computational mathematics

MAT802 Numerical Methods for Elliptic and Parabolic Partial Differential Equations

Prof. Dr. Stefan Sauter
Institut für Mathematik
Universität Zürich

Sheet 7

Deadline: 12:00, 10.11.2017

Exercise 1 (6 Points)

As a continuation of Exercise 3 in Sheet 6, consider the l-th 2D Lagrange basis function of polynomial degree k over the reference element $\bar{\hat{\tau}}$

$$b_{k,l}(x_1, x_2) = \sum_{i=0}^{k} \sum_{j=0}^{k-i} a_{i,j}^{k,l} \, {x_1}^i {x_2}^j \text{ for } (x_1, x_2) \in \bar{\hat{\tau}}\,,$$

where $a_{i,j}^{k,l}$ is coefficient dependent of $k \geq 1$ and $1 \leq l \leq \binom{k+2}{2}$. For each pair of k and l, the coefficients $a_{i,j}^{k,l}$ are determined by the interpolation condition of the Lagrange basis function.

Write a MATLAB programm that takes k and l as the input and returns the coefficients $a_{i,j}$ as the output. Calculate the coefficients for $k = 3$ and $k = 4$ respectively.

Exercise 2 (4 Points)

Recall the knowledge of numerical integrations from Numerik I. Approximate the integral $\int_0^h f(x)\, dx$ by using the Gauss-Legendre quadrature of n nodes with the weight function $w(x) = 1$

a) Determine the degree of exactness of the quadrature formula with respect to n.

b) Derive an explicit error estimate for the integrand $f(x) = \frac{1}{x+1}$ with respect to n.

Exercise 3 (4 Points)

Let τ denote an arbitary triangle in the 2D mesh and let $\hat{\tau}$ denote the reference triangle. Consider the integrals $I_\tau(g) = \int_\tau g(x)\, dx$ and $J_\tau(g) = \int_\tau \nabla g(x)\, dx$ for $x \in \mathbb{R}^2$.

a) Transform the integrals I_τ and J_τ onto $\hat{\tau}$.

b) Find a bilienar map that transforms the unit square $[0, 1]^2$ onto $\hat{\tau}$, and transform the integrals I_τ and J_τ onto the unit square via $\hat{\tau}$.