Exercise Sheet 5

Techniques of Differentiation, Storrer 5

Hand in: Wednesday, **25.10.2017**, ahead of the lecture.

MUST

Exercise 1

a) What is the definition of continuity?

b) Is a differentiable function always continuous?

c) Determine the derivatives of the following functions:

(1) $f(x) = 0$ (2) $f(x) = 5$ (3) $f(x) = x$ (4) $f(x) = x^2$
(5) $f(x) = e^x$ (6) $f(x) = \ln(x)$ (7) $f(x) = \sin(x)$ (8) $f(x) = \cos(x)$

STANDARD

Exercise 2 (3 points)

Find a and b such that the function f is differentiable in x_0.
Hint: Differentiability implies continuity.

a) (1 point)

$$f(x) = \begin{cases} x^2 - ax & , \quad x \leq x_0 \\ -x^2 + b & , \quad x > x_0 \end{cases} \quad ; \; x_0 = 1$$

b) (1 point)

$$f(x) = \begin{cases} \frac{1}{2}x + \frac{1}{4} & , \quad x \leq x_0 \\ ax^a + bx & , \quad x > x_0 \end{cases} \quad ; \; x_0 = 1$$

c) (1 point)

$$f(x) = \begin{cases} e^{ax} & , \quad x \leq x_0 \\ \frac{b}{x+2} & , \quad x > x_0 \end{cases} \quad ; \; x_0 = 0$$

Exercise 3 (4 points)

Determine the first derivatives of the following functions:

a) (1 point) $f(x) = \ln(x) \cdot x^n \quad , \quad x > 0,\ n \in \mathbb{N}_{>0}$

b) (1 point) $f(x) = \sqrt[4]{\dfrac{x^2-1}{x}} \quad , \quad x \neq 0$

c) (1 point) $f(x) = \dfrac{\cos^2(x)}{e^{-x} - e^x}$

d) (1 point) $f(x) = \cos(\cos(\cos(x)))$

Exercise 4 (3 points)

a) (2 points) Determine the derivative of $f(x) = \sqrt{\sqrt{x^2 - a^2} - a^2}$

b) (1 point) Determine the derivative of $f(x) = \tan(\sqrt{x^2 - 1})$

HONOURS

Aufgabe 5 (3 Points) Determine the derivative of the function

$$f(x) = \arcsin(\sqrt{1 - x^2}), \qquad |x| < 1 \quad .$$