computational mathematics

MAT802 Numerical Methods for Elliptic and Parabolic Partial Differential Equations

Prof. Dr. Stefan Sauter
Institut für Mathematik
Universität Zürich

Sheet 9

Deadline: 12:00, 24.11.2017

Use the code from Exercise 3 in Sheet 8 to perform the numerical experiment of the convergence behaviour on the following problem

$$-\Delta u = f \text{ in } \Omega \in \mathbb{R}^2 \text{ with } u = 0 \text{ on } \partial\Omega\,.$$

Let $\mathcal{G}$ denote the finite element mesh of Ω and let $u_s \in S_0^{1,0}(\mathcal{G})$ denote the finite element approximation of u. The refinement of the mesh $\mathcal{G}$ can be done by reducing the mesh size constraint $h_{\max}$ in the function call of refine2($\cdots$,$\cdots$,$\cdots$,$\cdots$,$h_{\max}$) from the Mesh2d MATLAB package.

Exercise 1 (2 Points)

Write a MATLAB function that calculates the errors in L^2- and H^1-norm, namely $\|u - u_s\|_{L^2(\Omega)}$ and $\|u - u_s\|_{H^1(\Omega)}$. The exact solution u is assumed to be known and given. The integral over each finite element can be approximated by using the midpoint rule.

Exercise 2 (4 Points)

Consider the following setting

$$\begin{aligned}\Omega =&(0,1)^2\,,\\ u(x_1,x_2) =&\sin(\pi x_1)\sin(2\pi x_2)\,,\\ f(x_1,x_2) =&5\pi^2\sin(\pi x_1)\sin(2\pi x_2)\,.\end{aligned}$$

a) Solve u_s for $h_{\max} = 0.5, 0.25, 0.125, 0.0625$ and calculate the corresponding errors in L^2- and H^1-norm.

b) Plot the errors against $h_{\max}$ in a log-log graph and comment on the convergence rate.

Exercise 3 (4 Points)

Consider the following setting

$$\begin{aligned}\Omega =&(-1,1)^2\backslash[0,1]^2\,,\\ u(x_1,x_2) =&2xy(x^2+y^2)^{-2/3}(1-x^2)(1-y^2)\,,\\ f(x_1,x_2) :=&-\Delta u(x_1,x_2)\,.\end{aligned}$$

The derivation of $f(x_1,x_2)$ can be done by using MATLAB symbolic toolbox.

a) Solve u_s for $h_{\max} = 0.5, 0.25, 0.125, 0.0625$ and calculate the corresponding errors in L^2- and H^1-norm.

b) Plot the errors against $h_{\max}$ in a log-log graph, and comment on the convergence rate in comparison with that in Exercise 2b.