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Numerical Methods for Elliptic and Parabolic Partial Differential Equations

Sheet 1

Deadline: 29.09.2017, 12:00.

Exercise 1 (3 P.)
Show that for a function u in $C^2(\mathbb{R}^d)$ the ansatz

$$-\sum_{i,j=1}^{d} \tilde{\mathbf{A}}_{i,j}(\mathbf{x})\partial_j\partial_i u(\mathbf{x}) + \sum_{i=1}^{d} \tilde{\mathbf{b}}_i(\mathbf{x})\partial_i u(\mathbf{x}) + \tilde{c}(\mathbf{x})u(\mathbf{x})$$

can be transformed to the left-hand side in (1.9) (page 6 of the lecture notes).

Exercise 2 (4 P.)
Find the eigenfunctions $u : [0, L] \times [0, H] \to \mathbb{R}$ and the eigenvalues of the Laplace equation with zero boundary condition. That is: find $\lambda \in \mathbb{R}$ and u such that

$$\begin{aligned} -\Delta u &= \lambda u, && \text{in } [0, L] \times [0, H] \\ u(0, x_2) = u(L, x_2) = u(x_1, 0) = u(x_1, H) &= 0, && \text{for } x_1 \in [0, L], x_2 \in [0, H]. \end{aligned}$$

Hint: Use the method of separation of variables, that is $u(x_1, x_2) = v(x_1) \cdot w(x_2)$.

Exercise 3 (3 P.)

a) Let $\Omega = [0, 1]^2$ and $T > 0$. Use the method of separation of variables ($u(x, t) = v(x) \cdot w(t)$) to find the solutions of the wave equation

$$\begin{aligned} u_{tt}(x, t) - \Delta u(x, t) &= 0, && \text{for } (x, t) \in \Omega \times [0, T] \\ u(x, t) &= 0, && \text{for } (x, t) \in \partial\Omega \times [0, T]. \end{aligned} \tag{1}$$

b) $(*)$ Find the unique solution u of (1) that satisfies the initial conditions

$$\begin{aligned} u(x, 0) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} g_{n,m} \sin(n\pi x_1)\sin(m\pi x_2), \\ \partial_t u(x, 0) &= \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} h_{n,m} \sin(n\pi x_1)\sin(m\pi x_2). \end{aligned}$$

The deadline for the submission of the exercise sheets is Friday, 12:00.

Exercises that are marked with $(*)$ give extra points.