TOPOLOGY

PROF. DR. A. S. CATTANEO

HS 2023

Program

(1) Topological spaces.
(2) Topology of metric spaces.
(3) Categories, functors, and natural transformations.
(4) Review of set theory: empty set, singletons, Cartesian products, coproducts, exponentials.
(5) Subspace topology. Quotient topology. Product topology. Coproduct topology.
(6) Homotopy
(7) Connectedness, path-connectedness, local connectedness, local path-connectedness.
(8) Hausdorff spaces.
(9) Compactness and local compactness.
(10) Interior, closure, boundary.
(11) Sequences. First and second countability. Topological manifolds.
(12) Filters: characterization of the Hausdorff property, of closure, and of continuity.
(13) Ultrafilters and prime filters. The ultrafilter lemma. Filters and compactness. Tychonoff's theorem.
(14) Diagrams, (co)cones, (co)limits. Initial elements, terminal elements, (co)products, pullbacks, pushouts. Inverse and direct limits. Equalizers and coequalizers. Completeness and cocompleteness.
(15) Equivalences and adjunctions.
(16) Splitting and conjoining topologies. The compact–open topology. Path spaces.
(17) The fundamental groupoid.
(18) The fundamental group. The category of pointed topological spaces. The wedge and the smash products.
(19) The smash–hom adjunction. The suspension–loop adjunction. The reduced suspension of spheres and consequences. Fibrations. The based path space. The real line as a fibration over the circle.
(20) The fundamental group of S^1. Brouwer's fixed point theorem. The degree of a map. The fundamental theorem of algebra. The Borsuk–Ulam theorem.
(21) The Seifert–van Kampen theorem.

(22) Fundamental groups of spheres. The fundamental group of a wedge of circles and of the torus.
(23) Classification of compact oriented surfaces.

References

[1] T.-D. Bradeley, T. Bryson, and J. Terilla, *Topology: A Categorical Approach*, MIT Press, 2020.