# Exercise Sheet 12

## Some solution concepts (for DEs), Storrer 16

Hand in: Wednesday, **13.12.2017**, ahead of the lecture.

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MUST

**Exercise 1**

a) Explain the term separable differential equations.

b) Explain the terms homogenous and inhomogeneous linear differential equation referring to a linear DE of first order.

c) Which of the following differential equations are separable?

(1) $y' = xy$ (2) $y' = x - y$ (3) $y' = x^2\sqrt{y}$

(4) $\dfrac{1}{y'} = xy$ (5) $\dfrac{1}{y'+1} = xy$ (6) $a\dfrac{1}{y'+1} = \dfrac{1}{xy+1}$

STANDARD

**Exercise 2** (6 points)

a) (1 Punkt) Determine the solutions of the differential equation $y' = x^{\frac{3}{2}}y$ for the initial values

1) (1/2 Punkt) $y(0) = 0$

2) (1/2 Punkt) $y(0) = 1$

b) Determine the solutions of the following linear differential equations using the *variation of parameters* method (German: Methode der Variation der Konstanten).

1) (3 points) $y' = y + x^2$ with initial value $y(0) = -1$

2) (2 points) $y' = -2y + e^{-2x}$ with initial value $y(0) = 1$

**Exercise 3** (6 points)
Determine the solutions of the following linear differential equations.

1) (2 points) $y' - 3x^2y - 2xe^{x^3} = 0$ (Variation of parameters)
2) (2 points) $(1 + x^3)y' + x^2y = 0$ (Separation of variables)
3) (2 points) $y' - y\tan(x) = \frac{1}{\cos(x)}$ (Variation of parameters)

**Exercise 4** (3 points)

a) (2 points) Solve the differential equation $y' = 3 \cdot (x(y-2))^2$.
b) (1 Punkt) Are there any singular solutions?

HONOURS

**Exercise 5** (4 points)
Let $x$ be the amount of fertilizer scattered per hectare arable land and $E(x)$ the resulting revenue (e.g. of a certain fruit). Increasing the amount of fertilizer enhances the revenue, however this is not unconditionally true: having reached a maximal value $E_m$, the revenue cannot be further increased - in case of an overfertilization the reveue even decreases. During the revenue increase *Mitscherlich's law* holds:

$$E'(x) = \alpha(E_m - E(x)) \qquad (\alpha \text{ is a positive constant})$$

Based on this relation determine the revenue as a function of the amount of fertilizer.

a) (3 points) Determine $E(x)$ with $E(0) = E_0 < E_m$.
b) (1 point) Draw a qualitative graph of $E(x)$ for $E_0 = \frac{E_m}{2}$.