[MAT121] Analysis I
Extra Exercises

Prof. Dr. Rémi Abgrall

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(1) Taylor series

Consider the function $f(x) := \log(\frac{1+x}{1-x})$ for $x \in (-1, 1)$.

(a) Determine the Taylor series of f around the point $x_0 = 0$.

(b) Determine the convergence radius ρ of this Taylor series.

(c) Show that for $|x| < \rho$ the Taylor series coincides with the function f, i.e. the series of Taylor polynomials converges to $f(x)$ for every x with $|x| < \rho$.

(2) Riemann sums

Use the definition of the integral, i.e. Riemann sums, to determine the following integrals

(a) $\int_a^b e^{2t} dt$ for $a, b \in \mathbb{R}$ and $a < b$.

(b) $\int_a^b \frac{1}{t^2} dt$ for $a, b \in \mathbb{R}$ and $0 < a < b$.

It can be assumed that the functions are integrable and you may use Proposition 9.1.4.

(3) Partial fraction decomposition

Let $a, b \in \mathbb{R}$ and $2 < a < b$. Determine the following integrals

(a)

$$\int_a^b \frac{7t+1}{t^2+t-6} dt.$$

Hint: Determine A, B such that $\frac{7t+1}{t^2+t-6} = \frac{A}{t-2} + \frac{B}{t+3}$.

(b)

$$\int_a^b \frac{-t^2+5t-2}{t(t+1)(t-1)} dt.$$

Hint: Use a similar decomposition as in **(a)**.

(4) Monotone functions

Show that a monotone function $f : [a, b] \to \mathbb{R}$ is integrable.