FIELD THEORIES ON MANIFOLDS WITH BOUNDARY

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Herbstsemester 2013

1. Symplectic linear algebra

(1) Symplectic forms and presymplectic forms
(2) Normal form theorem
(3) Weak and strong infinite-dimensional symplectic spaces
(4) The symplectic orthogonal space
(5) Symplectic, isotropic, coisotropic, and Lagrangian subspaces
(6) Linear symplectic reduction
(7) Canonical relations, their composition and the extended linear symplectic category.
(8) Kähler structures

2. Symplectic manifolds

(1) Generalities (definitions, symplectomorphisms, Liouville's volume form)
(2) Lagrangian and Hamiltonian mechanics; the Legendre transformation of hyperregular Lagrangians
(3) Moser's trick
(4) Darboux and Darboux–Weinstein theorems
(5) Classification of compact symplectic surfaces
(6) Presymplectic (sub)manifolds and reduction
(7) Symplectic, isotropic, coisotropic, and Lagrangian submanifolds
(8) Reduction of Lagrangian submanifolds intersecting coisotropic submanifolds
(9) Canonical relations; composition thereof; the extended symplectic "category"
(10) Generating functions and Morse families
(11) Example: (deformed) conormal bundles in cotangent bundles
(12) (Almost) Kähler structures
(13) FIeld theories on manifolds with boundary

3. Poisson geometry

(1) Motivations, definitions and examples; Poisson algebras
(2) Digression: The Schouten–Nijenhuis bracket
(3) Poisson cohomology and its interpretation up to degree two
(4) Canonical actions of Lie groups and quotients
(5) Poisson and Hamiltonian vector fields
(6) Coisotropic submanifolds and reduction; algebraic description ("coisotropes"); examples

4. Canonical quantization

(1) Schrödinger's quantization of $T^*\mathbb{R}^n$
(2) Schrödinger's equation; position and momentum operators; the ordering problem
(3) The momentum description
(4) Problems of canonical quantization; "quantization as a functor"; Dirac's dream; the Groenewald–van Howe Theorem

5. Geometric quantization

(1) Prequantization and the integrality condition (Digression: line bundles, connection, curvature, Chern class)
(2) The integral Hall effect
(3) Prequantization of the Poisson algebra of functions
(4) Polarizations
(5) Holomorphic quantization and the Riemann–Roch Theorem
(6) The Schrödinger equation
(7) Perturbative quantization of nondegenrate field theories on manifolds with boundary

6. Path-integral quantization

(1) Motivations
(2) From the Schrödinger equation to the path integral
(3) The semiclassical limit
(4) Perturbation theory

7. Graded linear algebra

(1) Superspaces, graded vector spaces
(2) Morphisms and graded morphisms
(3) Supertrace and superdeterminant
(4) The graded symmetric algebra
(5) Left and right derivations

(6) The Berezinian

8. Supermanifolds

(1) Functions, vector fields, differential forms.
(2) Integration and change of variables
(3) The Berezinian bundle; Berezinian forms; densities; the divergence of a vector field
(4) The supermanifolds of maps
(5) Graded manifold and the graded Euler vector field

9. Graded symplectic geometry

(1) Graded symplectic manifolds; Darboux coordinates
(2) Main facts on symplectic forms and Hamiltonian vector fields
(3) Normal form of odd symplectic manifolds
(4) Symplectic manifolds of degree -1 and their induced cohomology
(5) Integral forms; integration on submanifolds

10. The BV method

(1) Symplectic manifolds of degree -1 and the canonical BV operator on half densities
(2) BV cohomology and integration on Lagrangian submanifolds
(3) The quantum master equation (QME)
(4) The classical master equation (CME)
(5) Reinterpretation of the BRST method
(6) The BV-BFV method for field theories on manifolds with boundary

References

[1] S. Bates, A. Weinstein, *Lectures on the Geometry of Quantization*, Berkeley Mathematics Lecture Notes **8** (AMS, 1997).
[2] A. Cannas da Silva, *Lectures on Symplectic Geometry*, Lecture Notes in Mathematics **1764** (Springer-Verlag, Berlin, 2001).
[3] A. Cannas da Silva, A. Weinstein, *Geometric Models for Noncommutative Algebras*, Berkeley Mathematics Lecture Notes **10** (AMS, 1999).
[4] R. P. Feynman, A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).
[5] N. M. J. Woodhouse, *Geometric quantization*, Second edition, Oxford Mathematical Monographs, Oxford Science Publications (The Clarendon Press, Oxford University Press, New York, 1992).

References

Textbooks.

(1) S. Bates, A. Weinstein, *Lectures on the Geometry of Quantization*, Berkeley Mathematics Lecture Notes **8** (AMS, 1997).
(2) A. Cannas da Silva, *Lectures on Symplectic Geometry*, Lecture Notes in Mathematics **1764** (Springer-Verlag, Berlin, 2001).
(3) A. Cannas da Silva, A. Weinstein, *Geometric Models for Noncommutative Algebras*, Berkeley Mathematics Lecture Notes **10** (AMS, 1999).
(4) R. P. Feynman, A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).
(5) N. M. J. Woodhouse, *Geometric quantization*, Second edition, Oxford Mathematical Monographs, Oxford Science Publications (The Clarendon Press, Oxford University Press, New York, 1992).
(6) J. Zinn-Justin, *Quantum field theory and critical phenomena*, Second edition, International Series of Monographs on Physics **85**, Oxford Science Publications, (The Clarendon Press, Oxford University Press, New York, 1993).
(7) A. S. Cattaneo, B. Keller, C. Torossian and A. Bruguières, *Déformation, Quantification, Théorie de Lie*, Panoramas et Synthèse **20** (2005), viii+186 pages; Part III.

Papers.

(1) A. S. Cattaneo, "On the BV formalism," unpublished, `http://www.math.unizh.ch/reports/07_05.pdf`
(2) ———, "From topological field theory to deformation quantization and reduction," in *Proceedings of the International Congress of Mathematicians, Madrid, Spain, 2006*, (ed. M. Sanz-Solé, J. Soria, J. L. Varona, J. Verdera), **Vol. III**, 338–365 (European Mathematical Society, 2006); `http://www.math.unizh.ch/fileadmin/user/asc/publikation/asc-v2.pdf`
(3) A. S. Cattaneo and F. Schätz, "Introduction to supergeometry," Rev. Math. Phys. **23**, 669–690 (2011).
(4) H. M. Khudaverdian and Th. Th. Voronov, "Geometry of differential operators, odd Laplacians, and homotopy algebras," `math.DG/0402292`
(5) A. Schwarz, "Geometry of Batalin–Vilkovisky quantization," Commun. Math. Phys. **155**, 249–260 (1993).
(6) P. Ševera, "On the origin of the BV operator on odd symplectic supermanifolds," `math.DG/0506331` `http://www.staff.science.uu.nl/~schat001/thesis.pdf`
(7) A. S. Cattaneo, P. Mnëv and N. Reshetikhin, "Classical and quantum Lagrangian field theories with boundary," 25 pages, in Proceedings of the "Corfu Summer Institute 2011 School and Workshops on Elementary Particle Physics and Gravity," PoS(CORFU2011)044.
(8) A. S. Cattaneo, P. Mnëv and N. Reshetikhin, "Classical BV theories on manifolds with boundaries," `http://arxiv.org/abs/1201.0290`