Exercises. XVII

A- Let k be a field, X a k-scheme and $x \in X(k)$ a rational point.

(1) Show that the morphism $\mathcal{O}_{X,x} \to \mathfrak{m}_x/\mathfrak{m}_x^2$ sending a germ f_x to $(f - f(x))_x + \mathfrak{m}_x^2$ is a derivation of the k-algebra $\mathcal{O}_{X,x}$.

(2) Deduce that the obvious morphism $\mathfrak{m}_x/\mathfrak{m}_x^2 \to (\Omega_{X/k})_x/\mathfrak{m}_x(\Omega_{X/k})_x$ sending $f_x + \mathfrak{m}^2$ to the class of $\mathrm{d}f$ is invertible.

B- Let k be an algebraically closed field, X a quasi-projective k-scheme and G a finite group acting on X. We assume that for every $x \in X(k)$, the stabilizer of x is trivial, i.e., no element $g \in G\backslash\{1\}$ takes x to x. Show that $X \to X/G$ is étale. (Hint: Assume that $X = \mathrm{Spec}(A)$ is affine. To show that A is a flat A^G-module, consider the morphism on formal completions

$$\varprojlim_n A^G/\mathfrak{p}^n \to \varprojlim_n A/\mathfrak{p}^n A$$

where p is a maximal ideal of A^G.)