solutions for Nonlinear Schrödinger equations and systems on bounded domains

Benedetta Noris, University of Picardy Jules Verne, France

Abstract. In this talk we discuss the existence and orbital stability of L^2-normalized solutions of the Nonlinear Schrödinger equation on bounded domains, with homogeneous Dirichlet boundary conditions. We also consider nonlinear Schrödinger systems of Gross–Pitaevskii type. We provide sufficient conditions for the existence of orbitally stable standing waves. Such waves correspond to (global or local) minimizers of the associated energy.

These are joint works with H. Tavares (Lisbon) and G. Verzini (Milan).

Parabolic variational constrained inequalities on star-shaped sets

Anna Ochal, Jagiellonian University, Poland

Abstract. We present the existence of a solution to a generalized evolution variational inequality relative to a closed region, which is star-shaped relative to a ball. This will yield a solution to a standard variational inequality, if the set is also convex. This result is worthwhile and it is done in a quite general setting involving an L-pseudomonotone operator. The proof of the existence result is based on the hemivariational inequality approach, a surjectivity theorem for multivalued pseudomonotone operators in reflexive Banach spaces, and a penalization method.

This presentation is a joint work with Leszek Gasinski, Stanislaw Migorski and Zijia Peng.

Eigenvalues of the Laplacian with moving mixed boundary conditions: sharp asymptotics in terms of boundary capacity

Roberto Ognibene, University of Milano-Bicocca, Italy

Abstract. In this talk I will consider the eigenvalue problem for the Laplace operator with Neumann boundary conditions and a perturbation of it, which consists in imposing Dirichlet boundary conditions in a small subset of the boundary. In this framework, I will state the sharp aymptotic behaviour of a perturbed eigenvalue in the case in which it is converging to a simple eigenvalue of the Neumann problem. The asymtoptics turn out to rely on the capacity of the subset where the perturbed eigenfunction is vanishing. Finally I will focus on the case of Dirichlet boundary conditions imposed on a subset which is scaling to a point.

Existence results for 1-Laplace Dirichlet problems with singular nonlinearities

Francescantonio Oliva, INdAM, Italy

Abstract. We deal with a problem involving the 1-laplacian operator and a possibly singular first order term as

$$\begin{cases} -\operatorname{div}\left(\dfrac{Du}{|Du|}\right) = g(u)|Du| + h(u)f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P}$$

where Ω is a open and bounded subset of $\mathbb{R}^N$ with regular boundary, $f \geq 0$ belongs to $L^N(\Omega)$, g, h are continuous nonnegative functions which may blow up at the origin. We discuss existence (when expected) of a bounded BV-solution to (P) under suitable assumptions on the data.

Minimal control time for one-dimensional first-order hyperbolic systems

Guillaume Olive, Jagiellonian University, Poland

Abstract. The goal of this talk is to present some recent results concerning the exact controllability of one-dimensional first-order linear hyperbolic systems when all the controls are acting on the same side of the boundary. We show that the minimal time needed to control the system is given by an explicit and easy-to-compute formula with respect to all the coupling parameters of the system. The proof relies on the introduction of a canonical UL-decomposition and the compactness-uniqueness method.

This is based on a joint work with Long Hu.

Homogenization and Dimension Reduction in Textiles

Julia Orlik, Fraunhofer ITWM, Germany

Abstract. In this work, we investigate periodic structures made of fibers or yarns, like textiles and derive their macroscopic properties via simultaneous homogenization and dimension reduction. As reference do- main we consider a canvas structure, which we assume to consist of periodically oscillating and isotropic beams with periodicity ε and radius r. Furthermore, the beams are in contact and thereby the elasticity problem is restricted on a cone fulfilling non-penetration and gap conditions. To obtain different compact- ness results for all components of the displacement, we apply the decomposition of displacements for beams, [1], yielding an elementary and a warping displacement. The derived estimates depend on the small parameters, the elastic energy, [2], and the contact. Moreover, we introduce an adapted unfolding operator, [3] with an incorporated dimension reduction from three to two dimensions. The properties of the unfolding operator together with the compactness results leads to its weak convergence, equivalent to the two-scale convergence. Consequently, the unfolded limits of the displacements, the strain tensor and contact condition yield homogenized 2D-model for a textile.

This is a joint work with G. Griso and S. Wackerle

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On Solvability of Fractional Models of Dynamics of Viscoelastic Continua

Vladimir Orlov, Voronezh State University, Russia

Abstract. The mathematical models of dynamics of viscoelastic continua with consti- tutive relations containing fractional derivatives are under consideration. We establish the existence of weak solutions of the corresponding initial-boundary value problems. In the planar case the uniqueness of weak solutions is proved. For the proofs of the main results we approximate the problems under consid- eration by a sequence of regularized systems of Navier-Stokes type. We use theory of fractional powers of positive operators, fractional calculus and clas- sical results on Navier-Stokes equations. In the planar case we investigate the strong solvability of some fractional models.

This is a joint work with Victor Zvyagin.

Hölder regularity for nonlocal double phase equations

Giampiero Palatucci, University of Parma, Italy

Abstract. We present some regularity estimates for viscosity solutions to a class of possible degenerate and singular integro-differential equations whose leading operator switches between two different types of fractional elliptic phases, according to the zero set of a modulating coefficient $a = a(\cdot,\cdot)$. The model case is driven by the following nonlocal double phase operator,

$$\int \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{n+sp}}\,\mathrm{d}y + \int a(x,y)\frac{|u(x)-u(y)|^{q-2}(u(x)-u(y))}{|x-y|^{n+tq}}\,\mathrm{d}y,$$

where $q \geq p$ and $a(\cdot,\cdot) \geqq 0$. Our results do also apply for inhomogeneous equations, for very general classes of measurable kernels. By simply assuming the boundedness of the modulating coefficient, we are able to prove that the solutions are Hölder continuous, whereas similar sharp results for the classical local case do require a to be Hölder continuous. To our knowledge, this is the first (regularity) result for nonlocal double phase problems.

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Work in collaboration with C. De Filippis. Available at
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Global existence of a two-dimensional chemotaxis-haptotaxis model with remodeling of non-diffusible attractant

Peter Pang, National University of Singapore, Singapore

Abstract. In this talk, we will consider a cancer invasion model involving chemotaxis and haptotaxis in two spatial dimensions. This model involves two parabolic PDEs and an ODE. The novelty of this study lies in: first, our treatment of the full parabolic model (whereas previous studies simplify one PDE to its elliptic version), and second, allowing for self-remodeling of the extracellular matrix (whereas previous studies normally assume its absence). The presence of the ODE presents particular mathematical challenges with regard to regularity. Under appropriate regularity assumptions on the initial data, by using adapted Lp estimate techniques, we are able to establish global existence and uniqueness of classical solutions in the high cell proliferation regime.

Simple Motion Evasion Differential Game of Many Pursuers and Evaders with Integral Constraints

Bruno Antonio Pansera, University Mediterranea of Reggio Calabria, Italy

Abstract. We study a simple motion evasion differential game of many pursuers and evaders. Control functions of players are subjected to integral constraints. If the state of at least one evader does not coincide with that of any pursuer forever, then evasion is said to be possible in the game. The aim of the group of evaders is to construct their strategies so that evasion can be possible in the game and the aim of the group of pursuers is opposite. The problem is to find a sufficient condition of evasion. If the total energy of pursuers is less than or equal to that of evaders, then it is proved that evasion is possible, and moreover, evasion strategies are constructed explicitly.

Global regularity for degenerate/singular parabolic equations involving measure data

Jung-Tae Park, Korea Institute for Advanced Study, South Korea

Abstract. In this talk, we consider degenerate and singular parabolic equations with p-Laplacian structure in bounded non-smooth domains when the right-hand side is a signed Radon measure with finite total mass. We introduce a suitable solution and the (intrinsic) fractional maximal function of a given measure and provide minimal conditions. We develop a new tool that allows global regularity estimates for the spatial gradient of solutions to such parabolic measure data problems.

This is joint work with Sun-Sig Byun and Pilsoo Shin.

Regularity results for minimizers of a class of degenerate convex functionals

Antonia Passarelli di Napoli, University of Naples Federico II, Italy

Abstract. I will present some regularity results for vectorial minimizers of integral functionals that are contained in [1] , [2], [3]. The functionals considered here are of the type

$$\int_\Omega f(x, Du(x))\,dx$$

with energy density $f(x,\xi)$ uniformly convex and with radial structure with respect to the gradient variable only at infinity. We assume that there exist $2 \le p \le q$ and $C \ge 0$ such that

$$\frac{1}{C}|\xi|^p \le f(x,\xi) \le C(1+|\xi|^q).$$

If the dependence of the energy density on the x variable is controlled with a function in a suitable Sobolev class, then we establish the higher differentiability and the higher integrability of the gradient of the minimizers.

Moreover, in case of standard growth conditions, i.e. if $p = q$, we are able to prove that the gradient of the minimizers belongs to L^r locally, for every $r \ge t; 1$.

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Exact solution of a Neumann boundary value problem for the stationary axisymmetric Einstein equations

Long Pei, Norwegian University of Science and Technology, Norway

Abstract. For a stationary and axisymmetric spacetime, the vacuum Einstein field equations reduce to a single nonlinear elliptic PDE in two dimensions called the Ernst equation. By solving this equation with a Dirichlet boundary condition imposed along the disk, Neugebauer and Meinel in the 1990s famously derived an explicit expression for the spacetime metric corresponding to the Bardeen–Wagoner uniformly rotating disk of dust. In this paper, we consider a similar boundary value problem for a rotating disk in which a Neumann boundary condition is imposed along the disk instead of a Dirichlet condition. Using the integrable structure of the Ernst equation, we are able to reduce the problem to a Riemann–Hilbert problem on a genus one Riemann surface. By solving this Riemann-Hilbert problem in terms of theta functions, we obtain an explicit expression for the Ernst potential. Finally, a Riemann surface degeneration argument leads to an expression for the associated spacetime metric.

Spectral optimization in weighted Neumann problems: Asymptotic spherical shapes

Benedetta Pellacci, University of Campania Luigi Vanvitelli, Italy

Abstract. We study the positive principal eigenvalue of a weighted problem associated with the Neumann Laplacian. This analysis is related to the investigation of the survival threshold in population dynamics. When trying to minimize such eigenvalue with respect to the weight, one is lead to consider a shape optimization problem, which is known to admit spherical optimal shapes only in very specific cases. We investigate whether spherical shapes can be recovered in general situations, in some singular perturbation limit.

This is a joint work with Dario Mazzoleni and Gianmaria Verzini.

Multiplicity of multi-bump type nodal solutions for a class of elliptic problems with exponential critical growth in $\mathbb{R}^2$.

Denilson Pereira, Federal University of Campina Grande, Brazil

Abstract. In this paper, we establish the existence and multiplicity of multi-bump nodal solutions for the following class of problems

$$-\Delta u + (\lambda V(x) + 1)u = f(u), \text{ in } \mathbb{R}^2,$$

where $\lambda \in (0, \infty)$, f is a continuous function with exponential critical growth and $V : \mathbb{R}^2 \to \mathbb{R}$ is a continuous function verifying some hypotheses.

This is a joint work with C. O. Alves.

Homogenization of linear elasticity with slip-displacement conditions

Malte A. Peter, University of Augsburg, Germany

Abstract. We consider the linearized elasticity problem with slip displacement conditions for a two-scale composite of two solids. Such interface jumps in displacement arise e.g. in contact problems. In order to upscale this problem, we assume that one material is connected whereas the other one is disconnected and periodically distributed over the whole domain. The methods of two-scale convergence and periodic unfolding are applied to determine the macroscopic limit problem rigorously.

This is joint work with Tanja Wolfer (Augsburg).

Obstacle problems in linearized elasticity

Paolo Piersanti, City University of Hong Kong, Hong Kong

Abstract. In my talk, I will present some recent results on obstacle problems for elliptic membrane shells. A specific obstacle problem for the three-dimensional model will be defined and a rigorous asymptotic analysis as the thickness approaches zero will be performed. We will finally compare the obtained result with what is obtained by letting the thickness approach zero in the corresponding Koiter's model subjected to an obstacle.

On existence and concentration of solutions to a class of quasilinear problems involving the 1-laplacian operator

Marcos Pimenta, São Paulo State University Brazil

Abstract. In this work we use variational methods to prove results on existence and concentration of solutions to a problem in $\mathbb{R}^N$ involving the $1-$Laplacian operator. A thorough analysis on the energy functional defined in the space of functions of bounded variation ($\mathbb{R}^N$) is necessary, where the lack of compactness is overcome by using the Concentration of Compactness Principle due to Lions.

Klein–Gordon–Maxwell systems under mixed boundary conditions

Lorenzo Pisani, University of Bari Aldo Moro, Italy

Abstract. In this talk I will present a few results concerning the system

$$\begin{cases} -\Delta u + m^2\, u = (\omega + q(x)\,\phi)^2 u & \text{in } \Omega, \\ \Delta\phi - \mu^2\phi = q(x)\,\big(\omega + q(x)\,\phi\big)\,u^2 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ \dfrac{\partial\phi}{\partial\nu} = \alpha & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^3$ is a bounded and smooth domain, $m, \omega, \mu \in \mathbb{R}$, $q \in L^6(\Omega) \setminus \{0\}$, $\alpha \in H^{1/2}(\partial\Omega)$.

The Neumann boundary conditions on the electrostatic potential ϕ play a major role; existence of solutions can be obtained by variational methods. The "Proca parameter" $\mu \neq 0$ drastically changes the functional setting, compared to the case $\mu = 0$.

This is joint work with Monica Lazzo.

Sharp estimates for the first Laplacian eigenvalue and the torsional rigidity on doubly connected open sets

Gianpaolo Piscitelli, University of Cassino and Southern Lazio, Italy

Abstract. We prove that the annulus is the shape that optimize the first eigenvalue and the torsional rigidity on double connected sets with fixed measure. In particular, if we fix the external perimeter, the annulus minimizes the first eigenvalue and maximizes the torsional rigidity with external Robin and internal Neumann boundary conditions. We use particular test functions, based on the value of the gradient on the level sets of the radial eigenfunctions on the annulus.

This is a joint work with G. Paoli and L. Trani.

Bubbling nodal solutions for a large perturbation of the Moser–Trudinger equation on planar domains

Angela Pistoia, Sapienza University of Rome, Italy

Abstract. I will discuss some results obtained in collaboration with Massimo Grossi, Gabriele Mancini and Daisuke Naimen concerning the existence of nodal solutions for the problem

$$-\Delta u = \lambda u e^{u^2+|u|^p} \text{ in } \Omega, u = 0 \text{ on } \partial\Omega,$$

where $\Omega \subseteq \mathbb{R}^2$ is a bounded smooth domain and $p \to 1^+$. If Ω is ball, it is known that the case $p = 1$ defines a critical threshold between the existence and the non-existence of radially symmetric sign-changing solutions with λ close to 0. In our work we construct a blowing-up family of nodal solutions to such problem as $p \to 1^+$, when Ω is an arbitrary domain and λ is small enough. To our knowledge this is the first construction of sign-changing solutions for a Moser–Trudinger type critical equation on a non-symmetric domain.

Asymptotic behavior of the $W^{1/q,q}$-norm of mollified BV functions and applications to singular perturbation problems

Arkady Poliakovsky, Ben Gurion University of the Negev, Israel

Abstract. Motivated by results of Figalli and Jerison and Hernãndez, we prove that:

$$\lim_{\epsilon\to 0^+} \frac{1}{|\ln\epsilon|}\|\eta_\epsilon * u\|^q_{W^{1/q,q}(\Omega)} = C_0 \int_{J_u} \left|u^+(x) - u^-(x)\right|^q d\mathcal{H}^{N-1}(x),$$

where $\Omega \subset \mathbb{R}^N$ is a regular domain, $u \in BV(\Omega) \cap L^\infty$, $q \geq 1$ and $\eta_\epsilon(z) = \epsilon^{-N}\eta(z/\epsilon)$ is a smooth mollifier. In addition, we apply the above formula to the study of certain singular perturbation problems.

Interior Schauder estimates for degenerate Kolmogorov operators

Sergio Polidoro, University of Modena and Reggio Emilia, Italy

Abstract. We give an elementary proof of interior Schauder estimates for a family of degenerate linear second order operators that includes as a prototype the following one

$$\mathcal{L}u := \sum_{i,j=1}^{n} a_{ij}(x,y,t)\partial^2_{x_i x_j} u(x,y,t) + \sum_{i=1}^{n} x_i \partial_{y_i} u(x,y,t) - \partial_t u(x,y,t).$$

Here (x,y,t) is a point of $\mathbb{R}^{2n+1}$, and $(a_{ij})_{i,j=1,\dots,n}$ is an uniformly positive symmetric matrix with Hölder continuous coefficients.

The proof relies on the method introduced by by Xu-Jia Wang for uniformly elliptic and parabolic operators, and restores known results due to A. Lunardi and to M. Manfredini. It also provides us with weaker a priori estimates if we assume that the coefficients a_{ij}'s of the ooperator $\mathcal{L}$ and the right-hand side f of the equation $\mathcal{L}u = f$ are Dini continuous.

This is a joint work with M. Eleuteri and B. Stroffolini.

Oscillating solutions for nonlinear equations involving the Pucci's extremal operators

Alessio Pomponio, Polytechnic University of Bari, Italy

Abstract. In this talk we deal with the following nonlinear equations

$$\mathcal{M}^{\pm}_{\lambda,\Lambda}(D^2u) + g(u) = 0 \qquad \text{in } \mathbb{R}^N, \tag{$\mathcal{P}^{\pm}$}$$

where $\mathcal{M}^{\pm}_{\lambda,\Lambda}$ are the Pucci's extremal operators, with $0 < \lambda \leq \Lambda$, for $N \geq 1$ and under the assumption $g'(0) > 0$.

We show the existence of oscillating solutions, namely with an unbounded sequence of zeros, for equations ($\mathcal{P}^{\pm}$). Moreover these solutions are periodic, if $N = 1$, while they are radial symmetric and decay to zero at infinity with their derivatives, if $N \geq 2$.

The results have been obtained in a joint work with Pietro d'Avenia (Politecnico di Bari, Italy).

Optimal integrability from Reverse Hölder's Inequalities in dimension one

Arturo Popoli, University of Naples Federico II, Italy

Abstract. We describe a general approach on the sharp integrability property of Reverse Hölder Inequalities in dimension one which unifies the self-improving properties and the mutual embeddings of *Muckenhoupt classes* and *Gehring classes*. It is shown that the optimal exponents of integrability as well as the best constants in the improved integral estimates can be obtained by mean of a unique continuous function.

Controllability of Hamilton–Jacobi equations

Alessio Porretta, University of Rome Tor Vergata, Italy

Abstract. In this talk I will discuss the question whether solutions of Hamilton–Jacobi equations can be controlled from the boundary in order to catch in finite time some stationary profile. I also address the question whether the corresponding vanishing viscosity approximation can be exactly controlled accordingly. Those questions are extensively investigated for conservation laws, and a comparison is possible in the one-dimensional case.

The talk is based on a work in progress jointly with E. Zuazua.

Qualitative properties of solutions to a class of nonlinear parabolic PDE's

Maria Michaela Porzio, Sapienza University of Rome, Italy

Abstract. We will show some qualitative properties of a class of nonlinear parabolic equations appearing in many physical applications being a nonlinear version of the heat equation. We will describe the regularity properties and the behavior in time of the solutions with special attention to the autonomous case when the influence of the solutions to suitable elliptic problems appears.

Level convexity for supremal functionals

Francesca Prinari, University at Ferrara, Italy

Abstract. The existence of a level convex supremand is crucial in the problems involving supremal functionals. In the gradient case, under a mild assumption on the sublevel sets of a supremal functional F, we show that the lower semicontinuous envelopes of F with respect to the weak* topology and the uniform convergence are level convex (i.e. they have convex sub-level sets).

On the homogenization of nonlinear plate

Erick Pruchnicki, University of Lille, France

Abstract. In this paper we propose a multiscale finite-strain shell theory for simulating the mechanical response of highly heterogeneous plate. To resolve this issue a higher-order stress-resultant plate formulation based on multiscale homogenization is considered. At the macroscopic scale level, we approximate the displacement field by a fourth-order Taylor-Young expansion in thickness. We take account of the microscale fluctuations by introducing a boundary value problem over the domain of a three-dimensional representative volume element (RVE). The geometrical form and the dimensions of the RVE are determined by the representative microstructure of the heterogeneity. By considering a specific form of Hill-Mandel condition, we deduce that the macroscopic stress resultants are the volume averages through RVE of microscopic stress. All microstructural constituents are modeled as hyperelastic material.

Keywords: nonlinear elasticity, high-order plate theory, homogenization, macro micro Hill-Mandel condition, plate.

Low-up and global existence for the porous medium equation with reaction on a class of Cartan–Hadamard manifolds

Fabio Punzo, Polytechnic University of Milan, Italy

Abstract. The talk is concerned with the porous medium equation with power-type reaction terms up on negatively curved Riemannian manifolds, and solutions corresponding to bounded, nonnegative and compactly supported data. If $p > m$, small data give rise to global-in-time solutions while solutions associated to large data blow up in infinite time. If $p < m$, large data blow up at worst in infinite time, and under the stronger restriction $p \in (1, (1+m)/2]$ all data give rise to solutions existing globally in time, whereas solutions corresponding to large data blow up in infinite time. The results are in several aspects significantly different from the Euclidean ones, as has to be expected since negative curvature is known to give rise to faster diffusion properties of the porous medium equation.

Partial Regularity of Minimizers of Asymptotically Convex Functionals with Sobolev Coefficients

Teresa Radice, University of Naples Federico II, Italy

Abstract. We consider minimizers of the functional

$$\int_\Omega f(x,u,Du)\ dx,$$

where $f : \Omega \subseteq \mathbb{R}^n \times \mathbb{R}^N \times \mathbb{R}^{N\times n} \to \mathbb{R}$ is only asymptotically convex. More precisely, we suppose that f is asymptotically related to the function $x \mapsto a(x)g\big(|Du|\big)$, where $a : \Omega \to \mathbb{R}$ is only required to belong to the Sobolev class $W^{1,q}_{\rm loc}(\Omega)$ for some $q > 1$. Thus, loosely speaking, the coefficient of the fastest growing term of f may be discontinuous and only weakly differentiable. We demonstrate that a minimizer u of this problem is Hölder continuous outside a set of zero Lebesgue measure.

This is a joint work with C. S. Goodrich and A. Passarelli di Napoli.

Inequalities and consequences of some results of Calculus of Variations

Maria Alessandra Ragusa, University of Catania, Italy

Abstract. We review the advances on the regularity problem and present recent results related to minimizers of quadratic and non quadratic growth functionals. We would like to point out that the integrand could be discontinuous because the vanishing mean oscillation hypothesis is assumed.

Homogenization of a class of singular elliptic problems in perforated domains

Federica Raimondi, University of Rouen Normandy, France

Abstract. In this talk we investigate the asymptotic behaviour of a class of quasilinear elliptic problems posed in a domain perforated by ε-periodic holes of ε-size. The quasilinear equation presents a nonlinear singular lower order term, which is the product of a continuous function ζ (singular in zero) and a function f whose summability depends on the growth of ζ near its singularity. We prescribe a nonlinear Robin condition on the boundary of the holes and a homogeneous Dirichlet condition on the exterior boundary.

The main tool for proving the homogenization result is a convergence result stating that the gradient of the solution u_ε behaves like that of the solution of a suitable linear problem associated with a weak cluster point of the sequence $\{u_\varepsilon\}$, as $\varepsilon \to 0$. This idea was originally introduced in the literature for the homogenization of nonlinear problems with quadratic growth with respect to the gradient. In our case, this allows us not only to pass to the limit in the quasilinear term, but also to study the singular term near its singularity, via an accurate a priori estimate. We also obtain a corrector result for our problem.

The difficulties due to the singular term and the boundary nonlinear term are treated by menas of the periodic unfolding method.

Based on joint work with P. Donato and S. Monsurrò.

A multiparameter semipositone fractional laplacian problem involving critical exponent

Dhanya Rajendran, Indian Institue of Technology Goa, India

Abstract. In this talk we shall discuss the existence of at least one positive solution for nonlocal semipositone problem of the type

$$(P^\mu_\lambda)\begin{cases} (-\Delta)^s u &= \lambda(u^q-1)+\mu u^r \text{ in } \Omega \\ u &> 0 \text{ in } \Omega \\ u &\equiv 0 \text{ on } \mathbb{R}^N \setminus \Omega. \end{cases}$$

when the positive parameters λ and μ belongs to certain range. Here $\Omega \subset \mathbb{R}^N$ is assumed to be a bounded open set with smooth boundary, $s \in (0,1), N > 2s$ and $0 < q < 1 < r \leq \frac{N+2s}{N-2s}$. The proof relies on the construction of a positive subsolution for (P^0_λ) for $\lambda > \lambda_0$. Now for each $\lambda > \lambda_0$, for all small $0 < \mu < \mu_\lambda$ we establish the existence of at least one positive solution of (P^μ_λ) using variational method. Also in the subcritical case, i.e., for $1 < r < \frac{N+2s}{N-2s}$, we show the existence of second positive solution via mountain pass argument.

Prestressed thin structures

Annie Raoult, Paris Descartes University, France

Abstract. We consider thin structures that aim at achieving a prescribed metric. We will see that the Riemannian curvature tensor plays an important role in deriving the limit energy.

Optimization of a supply chain via a genetic algorithm

Luigi Rarità, University of Salerno, Italy

Abstract. Supply systems represent important issues in industrial applications, as the principal aim is to control unwished phenomena, such as dead times, bottlenecks and so on. Suppliers, manufacturers, warehouses and stores are components of supply chains and networks, whose dynamics is here studied via differential equations. In particular, we focus on supply networks modelled by a continuous model ([1]), based on partial differential equations for densities of goods on arcs and ordinary differential equations for the dynamics of queues among arcs. A suitable cost functional ([2], [3]), defined in terms of processing velocities of suppliers, is minimized with the aim of decreasing queues and achieving an a priori prescribed outflow. The minimization is allowed by a genetic algorithm that foresees iterations in which velocity of suppliers change via mechanisms of selection, crossover and mutation. A case study is presented and the obtained results also prove that the adoption of Artificial Intelligence techniques allows simplifying the analysis of computational times for various space/time discretizations.

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Properties of solutions of integro-differential equations arising in viscoelasticity theory

Nadezhda Rautian, Lomonosov Moscow State University, Russia

Abstract. The main goal of our work is studying of the asymptotic behavior of the solutions of Gurtin–Pipkin type integro-differential equations on the base of spectral analysis of their symbols. The Gurtin–Pipkin equation arises in such domains of mechanics and physics as heat transfer theory, theory of viscoelastic media, and kinetic theory of gases. Since the Gurtin–Pipkin type integro-differential equations arise in numerous applications, it is reasonable and natural to study integro-differential equations with unbounded operator coefficients in a Hilbert that are the operator models of such type partial integro-differential equations and have the following type

$$\frac{d^2u(t)}{dt^2} + A^2u(t) - \int_0^t \mathcal{K}(t-s)A^2u(s)ds = f(t), \quad t \in \mathbb{R}_+,$$

where A is a self-adjoint positive operator acting in the separable Hilbert space H and having a compact inverse operator. Assume that kernel $\mathcal{K}(t)$ is scalar function that determined empirically and admits the representation

$$\mathcal{K}(t) = \int_0^\infty \frac{e^{-t\tau}}{\tau} d\mu(\tau),$$

where $d\mu$ is a positive measure corresponding to an increasing right-continuous distribution function μ. The integral is understood in the Stieltjes sense.

The correct solvability of initial-value problems for integrodifferential equations with unbounded operator coefficients in Hilbert spaces is determined. A spectral analysis of the operator functions, which are the symbols these integro-differential equations is provided. Strong solutions of these equations are represented as a sum of terms corresponding to the real and nonreal parts of the spectrum of the operator functions that are the symbols of these equations (see [1]). The resulting representations are new for the given class of integro-differential equations.

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Gradient flow formulations of discrete and continuous evolutionary models

Ana Margarida Ribeiro, New University of Lisbon, Portugal

Abstract. Biological evolution is a complex phenomena comprising several mechanisms, as natural selection, mutation or genetic-drift, among others. Having these mechanisms in mind, several models in evolutionary dynamics were developed by means of discrete time Markov chains, continuous time stochastic processes, and systems of ordinary differential equations. In this talk, I'll introduce gradient flow formulations to some classes of these three types of models relating also the three perspectives.

This is a joint work with F. Chalub, L. Monsaingeon, and M. Souza.

Hopf Lemma and regularity estimates for anisotropic elliptic equations

Giuseppe Riey, University of Calabria, Italy

Abstract. We present the results in [1]. For $n \geq 2$, let $\Omega \subset \mathbb{R}^n$ be a smooth bounded domain. We consider the functional $I(u) = \int_\Omega [B(H(\nabla u)) - F(u)]\, dx$, whose Euler-Lagrange equation is given by

$$-\operatorname{div}\left(B'(H(\nabla u))\nabla H(\nabla u)\right)) = f(u), \tag{1}$$

where H is a Finsler norm, f is a positive continuous function on $[0,\infty)$, locally Lipschitz continuous on $(0,\infty)$ and B satisfies

(i) $B \in C^{3,\beta}_{loc}((0,+\infty)) \cap C^1([0,+\infty))$, with $\beta \in (0,1)$
(ii) $B(0) = B'(0) = 0, \quad B(t), B'(t), B''(t) > 0 \ \forall t \in (0,+\infty)$
(iii) $\exists\, p > 1, k \in [0,1], \gamma > 0, \Gamma > 0$:
$\gamma(k+t)^{p-2}t \leq B'(t) \leq \Gamma(k+t)^{p-2}t\,, \quad \gamma(k+t)^{p-2} \leq B''(t) \leq \Gamma(k+t)^{p-2}\,.$

Taking $H(\xi) = |\xi|$ and $B(t) = \frac{t^p}{p}$, the operator at left-hand side of (1) becomes the usual p-Laplace operator. If $H(\xi) = |\xi|$ and $B(t) = \sqrt{1+t^2}$, $I(u)$ is the euclidean area functional.

We prove local regularity estimates for positive weak solutions of (1), namely a weighted integral hessian estimate as well as the integrability of the inverse of the gradient.

Adding a suitable hypothesis on f, we also prove a Hopf type Lemma and, thanks to this result, the local regularity estimates are then extended to the whole Ω.

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Error estimates for the convergence in unilateral contact problems for linearly elastic shells: The elliptic membrane case

Ángel Daniel Rodríguez Arós, University of A Coruña, Spain

Abstract. We consider a family of linearly elastic shells all sharing the same middle surface, in unilateral contact with a rigid foundation on the lower face. The shells are elliptic and their lateral face is clamped. Under these conditions, when the thickness tends to zero, the solution of the three-dimensional contact problem converges to the solution of a two-dimensional obstacle problem for an elastic membrane shell. We provide error estimates for the convergence.

Stability analysis for parabolic problems in general unbounded domains

Luca Rossi, EHESS, France

Abstract. We present different notions of stability in the framework of elliptic and parabolic problems in unbounded domains. Necessary and sufficient conditions for them to hold will be derived using some generalizations of the principal eigenvalue for an elliptic operator, inspired by a series of works in collaboration with H. Berestycki and F. Hamel. As an application, we will derive the validity of the "hair-trigger" effect for the Fisher-KPP equation under Neumann boundary condition.

Balanced Viscosity solution to a multi-rate system for damage and plasticity

Riccarda Rossi, University of Brescia, Italy

Abstract. Several mechanical systems are modeled by the static momentum balance for the displacement coupled with rate-independent flow rules for some internal variables. In this context, we consider a rate-independent model for damage and plasticity, and we regularize both the elastic equilibrium equation and the rate-independent flow rules for plasticity and damage by adding viscous dissipation terms modulated by coefficients that vanish to zero with different rates. We thus obtain a system where the displacement and the internal variables for plasticity and damage relax to elastic equilirium and to rate-independent evolution with different relaxation rates. We address its vanishing-viscosity analysis and prove convergence to a Balanced Viscosity solution to the origina lrate-independent system. We show that this concept provides an accurate description of the system behavior at jumps.

This is a joint work with V. Crismale and G. Lazzaroni.

Morse–Sard theorem and Lusin N-property: a new synthesis result for Sobolev spaces

Alba Roviello, University of Campania Luigi Vanvitelli

Abstract. For a vector-valued mapping v defined on $\mathbb{R}^n$, we study the following problem: if S is a subset of the m-critical set of v (i.e. rank $(\operatorname{grad} v)(x)$ is not maximum) and S is negligible respect to the Hausdorff measure, can we say the same for $v(S)$? The classes of functions C^k and $C^{k,\alpha}$ were studied by Baites and Moreira. We solve the problem for Sobolev spaces $W^{k,p}$ and fractional Sobolev space $W^{k+\alpha,p}$. The result allows to have some information on the dimension of the distorted set $v(S)$, and puts in relation the Morse–Sard theorem with the classical Lusin N-property, which are classical tools in differentiation theory and theory of partial differential equations.

"Gas Dynamics" mean field games: energy estimates

Olga Rozanova, Moscow State University, Russia

Abstract. We consider a class of the mean field game (MFG) equations, which have a structure of the equations of gas dynamics. They can be characterized in terms of mass, momentum and energy. We study the integral quantities related to the parabolic and hyperbolic version of the MFG equations and show that the balance of the energy in the parabolic version is closely connected with the formation of singularities of solutions to the hyperbolic version.

Modelling of bonded elastic structures by a variational method: theoretical analysis and computational algorithm

Evgeny Rudoy, Lavrentyev Institute of Hydrodynamics SB RAS & Novosibirsk State University, Russia

Abstract. We deal with an equilibrium problem of two bodies joined (glued) with each other along a part of their common interface. There exists a crack on the rest part of the interface. Surface loadings are applied to both bodies. We assume that the interface is "spring type interface", modeling a soft and thin material between bodies. We impose a non-penetration condition and Treska's friction on the common interface including both the adhesive layer and the crack. The non-penetration condition excludes mutual penetration of bodies. A formula for the derivative of the energy functional with respect to the crack length is obtained. It is shown that the derivative can be represented as a path-independent integral (J-integral). Moreover, we propose a non-overlapping domain decomposition method for the joined structure and study its convergence theoretically and numerically. Numerical examination shows the efficiency of the proposed method and importance of the non-penetration condition.

Ways to treat a diffusion problem with the fractional Caputo derivative

Piotr Rybka, University of Warsaw, Poland

Abstract. The problem

$$u_t = (D^\alpha u)_x + f$$

augmented with initial and boundary data appear in model of subsurface flows. Here,$D^\alpha u$ denotes the fractional Caputo derivative of order $\alpha \in (0,1)$.

We offer three approaches:

(1) from the point of view of semigroups;
(2) from the point of view of the theory of viscosity solutions;
(3) from the point of view of numerical simulations.

This is a joint work with T. Namba, K. Ryszewska, V. Voller.

Dynamical system theoretic approach to turnpike in optimal control theory

Noboru Sakamoto, Nanazan University / DeustoTech, Japan

Abstract. In this talk, I will present an alternative framework to analyze the turnpike property in optimal control problems. First, it is shown that the turnpike property naturally appears in general dynamical systems having hyperbolic equilibrium. It is then applied to optimal control problems. This framework is, compared with existing one, more geometric and gives a good intuition for the understanding of the turnpike.

Unique Solvability of Some Nonlinear Inequalities with Fractional Laplacian

Olga Salvieva, Moscow State Technological University "Stankin", Russia &
Evgeny Galakhov, Peoples' Friendship University of Russia

Abstract. Let $s = [s] + \{s\} \in \mathbb{R}_+$, where $[s]$ is the integer part of s, and $\{s\}$ the fractional one. We define the operator $(-\Delta)^s$ in the standard way by the formula

$$(-\Delta)^s u(x) \stackrel{\text{def}}{=} c_{n,s} \cdot (-\Delta)^{[s]} \left(\text{p.v.} \int\limits_{\mathbb{R}^n} \frac{u(y) - u(x)}{|x-y|^{n+2\{s\}}}\, dy \right), \tag{1}$$

where $c_{n,s}$ is an appropriate positive constant, for all functions such that the right-hand side of (1) makes sense at least in the distributional setting.

We consider the nonlinear elliptic inequality

$$(-\Delta)^s u \geq c|u|^q (1+|x|)^\alpha \quad (x \in \mathbb{R}^n), \tag{2}$$

where $q > 1$ and $\alpha \in \mathbb{R}$.

Using a modification of the test function method [1], we obtain the following theorem.

Theorem 1. *The trivial solution of* (2) *is unique in the class of nonnegative weak solutions for* α *and* q *such that*

$$\alpha > -2s \text{ and } 1 < q < \frac{n+\alpha}{n-2s}. \tag{3}$$

Similar results were obtained for some other types of nonlinear problems with fractional Laplacian, including systems of elliptic inequalities and respective parabolic ones. Some of these results were published in [2].

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Differential Game of Pursuit Evasion

Mehdi Salimi, University Mediterranea of Reggio Calabria, Italy

Abstract. Many works have been devoted to pursuit evasion games [1, 2, 3]. In this research we study a pursuit evasion differential game with finite number of pursuers and one evader in Hilbert space. The control functions of players satisfy the geometric constraints. Evasion for evader from pursuers is possible if there exists a strategy of the evader such that geometric point of evader and pursuers are different at any time for any admissible controls of the pursuers. We make an explicit strategy for the evader that ensures its evasion.

This is a joint work with Massimiliano Ferrara.

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Minimization problems associated with embeddings of the critical Sobolev spaces

Megumi Sano, Hiroshima University, Japan

Abstract. We consider minimization problems associated with best constants of functional inequalities in the critical Sobolev spaces. Especially, we study an open problem about the existence of the minimizer, which is mentioned by Horiuchi and Kumlin in 2012. Symmetry breaking property of the minimizer is also studied.

Multiplicity of positive solutions for an equation with degenerate nonlocal diffusion

João R. Santos Júnior, Federal University of Pará, Brazil

Abstract. A multiplicity result of positive solutions with ordered $L^p(\Omega)$-norms is provided, which relates the number of positive solutions of

$$\begin{cases} -a(\int_\Omega u^p dx)\Delta u = f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P}$$

with the number of zeroes of the continuous function a. Where Ω is a bounded domain and a, f are continuous real functions with a vanishing in many positive points.

On the damped wave equation with constraints and applications

Riccardo Scala, Sapienza University of Rome, Italy

Abstract. We will discuss a type of weak solution to the damped wave equation with unilateral constraint. This constraint arises in a nonlinear term, which will belong to the subdifferential of a suitable potential. Consequently we will see how to apply the same technique to analyze systems of viscoelastic bodies, and the so-called locking materials.

Fractional integrals associated to operators with gaussian kernel bounds on anisotropic Morrey spaces

Andrea Scapellato, University of Catania, Italy

Abstract. Aim of this talk is to show new results related to fractional integrals associated to operators with Gaussian kernel bounds in the framework of anisotropic Morrey spaces. Precisely, we obtain boundedness properties for the above operators and their commutators with functions having bounded mean oscillation in the context of Morrey spaces with mixed norm.

Existence of Variational Solutions for Doubly Nonlinear Equations of Porous Medium Type

Leah Schätzler, University of Erlangen-Nuremberg, Germany

Abstract. I will discuss the existence of variational solutions to doubly nonlinear parabolic equations of the type

$$\partial_t u^m - div(|\nabla u|^{p-2}\nabla u) = 0$$

with parameters $m \in (0, \infty)$ and $p \in (1, \infty)$. The existence result relies on the method of minimizing movements.

A semilinear curl-curl problem in $\mathbb{R}^3$

Jacopo Schino, Polish Academy of Sciences, Poland

Abstract. We look for nontrivial solutions to the semilinear problem

$$\nabla \times \nabla \times u = f(x,u) \text{ in } \mathbb{R}^3.$$

We prove the existence of a nontrivial least-energy solution and of infinitely many geometrically distinct solutions.

The nonlinearity is modelled by an N-function being subcritical at inifnity and supercritical at 0.

The main difficulty arises from the infinite dimension of the kernel of the operator $\nabla\times$.

On the Hopf Boundary Lemma for quasilinear problems involving singular nonlinearities

Berardino Sciunzi, The University of Calabria, Italy

Abstract. We consider positive solutions to quasilinear elliptic problems with singular nonlinearities. We provide a Hopf type boundary lemma via a suitable scaling argument that allows to deal with the lack of regularity of the solutions up to the boundary.

The semirelativistic Choquard equation with a local nonlinear term

Simone Secchi, University of Milano-Bicocca, Italy

Abstract. We propose an existence result for a semirelativistic Choquard equation with a local nonlinearity in $\mathbb{R}^N$. An external potential is decomposed as the sum of a periodic part and of a decaying part.

This is a joint work with B. Bieganowski.

The reflection principle in the control problem of the heat equation

Albrecht Seelmann, TU Dortmund, Germany

Abstract. We consider the control problem for the generalized heat equation for a Schrödinger operator on a domain with a reflection symmetry with respect to a hyperplane. We show that if this system is null-controllable, then so is the system on its respective parts. Moreover, we show that the corresponding control cost does not exceed the one on the whole domain.

This talk is based on joint work with M. Egidi.

The limit as $p \to 1$ of the radial spectrum of the p-Laplacian operator

Sergio Segura de León, University of Valencia, Spain

Abstract. In this talk the radial spectrum of the 1-Laplace operator under Dirichlet boundary conditions in a ball of $\mathbb{R}^N$ is analized. To this end, we first provide a direct proof of the existence for each $n \in \mathbb{N}$ of the limit $\lambda_{(1),n} := \lim_{p\to 1} \lambda_{(p),n}$ of the n–th Ljusternik–Schnirelman Dirichlet eigenvalue $\lambda_{(p),n}$ of $-\Delta_p$ in a bounded Lipschitz domain $\Omega \subset \mathbb{R}^N$. More fruitful, it is shown that $\lambda_{(1),n}$ defines an eigenvalue of the 1–Laplacian operator $-\Delta_1$, with a well–defined strong associated eigenfunction $u_n \in BV(\Omega)$.

In the main results of the paper, we prove that radial LS eigenvalues of $-\Delta_p$, with $p > 1$, jointly with its associated eigenfunctions converge, and their limits solve the limit problem for radial eigenpairs of $-\Delta_1$. Moreover, we prove the uniqueness for this limit problem. In this way, the radial LS eigenvalues of $-\Delta_1$ are fully described, together with a detailed account on the profiles of their associated eigenfunctions.

This is a joint work with José C. Sabina de Lis (University of La Laguna).

The wave equation in an interval with moving endpoints: Exact solution and boundary observability

Abdelmouhcene Sengouga, University of M'sila, Algeria

Abstract. We consider the wave equation in an interval with two linearly moving endpoints $(-\ell_1 t, \ell_2 t)$, with $0 \leq \ell_1, \ell_2 \leq 1$. We give the exact solution by a generalized Fourier series. Then, we show that the energy decays at the precise rate $1/t$. Moreover, denoting by L_0 the initial length of the interval, we establish the observability of the wave equation at one endpoint (resp. at the two endpoints) in a sharp time $2L_0/(1-\ell_1)(1-\ell_2)$, (resp. $2L_0/(1-max\{\ell_1,\ell_2\})$).

Keywords: 1-d wave equation, time-varying domains, boundary observability, generalized Fourier series.

Asymptotic models for multiphysic imperfect interfaces

Michele Serpilli, Marche Polytechnic University, Italy

Abstract. In the present work we describe the mechanical behavior of a composite constituted of two solids, perfectly bonded together by a thin adhesive layer in a general multiphysic and multifield framework, by means of an asymptotic analysis. The so-called multiphysic materials represent a class of materials in which different physical behaviours are coupled together, such as piezoelectricity. After defining a small parameter ε, which will tend to zero, associated with the thickness and the constitutive coefficients of the intermediate layer, we characterize three different limit models and their associated limit problems: the soft interface model, in which the constitutive coefficients depend linearly on ε; the hard interface model, in which the constitutive properties are independent of ε; the rigid interface model, in which they depend on $1/\varepsilon$. The asymptotic expansion method is reviewed by taking into account the effect of higher order terms and by defining a general multiphysic interface law which comprises the above aforementioned models. Finally, we prove that the solution of the original problems strongly converges towards the solution of the limit problems, as ε tends to zero.

Multiplicity of solutions for fractional problems: the effect of the domain topology

Raffaella Servadei, University of Urbino Carlo Bo, Italy

Abstract. In this talk we are concerned with the multiplicity of solutions for fractional Laplace problem in presence of critical term. Using the Lusternik–Schnirelman theory, we relate the number of nontrivial solutions of the problem under consideration with the topology of the domain, extending the validity of well-known results for the classical Laplace equation to the fractional nonlocal setting.

Regularity features for integrodifferential equations

Daniela Sforza, Sapienza University of Rome, Italy

Abstract. Our aim is to describe some regularity results for solutions of wave equations with integral terms of convolution type. Under rather general assumptions on the integral kernel we can define the trace of the normal derivative for weak solutions, having regularity properties. Consequently, we can extend to integrodifferential equations some results, known in the literature for wave equations without integral terms as hidden regularity results.

This is a joint work with Paola Loreti.

A time dependent term arising in transmission condition in the critical scale homogenization of the diffusion equation with a dynamical condition

Tatiana Shaposhnikova, Lomonosov Moscow State University, Russia

Abstract. In the present talk we consider a homogenization problem for the diffusion equation in perforated along manifold domain with anonlinear dynamical boundary conditions containing a parameter, which has a critical value, on the boundary of perforations of critical size.

A viscous approximation of crack propagation in elastic bodies

Viktor Shcherbakov, Lavrentyev Institute of Hydrodynamics SB RAS & Novosibirsk State University, Russia

Abstract. We discuss a rate-independent model for crack propagation in 2D elastic bodies without prescribing a priori the crack path. Due to the dependence on the crack path the energy is nonconvex; therefore, solutions may have jumps as a function of time. We employ a viscous approximation of the model and consider it as a limit of systems driven by viscous, rate-dependent dissipation in order to prove the existence of solutions that satisfy the Griffith fracture criterion and to describe accurately the behavior of the solutions at jumps.

The time-dependent generalized membrane shell model and its numerical computation

Xiaoqin Shen, Xi'an University of Technology, China

Abstract. In this paper, we discuss the time-dependent generalized membrane shell model, which has not been addressed numerically in literature. We show that the solution of this model exists and is unique. We first provide a numerical method for the time-dependent generalized membrane shell. Concretely, we semi-discretize the space variable and fully discretize the problem using time discretization by the Newmark scheme. The corresponding numerical analyses of existence, uniqueness, stability and convergence with a priori error estimates are given. Finally, we present numerical experiments with a portion of the conical shell and a portion of the hyperbolic shell to verify theoretical convergence results and demonstrate the effectiveness of the numerical scheme.

On generalised Kirchhoff type problems

Gaetano Siciliano, University of São Paulo, Brazil

Abstract. In the talk we discuss a result of existence of solutions for a class of generalised Kirchhoff equation in a bounded domain under homogeneous Dirichlet boundary conditions.

On The Extremal Parameters Curve of a Quasilinear Elliptic System of Differential Equations

Kaye Silva, Federal University of Goiás, Brazil

Abstract. We study the following system of quasilinear elliptic equations

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2}u + \alpha f |u|^{\alpha-2}|v|^{\beta}u & \text{in } \Omega, \\ -\Delta_q v = \mu |v|^{q-2}v + \beta f |u|^{\alpha}|v|^{\beta-2}v & \text{in } \Omega, \\ (u,v) \in W_0^{1,p}(\Omega) \times W_0^{1,q}(\Omega). \end{cases} \tag{$P_{\lambda,\mu}$}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with regular boundary, $\lambda, \mu \in \mathbb{R}$, $1 \leq p$, $q \leq \infty$ and

$$\frac{\alpha}{p} + \frac{\beta}{q} \geq 1, \quad \alpha \geq p \text{ or } \beta \geq 0, \quad \frac{\alpha}{p^*} + \frac{\beta}{q^*} \leq 1. \tag{α,β}$$

By using the Nehari manifold and the notion of extremal parameter, we extend some results of Bobkov and Ilyasov concerning existence of positive solutions.

Controllability in Population Models Structured by Age, Size, and Spatial Position

Yacouba Simpore, University of Ouagadougou, Burkina Faso

Abstract. We introduced the controllability in Population Models Structured by Age, Size, and Spatial Position. We formulated some hypothesis on the kernel β, and establish the observability inequalities of the adjoint system using the technique introduced by D. Maity, M. Tucsnak and E. Zuazua combining final-state observability estimates with the use of characteristics and with L^∞ estimates of the associated semigroup.

Computational method based on wavelets for singular partial differential equation arising from viscoelasticity

Somveer Singh, Indian Institute of Technology, India

Abstract. We propose and analyze a Legendre wavelet collocation method(LWCM)for the nonlinear weakly singular partial integro-differential equation (SPIDE) arising from viscoelasticity subject to the given initial and boundary conditions. This problem can be found in the mathematical modeling of physical phenomena involving viscoelastic forces. Operational matrix of integration of Legendre wavelets along with collocation method are utilized to reduce the original SPIDE into the nonlinear system of algebraic equations. Some numerical results are presented to simplify applications of operational matrix formulation and reduce the computational cost. Convergence analysis, numerical stability and rate of convergence (C-order) of the proposed method are also investigated by considering a test function.

Quasilinear elliptic equations with degenerate coerciveness and measure data

Flavia Smarrazzo, University Campus Bio-Medico of Rome, Italy

Abstract. In this talk I will discuss existence of measure-valued solutions for a class of degenerate elliptic equations with measure data. The notion of solution is natural, since it is obtained by a regularization procedure which also relies on a standard approximation of the datum. In addition, partial uniqueness results and qualitative properties of the constructed solutions will be provided, concerning in particular the structure of their diffuse part with respect to the harmonic-capacity.

Solutions of the self-dual O(3) Maxwell–Chern–Simons–Higgs equations on a 2-D flat torus

Kyungwoo Song, Kyung Hee University, South Korea

Abstract. We consider a semi-linear elliptic system, called the self-dual O(3) Maxwell–Chern–Simons–Higgs equations, on a two-dimensional flat torus arising from the O(3) sigma gauge field model. The system has three important parameters $\tau \in [0, 1]$,$\kappa \geq 0$ and $q \geq 0$. We focus on the case of $\tau = 1$ which yields a different structure from that of $0 \geq \tau \leq 1$. Then, we show that the system possesses a solution for a sufficiently small κ and a very large q on a torus. In the proof, we develop a new method in order to use the topological degree theory. We also prove the Chern–Simons limit for our solutions.

A distributional Approach to Fractional Sobolev Spaces and Fractional Variation

Giorgio Stefani, Scuola Normale Superiore, Italy

Abstract. We introduce the new space $BV^\alpha(\mathbb{R}^n)$ of functions with bounded fractional variation in $\mathbb{R}^n$ of order $\alpha \in (0,1)$ via a new distributional approach exploiting suitable notions of fractional gradient and fractional divergence already existing in the literature. In analogy with the classical BV theory, we give a new notion of set E of (locally) finite fractional Caccioppoli α-perimeter and we define its fractional reduced boundary. Thanks to the continuous embedding $W^{\alpha,1}(\mathbb{R}^n) \subset BV^\alpha(\mathbb{R}^n)$, our theory provides a natural extension of the known fractional framework. Our main result partially extends De Giorgis Blow-up Theorem to sets of locally finite fractional Caccioppoli α-perimeter, proving existence of blow-ups and giving a first characterisation of these (possibly non-unique) limit sets. Time permitting, we also study the limiting behaviour of the fractional α-variation as $\alpha \to 1^-$.

This is a joint work with G. E. Comi.

Regularity theory for nonlocal space-time master equations

Pablo Raúl Stinga, Iowa State University, USA

Abstract. We report on the novel regularity theory for fractional powers of parabolic operators in divergence form we developed recently. Our results include local degenerate parabolic extension problems, interior and boundary Harnack inequalities and sharp interior and global parabolic Schauder estimates. For the latter, we also prove a characterization of the correct intermediate parabolic Hölder spaces in the spirit of Sergio Campanato.

This is joint work with Animesh Biswas (Iowa State University).

Kernel estimates for elliptic operators with unbounded diffusion, drift and potential terms

Cristian Tacelli, University of Salenro, Italy

Abstract. We prove that the heat kernel k associated to the operator $A := (1 + |x|^\alpha)\Delta + b|x|^{\alpha-1}\frac{x}{|x|}\cdot\nabla - |x|^\beta$ satisfies

$$\begin{aligned} k(t,x,y) \;\leq\; & c_1 e^{\lambda_0 t + c_2 t^{-\gamma}} \left(\frac{1+|y|^\alpha}{1+|x|^\alpha}\right)^{\frac{b}{2\alpha}} \frac{(|x||y|)^{-\frac{N-1}{2}-\frac{1}{4}(\beta-\alpha)}}{1+|y|^\alpha} \\ & \times \exp\left(-\frac{\sqrt{2}}{\beta-\alpha+2}\left(|x|^{\frac{\beta-\alpha+2}{2}} + |y|^{\frac{\beta-\alpha+2}{2}}\right)\right) \end{aligned}$$

for $t > 0$, $|x|$, $|y| \geq 1$, where $b \in \mathbb{R}$, c_1, c_2 are positive constants, λ_0 is the largest eigenvalue of the operator A, and $\gamma = \frac{\beta-\alpha+2}{\beta+\alpha-2}$, in the case where $N > 2$, $\alpha > 2$ and $\beta > \alpha - 2$. The proof is based on the relationship between the log-Sobolev inequality and the ultracontractivity of a suitable semigroup in a weighted space.

Equations and systems with neutral delays

Nasser-eddine Tatar, King Fahd University of Petroleum and Minerals, Saudi Arabia

Abstract. Time-delay systems are frequently employed to describe phenomena when the repercussion resulting from the reaction to an applied force is differed in time. The state and/or the rate of change of the state depends in this case on the elapsed states. The study of such differential equations is not only of theoretical importance but also of practical interest. Indeed, such problems arise in: population dynamic, signal theory, control theory, biomathematics, etc. It is well-known that the insertion of delays in a system may produce undesirable fluctuations and disturbances. This is one of the main difficulties prompted by delays. Besides being easily affected by 'small' delays, differential equations may, on the contrary, be brought back to steady state by 'large' neutral delays in other circumstances. Indeed, neutral delays are inserted purposely in some cases to effectively lift the performance of certain structures. Many ways have been already found and a lot of efforts are being devoted. In this presentation some kinds of neutral delays in some equations and systems appearing in the study of wave propagation in viscoelastic media and in acoustic wave propagation will be discussed. In particular, it will be proved that, despite the destructive nature of delays in general, solutions may go back to the equilibrium state in an exponential manner as time goes to infinity. Reasonable conditions on the distributed neutral delays are established.

Simplicity of Spectral Edges and Applications to Homogenization

Vivek Tewary, Indian Institute of Technology Bombay, India

Abstract. We consider the spectrum of a second-order elliptic operator in divergence form with periodic coefficients, which is known to be completely described by Bloch eigenvalues. We show that under small perturbations of the coefficients, a multiple Bloch eigenvalue can be made simple. The Bloch wave method of homogenization relies on the regularity of spectral edge. The spectral tools that we develop, allow us to obtain simplicity of an internal spectral edge through perturbation of the coefficients. As a consequence, we are able to establish Bloch wave homogenization at an internal edge in the presence of multiplicity by employing the perturbed Bloch eigenvalues. We show that all the crossing Bloch modes contribute to the homogenization at the internal edge and that higher and lower modes do not contribute to the homogenization process.

Non-local elliptic problems with variable exponents

Sweta Tiwari, Indian Institute of Technology Guwahati, India

Abstract. In this talk, first we introduce the fractional Sobolev spaces with variable order and variable exponents. Then we discuss the Sobolev type embedding results for these variable order fractional spaces. We also discuss the multiplicity and regularity of the solutions for a class of nonlocal problems involving variable order fractional $p(\cdot)$- Laplacian of the form

$$\begin{aligned}(-\Delta)^{s(\cdot)}_{p(\cdot)}u(x) &= f(x,u), \quad x \in \Omega,\\ u &= 0, \qquad x \in \mathbb{R}^{\kappa} \setminus \Omega,\end{aligned}$$

where Ω is a smooth and bounded domain in $\mathbb{R}^n$, $n \geq 2$, $p \in C(\overline{\Omega} \times \overline{\Omega}, (1,\infty))$, $s \in C(\overline{\Omega} \times \overline{\Omega}, (0,1))$ and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function of concave-convex type with variable growth. We show the existence of the multiple solutions and uniform estimate for the solutions of the above problem.

On the Cauchy problem for the wave equation on time-dependent domains

Rodica Toader, University of Udine, Italy

Abstract. We introduce a notion of solution to the wave equation on a suitable class of time-dependent domains and compare it with a previous definition. We comment on existence, uniqueness and continuous dependence properties.

The talk is based on results obtained together with G. Dal Maso (SISSA,Trieste).

The existence of trajectory and global attractors for the 3D Bingham fluid model

Mikhail Turbin, Voronezh State University, Russia

Abstract. We consider the qualitative dynamics of weak solutions in the model of Bingham fluid motion on 3D torus using the theory of trajectory and global attractors for trajectory spaces. We establish the existence of weak solutions for this model. For this purpose we demonstrate the solvability of an approximating problem, using some a priori estimates and the topological degree theory. Then the convergence (in some generalized sense) of solutions of approximating problems to a solution of the given problem is proved. Then we define a family of trajectory spaces, introduce the notions of a trajectory and global attractors and prove the existence of these attractors.

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Co-author: Dolgikh Anton

The existence theorem on the weak solution for the initial-boundary value problem for the Oskolkov system of equations

Anastasiia Ustiuzhaninova, Voronezh State University, Russia

Abstract. We prove the existence of weak solution for the Oskolkov system of equations. For proof at first, we consider the problem that approximates the original problem. Then, on the base of a priori estimates of solutions and the Leray-Schauder degree theory we prove the existence of solutions for the operator equation which is equivalent to the approximation problem. Further, on the base of a priori estimates which do not depend on the parameter of approximation, it is shown that from the sequence of solutions for the approximation problem, it is possible to extract a subsequence which converges to the weak solution for the original problem as the approximation parameter tends to zero.

Acknowledgments: This research was supported by the Ministry of Science and Higher Education of the Russian Federation (grant 14.Z50.31.0037).

On an elliptic equation with critical growth and Hardy potential

Giusi Vaira, University of Campania Luigi Vanvitelli, Italy

Abstract. I will discuss classification results for the critical p-Laplace equation in the whole space. In particular I shall present some new results in collaboration with F. Oliva and B. Sciunzi regarding the doubly critical equation involving the Hardy potential.

Stochastic homogenization of high-contrast media

Igor Velčić, University of Zagreb, Croatia

Abstract. Using a suitable stochastic version of the compactness argument of [V. V. Zhikov, 2000. On an extension of the method of two-scale convergence and its applications. Sb. Math., 191(78), 9731014], we develop a probabilistic framework for the analysis of heterogeneous media with high contrast. We show that an appropriately defined multiscale limit of the field in the original medium satisfies a system of equations corresponding to the coupled macroscopic and microscopic components of the field, giving rise to an analogue of the "Zhikov function", which represents the effective dispersion of the medium. We demonstrate that, under some lenient conditions within the new framework, the spectra of the original problems converge to the spectrum of their homogenisation limit. We discuss the case of bounded and unbounded domains.

This is a joint work with M. Cherdantsev (University of Cardiff) and K. Cherednichenko (University of Bath).

H-Systems in higher Dimensions

Anna Verde, University of Naples Federico II, Italy

Abstract. Within the regularity theory for nonlinear elliptic systems in dimension $n \geq 3$, with a critical nonlinearity, I will discuss a recent result (obtained in collaboration with N. Fusco, J. Kristenses and C. Leone) concerning the so called H-systems, equations of hypersurfaces of prescribed mean curvature.

On the solvability of a two-dimensional chemotaxis system with singular sensitivity

Giuseppe Viglialoro, University of Cagliari, Italy

Abstract. We study the zero-flux chemotaxis-system

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (\frac{u}{v} \nabla v) \\ v_t = \Delta v - f(u)v \end{cases}$$

in a smooth and bounded domain Ω of $\mathbb{R}^2$, with $\chi > 0$ and $f \in C^1(\mathbb{R})$ essentially behaving like u^β, $0 < \beta < 1$. Precisely for $\chi < 1$ and any sufficiently regular initial data $u(x,0) \geq 0$ and $v(x,0) > 0$ on $\bar{\Omega}$, we show the existence of global classical solutions. Moreover, if additionally $m := \int_\Omega u(x,0)$ is sufficiently small, then also their boundedness is achieved. This is a joint paper with J. Lankeit: see [1].

References

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The Pediodic Unfolding Method in thin domains

Manuel Villanueva-Pesqueira, Comillas Pontifical University, Spain

Abstract. We will provide a presentation of the unfolding method for thin domains with an oscillatory boundary. The motivation is to analyze the limit behavior, as the thickness parameter tends to zero, of the solutions of second-order partial differential equations posed in oscillating thin domains even with non-periodic oscillatory boundaries. In particular, by using the unfolding method we will obtain the homogenized limit problem and some corrector results for the Poisson equation with Neumann boundary conditions. Moreover, as an application to fluid dynamics we will show the asymptotic behavior of a Bingham fluid in a thin domain with a rough boundary.

Spectral analysis and correct solvability of Volterra integro-differential equations

Victor Vlasov, Lomonosov Moscow State University, Russia

Abstract. We study Volterra integro-differential equations with unbounded operator coefficients in Hilbert space. The principal part of these equations is an abstract hyperbolic operator perturbed by summands of Volterra integral operators. Operator models of such type have many applications in the linear viscoelasticity theory, homogenezation theory, heat conduction theory in media with memory, etc. In particular these integro-differential equations can be realized as the system of integro-partial differential equations:

$$\rho\ddot{u}(x,t) - Lu(x,t) + \int_0^t \Gamma_1(t-s)L_1u(x,s)ds + \int_0^t \Gamma_2(t-s)L_2u(x,s)ds = f(x,t),$$

were $u = \vec{u}(x,t) \in \mathbb{R}^3$ is displacement vector of viscoelastic anisotropic media, $t \geq 0$, $x \in \Omega \subset \mathbb{R}^3$ is bounded domain, u satisfy Dirichlet conditions in a domain with smooth boundary Ω, $L_1 = \mu\cdot(\Delta u + 1/3\cdot\text{grad div}u)$, $L_2 = \lambda\cdot\text{grad div}u$, $Lu = (L_1+L_2)u$ is Lame operator of elasticity theory, Γ_1, Γ_2 are memory relaxation functions that are the series of decreasing exponents with positive coefficients.

A spectral analysis of the operator-valued functions, which are the symbols of considered integro-differential equations is provided. The structure and localization of spectra for these operator-valued functions are analyzed (see [1], [2]).

These results are the natural generalization of our results obtained in [3].

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On a parabolic-parabolic system with gradient dependent chemotactic coefficient and consumption

Hengling Wang, Southeast University, China

Abstract. In this talk, I will introduce a parabolic-parabolic system with gradient dependent chemotactic coefficient and consumption of chemoattractant under homogeneous boundary conditions of Neumann type, in a bounded domain with smooth boundary. It is proved that if initial data satisfy certain conditions, then the model admits at least one global weak solution for lower dimensional and possesses at least one global renormalized solution for high dimensional.

Minimizers of interaction functional with exogenous potential

Wanwan Wang, Southeast University, China

Abstract. We consider the minimization problem of interaction functional with exogenous potential in higher dimensional space. The existence of minimizers in various cases of potentials are established based on the concentration compactness principle. Especially, for the particular potentials, the global minimizer is given explicitly by the method of calculus of variation.

Remarks on Krein–Rutman Theorems and PDE applications

Xuefeng Wang, Southern University of Science and Technology, China

Abstract. The classical (weak) Krein–Rutman Theorem states that if a positive, compact operator from an ordered Banach space to itself has a positive spetral radius, then the spetral radius is the "principal eigenvalue" of the operator. In the mid 80's, de Pagter proved that if the Banach space is a "Banach lattice", then any positive, compact operator has a positive spectral radius if it is irreducible. Combining these two theorems we have that the spectral radius of such an operator is a principal eigenvalue which is algebraically simple. On the other hand, Krein and Rutman's original paper also contains a strong version: if the positive cone of the Banach space has interior and if the operator is "strongly positive" (i.e., mapping non-zero points in the cone into the interior of the cone), then more can be said about the spectrum of the operator.

In this talk, I will (i) first introduce the concept of "semi strong positive" operators on ordered Banach spaces whose positive cones have non-empty interior, and then show, in a totally elementary fashion, that the afore-mentioned results hold true for these operators, generalizing the strong Krein–Rutman theorem; (ii) show the equivalence of semi strong positivity and irreducibility in a Banach lattice; (iii) discuss the case of reducible operators in Banach lattices; (iv) demonstrate the full power of Krein–Rutman theorems, by using as few PDE tools as possible, on some PDE examples such as elliptic eigenvalue problems on non-smooth domains, cooperative systems which may or may not be fully coupled. One of the things we emphasize is to use "upper and lower spectral radii" to characterize the principal eigenvalue (more generally, eigenvalues pertaining to positive eigenfunctions) in two max-min and min-max fashions.

This is a joint work with K. C. Chang and Xie Wu.

Greedy algorithm for parameter dependent Lyapunov equations. Application to control problems

Jerome Weston, University of Dubrovnik, Croatia

Abstract. We adapt and apply greedy methods to approximate in an efficient way the solutions to parameter dependent, operator Algebraic Lyapunov equations. The algorithm identifies the most distinguished values of parameters (offline part) so to better describe or approximate the whole manifold of solutions (online part). The results are applied to control problems for dissipative PDE-s depending on a parameter, where the controllability Gramian is approximated by the solution to corresponding Lyapunov equation. The algorithm provides an approximating space for the Gramian operators which is independent of particular control data (initial and target state). This allows for a robust approach which is not datum sensitive. Both the computational cost and the numerical experiments confirm the efficiency of the method. Methodology and numerical results will be discussed.

This is a joint work with Martin Lazar.

Homogenization of elliptic and parabolic soft inclusions

Minha Yoo, National Institute for Mathematical Sciences, South Korea

Abstract. In this talk, we consider periodic Soft Inclusion problems of parabolic equations of non-divergence form in a domain having non-conducting grains. In our problem, non-conducting grains are described by union of disjoint holes with periodicity epsilon and we assume that the solution satisfy Oblique type boundary conditions on the boundary of such holes. For each time scaling k=1,2,3, we obtain different effective equations and uniform convergences of solutions for each epsilon problems to the solutions of effective equations.

Fully anisotropic elliptic problems with minimally integrable data

Anna Zatorska-Goldstein, Poland

Abstract. We investigate nonlinear elliptic Dirichlet problems whose growth is driven by a general anisotropic N-function, which is not necessarily of power type and need not satisfy the Δ_2 nor the ∇_2-condition. Fully anisotropic, non-reflexive Orlicz–Sobolev spaces provide a natural functional framework associated with these problems. Minimal integrability assumptions are detected on the datum on the right-hand side of the equation ensuring existence and uniqueness of weak solutions. When merely integrable, or even measure, data are allowed, the existence of suitably further generalized solutions – in the approximable sense – is established. Their maximal regularity in Marcinkiewicz-type spaces is exhibited as well. The uniqueness of approximable solutions is also proved in the case of L^1–data.

The talk is based on the joint results with Angela Alberico, Andrea Cianchi, and Iwona Chlebicka.

Nonlinear Problems with unbounded coefficients

Gabriella Zecca, University of Naples Federico II, Italy

Abstract. In this talk we will consider a class of nonlinear problems with unbounded coefficients whose model appears in the diffusion convection phenomena. For these problems we derive existence uniqueness and regularity results.

Stability analysis and Hopf bifurcation in a combustion model with free interface

Mingmin Zhang, University of Science and Technology of China, China

Abstract. In this talk, we consider a one-dimensional thermal-diffusional combustion model with a first-order, stepwise ignition-temperature kinetics. In contrast to the classical Arrhenius kinetics, the temperature gradient is continuous at the free interface (the flame front). The main issue is the stability of the traveling wave solution. When the Lewis number is large, we introduce a perturbation parameter ε and a bifurcation parameter m related to the ignition temperature. Via Hurwitz theorem associated with asymptotic methods, we prove in particular that a Hopf bifurcation occurs at a critical value $m(\varepsilon)$. This is a joint work with Claude-Michel Brauner and Luca Lorenzi.

Transition semi-wave solutions of reaction diffusion equations with free boundaries

Tao Zhou, University of Science and Technology of China, China

Abstract. In this talk, we define the transition semi-wave solutions of the following free boundary problem

$$\begin{cases} u_t = u_{xx} + f(t,x,u), \ \ t \in \mathbb{R}, x < h(t), \\ \qquad u(t,h(t)) = 0, \ \ t \in \mathbb{R}, \\ h'(t) = -\mu u_x(t,h(t)), \ \ t \in \mathbb{R}. \end{cases} \tag{1}$$

If $f(t,x,u) = f(u) \in C^1([0,1])$ with $f(0) = f(1) = 0, f'(1) < 0$ and $f(u) < 0$ for $u > 1$, then we prove that the semi-wave connecting 1 and 0 of (1) is unique provided it exists and any bounded transition semi-wave connecting 1 and 0 is exactly the semi-wave.

If f is KPP-Fisher type and a.p. in t (resp. x), i.e., $f(t,x,u) = u(c(t) - u)$ (resp. $u(a(x) - u)$) with $c(t)$ (resp. $a(x)$) being a.p., then by totally different method, any bounded transition semi-wave connecting the unique a.p. positive solution of $u_t = u(c(t) - u)$ (resp. $u_{xx} + u(a(x) - u) = 0$) and 0 is exactly the unique a.p. semi-wave of (1).

On the optimization of heat rectification in graded materials

Federico Zullo, University of Brescia, Italy

Abstract. We consider the steady state Fourier equation of heat transfer for functionally graded materials, presenting a method to optimize the performances of a thermal rectifier. The approach allows to find the particular spatial distribution of the composition along the material giving a reasonably high value of the rectification coefficient. We give two examples by applying the proposed methodology to porous silicon devices and to silicon germanium alloys.

Thermoviscoelastic model of polymer solutions motion

Andrey Zvyagin, Voronezh State University, Russia

Abstract. This mathematical model of polymer solutions motion will be considered with constitutive law which is independent of the observer, i.e. that do not change under the Galilean transformation. Also in this mathematical model the viscosity will be depend from a temperature, which leads to emergence of additional heat balance equation (it is a parabolic equation with non- smooth coefficients and with right part from $L1(0,T;\Omega)$). To establish the existence of weak solution, the topological approximation method for studying problems of hydrodynamics will be used.

Solvability of a parabolic problem with non-smooth data

Victor Zvyagin, Voronezh State University, Russia

Abstract. We study a weak solvability of one linear parabolic problem with a summable right hand side and non-smooth coefficients. We consider this problem as appropriate operator equation in some Banach space. Methods of the semigroups theory, fractional powers of operators and integral operators for solvability of this operator equation are used.