[MAT121] Analysis I
Homework 1

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Deadline: 29.09.2017, 12:00

(1)[7p] Propositions versus statement forms

Which of the following expressions (**Ausdrücke**) are propositions (**Aussagen**), which are statement forms (**Aussageformen**) and which are terms (**Terme**), and, finally, which do not belong to any of these three groups? Explain your answers and motivate, in case of propositions (Aussagen), if they are true or false.

a)[1p] The number $\sqrt{2}$ has the value $\pi/57 + e/2$.

b)[1p] $a + 2 \cdot b - x$.

c)[1p] $x + y$ is smaller $x - z$.

d)[1p] The smallest number.

e)[1p] $((x + (1 \cdot (y + 1) \cdot y)) + y) + 1)$

f)[1p] For two different numbers n and m we have that either $m \leq n$ or $m^2 > n^2$.

g)[1p] If $1 + 2 = 4$, then $3 - 1 = 1$.

(2)[8p] (1/2 point each item) Sets I

Consider the set $M := \{a, b\}$. Which of the following relations is fulfilled?

a) $\emptyset \in \emptyset$	**e)** $\emptyset \subset \{\emptyset\}$	**i)** $a \subset M$	**m)** $\emptyset \in M$
b) $\emptyset \in \{\emptyset\}$	**f)** $\emptyset = \{\emptyset\}$	**j)** $M \in \{M\}$	**n)** $\{a, b\} \in M$
c) $\emptyset \subset \emptyset$	**g)** $\{b, a\} = M$	**k)** $M \subset \{M\}$	**o)** $b \in M \cup \{c\}$
d) $\emptyset \subseteq \emptyset$	**h)** $M \subset \{a, a, b, b\}$	**l)** $a \in \{\{\emptyset\}, \{M\}\}$	**p)** $\{b\} \in M$

(3)[9p] Sets II

Consider X to be a set, and A,B, $C \subset X$ subsets of X.
The symmetrical difference (**symmetrische Differenz**) $A\Delta B$ is defined as $A\Delta B = (A \cup B) \backslash (A \cap B)$.
Show that for A,B, C we have the associative property (**Assoziativgesetz**) that reads:

$$(A\Delta B)\Delta C = A\Delta(B\Delta C)$$

(4)[9p] Boolean tables

Check if the following logical conclusion is correct by using a boolean (truth) table (**Wahrheitstafel**):

Assumption (Prämisse) 1: If I study I get cake

Assumption 2: I get cake or I get carrots.

Conclusion: When I study I get no carrots.

(5)[9p] Composition of Functions I

Consider the following functions

$$\begin{aligned} f : [0,2] &\to [0,4], \qquad x \mapsto 2x \\ g : [0,4] &\to [2,6], \qquad x \mapsto x+2 \\ h : [2,6] &\to [-32,0], \qquad x \mapsto x^2 - 36 \end{aligned}$$

Observe the following two compositions (**Verknüpfungen**):

$$p(x) = (g \circ f)(x) \qquad \text{and} \qquad q(x) = (h \circ g)(x).$$

a)[1p] Write a simplified representation of the functions p and q.

b)[2p] Write the domain of definition (**Definitionsbereich**) and the co-domain (**Wertebereich**) of p and q.

c)[3p] Write and *prove* through the definition, if the functions f, g, and h are surjective (**surjektiv**), injective (**injektiv**) and bijective (**bijektiv**).

d)[3p] Write and *prove* through the definition, if the functions p and q are surjective, injective and bijective.

(6)[6p] Composition of Functions II

Consider the functions $j : [3,\infty[\to [0,\infty[$ defined by $j(x) := \sqrt{x^2-9}$.

a)[3p] Write j as the composition of two functions f and g, such that $j = f \circ g$.

b)[3p] Find the inverse of the function j.

(7)[5p] Relations

Consider a given relation R applied to the cities of Switzerland, and defined by

$$a\mathrm{R}b :\Leftrightarrow \text{City } a \text{ is not more than 30km in linear distance from } b \text{ away}$$

Geometrically you can imagine this relation, as the link between points on a plane: $a\mathrm{R}b$ *means a circle of radius* 30km *around* a *contains* b*.*

a) [3p] Prove that R is not an equivalence relation (**Äquivalenzrelation**).

b) [2p] Is it an ordering relation (**Ordnungsrelation**)?

(8)[7p] Supremum and infimum

Determine the supremum and infimum (**obere und untere Schranken**) for the following sets and determine the maximum and minimum:

a)[3p] $A_1 = \{x \in \mathbb{R} \,|\, |x| \leq 2x\}$,

b)[4p] $A_2 = \{n \in \mathbb{N} \,|\, n^2 \leq 4n\} \cup]-2,2]$